Free Vibration Analysis of Fixed Free Beam with Theoretical and Numerical Approach Method

J.P. Chopade
P.G.Student, Department of Mechanical Engineering,
Shri Sant Gadge Baba College of Engineering and Technology
Bhusawal, Maharashtra, India

R.B. Barjibhe
Department of Mechanical Engineering,
Shri Sant Gadge Baba College of Engineering and Technology
Bhusawal, Maharashtra, India

***Abstract*— This paper focuses on the theoretical analysis of transverse vibration of fixed free beam and investigates the mode shape frequency. All the theoretical values are analyzed with the numerical approach method by using ANSYS program package and co relate the theoretical values with the numerical values to find out percentage error between them.**

Keywords— Fixed Free Beam, Mode Shape Frequency, Finite Element Method.

I. INTRODUCTION

Vibration analysis is one of the vital tasks in designing of structural and mechanical system. The effect of vibration absorber on the rotating machinaries, vehicle suspension system and the dynamic behaviour of machine tool structures due to excitation are the important information that design engineer wants to obtain. This information helps to design system to control the excessive amplitude of the vibration. J.J.Wu and A.R Whittaker [2] have studied a uniform cantilever beam with spring mass system to investigate the natural frequencies and mode shapes of the cantilever vibration. [6] S.Mohammadi , A.hassanirad have explained theoretical model of cantilever beam and introduced vibration measuring instrument RFID technology which grouped under the term of Automatic Identification(Auto ID) to analysed the vibration of beam experimentally.[1]

II. THEORETICAL ANALYSIS OF TRANSVERSE VIBRATION OF FIXED FREE BEAM

A beam which is fixed at one end and free at other end is known as cantilever beam. [1] From elementary theory of bending of beams also known as Euler-Bernoulli, the relationship between the bending moment and deflection can be expressed as

$$M = EI\frac{d^2y}{\partial x^2} \quad (1)$$

Where, E is Young's Modulus and I is the moment of inertia of the beam. For uniform beam we can obtain equation of motion as

$$\frac{EI}{\rho A}\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} = 0 \quad (2)$$

Where, ρ is the mass density and A is cross sectional area of beam.

#

$$C^2\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = 0 \quad (3)$$

Where,

$$C = \sqrt{\frac{EI}{\rho A}}$$

The solution of Eq. (2) is too separate the variables one depends on position and another on time.

$$y = W(x)\,T(t) \qquad (4)$$

By substituting Eq. (4) to Eq. (3) and simplifying it we get,

$$\frac{C^2}{w(x)}\frac{\partial^4 y}{\partial x^4} = -\frac{1}{T(t)}\frac{d^2T(t)}{dt^2} \qquad (5)$$

The Eq. (5) can be written as two separate differential equation.

$$\frac{\partial^4 W}{\partial x^4} - \beta^4 W(x) = 0 \qquad (5.a)\#$$

$$\frac{\partial^2 T}{\partial t^2} + \omega^2 T(t) = 0 \qquad (5.b)$$

Where, $\beta^4 = \frac{\omega^2}{c^2} = \frac{\rho A \omega^2}{EI}$ (6)

To find out the solution of (5.a) Consider the equation

$$W(x) = C_1 \cos h\,\beta x + C_2 \sin h\,\beta x + C_3 \cos \beta x + C_4 \sin \beta x \qquad (7)$$

In order to solve Eq. (7) the following boundary conditions for cantilever beam are needed:

$x = 0 \qquad w = 0$

$x = 0 \qquad w' = 0$

$x = L \qquad w'' = 0$

$x = L \qquad w''' = 0$

By substituting the boundary conditions in to Eq. (7) w, w', w'', w''' we obtain the value of C1, C_2, C_3, and C_4.

The Eq. (7) becomes after differentiating and pitting values of C1, C_2, C_3, and C_4 in it.#

$$[\cos h\,\beta l + \cos \beta l]\,C_3 + [\sin h\,\beta l + \sin \beta l]\,C_4 = 0$$

$$[\sin h\,\beta l - \sin \beta l]\,C_3 + [\cos h\,\beta l + \cos \beta l]\,C_4 = 0 \qquad (8)$$

We can write Eq. (8) in matrix form as follows

$$\begin{bmatrix} \cos h\,\beta l + \cos \beta l & \sin h\,\beta l + \sin \beta l \\ \sin h\,\beta l - \sin \beta l & \cos h\,\beta l + \cos \beta l \end{bmatrix} \begin{bmatrix} C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (9)$$

For solving above matrix Eq. (9) we get determinant

$$(\cos h\,\beta l + \cos \beta l)^2 - (\sin h\,\beta l + \sin \beta l)(\sin h\,\beta l - \sin \beta l) = 0$$

or

$$\cos h^2 \beta l + 2\cos h\,\beta l.\cos \beta l + \cos^2 \beta l - \sin h^2 \beta l + \sin^2 \beta l = 0$$

But it is well known that

$$\cos h^2 \beta l - \sin h^2 \beta l = 1$$

$$\cos^2\beta l + \sin^2\beta l = 1$$

Hence we get,

$$\cos\beta l \cosh\beta l = -1 \qquad (10)$$

This transcendental equation has an infinite number of solution βi = 1, 2, 3 ...n.

Corresponding giving an infinite number of natural frequencies,

$$\omega_i = (\beta_i l)^2 \sqrt{\frac{EI}{\rho A l^4}} \qquad (11)$$

The first five roots of Eq. (10) are shown in Table 1.

Table -1 Value of roots

Roots i	**$\beta_i l$**
1	**1.875104**
2	**4.69409**
3	**7.85475**
4	**10.99554**
5	**14.13716**

2.1 Fixed free beam

The dimensions and the material constant for a uniform fixed free beam (cantilever beam) studied in this paper are: Material of beam = Al, Total length (L) = 0.5 m , width (B) = 0.045 m, height (H) = 0.005 m, moment of inertia (I) = 4.6875 x 10^{-10} m^4,Youngs Modulus (E) = 70 x 10^9 , mass per unit length m = 0.30375 kg, mass density ρ = 2700 kg/ m^3.

Putting all required data in Eq. (11) we get the five frequencies for five modes as shown in Table 2.

Table -2 Mode shape frequency

Mode	**Frequency in Hz**
1	**16.45**
2	**103.06**
3	**288.52**
4	**565.52**
5	**934.85**

III. Numerical Approach for transverse vibration of fixed free beam

We shall now investigate the free vibration of fixed free beam using the ANSYS program, a comprehensive finite element package. We use the ANSYS structural package to analyse the vibration of fixed free beam. Finite element procedures at present very widely used in engineering analysis. The procedures are employed extensively in the analysis of solid and structures and of heat transfer and fluids and indeed, finite element methods are useful in virtually every field of engineering analysis.

3.1 .Description of the finite element method

The physical problem typically involves an actual structure or structural component subject to certain loads. The idealization of the physical problem to a mathematical model requires certain assumptions that together lead to differential equations governing the mathematical model. Since the finite element solution technique is a numerical procedure, it is necessary to access the solution accuracy. If the accuracy criteria are not met, the numerical solution has to be repeated with refined solution parameters until a sufficient accuracy is reached.

3.2 .Important features of finite element method

The following are the basic features of the finite element method:

Division of whole in to parts, which allows representation of geometrically complex domains as collection of simple domains that, enables a systematic derivation of the approximation functions. Derivation of approximation functions over each element the approximation functions are algebraic polynomials that are derived using interpolation theory. Assembly of elements, which are based on continuity of the solution and balance of internal fluxes.

3.3 .Numerical Results

The numerical results were found out by using the ANSYS program as shown in Table 3.

Table -3 Mode shape frequency using Ansys program

Mode	Frequency in Hz
1	16.45
2	103.06
3	288.52
4	565.52
5	934.85

The mode shapes of free vibration of fixed free beam are shown Figure 1.

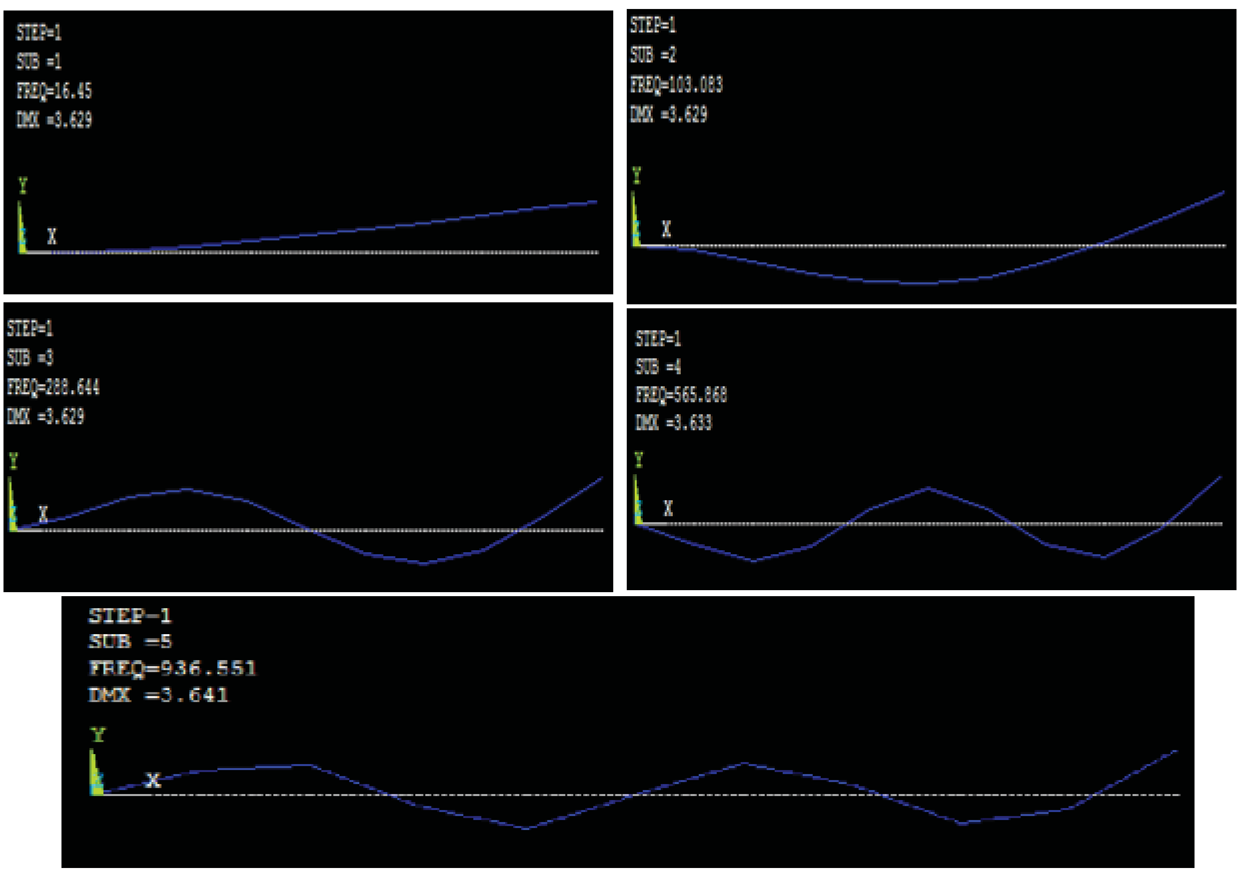

Figure1. Mode shapes of free vibration of fixed free beam

IV. RUSULT AND DISCUSSION

We have studied the free vibration of fixed free beam by using theoretical approach and the numerical approach using the ANSYS program, it has been found that the relative error between these two approaches are very minute. The percentage error between the numerical and theoretical methods is shown in Table 4.

Table -4 Percentage error

Mode	Theoretical frequency in Hz	Numerical frequency from ANSYS program in Hz	Percentage Error %
1	**16.45**	**16.450**	**0.0**
2	**103.06**	**103.08**	**0.0001**
3	**288.52**	**288.64**	**0.0004**
4	**565.52**	**565.87**	**0.0006**
5	**934.85**	**936.55**	**0.0018**

V. CONCLUSION

Firstly we obtained the equation for mode shape frequency theoretically and by analyzing this equation on the fixed free beam which we were used in this paper. The numerical study using the ANSYS program allows investigates the free vibration of fixed free beam to find out mode shape and their frequencies with high accuracy. Therefore it can be concluded that theoretical data is in good agreement with numerical results with negligible error.

ACKNOWLEDGEMENT

I have many to thank for successfully getting this far. By far the most important is Prof. R.B. Barjibhe who continually encourages me to pursue my passions. Further, I thank both Dr.R.P.Singh and Prof. A.V.Patil for their hard work and dedication toward my research and related research.

REFERENCES

[1] S.Mohammadi, A.Hassanirad, **"**Applied and Theoretical Cantilever Beam Free Vibration Analysis**"**, World Academy of Science, Engineering and Technology, 2012 Vol.61, PP 1619-1622.

[2] Tarsicio Beleândez, Cristian Neipp And Augusto Belea Ndez, **"**Numerical and Experimental Analysis of a Cantilever Beam: a Laboratory Project to Introduce Geometric Nonlinearity in Mechanics of Materials**"**, Int. J. Engng Ed., 2003, Vol. 19, No. 6, PP. 885 -892.

[3] J.-J. Wu and A. R. Whittaker**, "**The Natural Frequencies and Mode Shapes Of A Uniform Cantilever Beam With Multiple Two-DOF Spring Mass Systems**"**, Journal of Sound and Vibration, 1999, Vol.227 (2), PP 361-381.

[4] J –S Wu And H-M Chou**,"** Free Vibration Analysis Of A Cantilever Beam Carrying Any Number Of Elastically Mounted Point Masses With The Analytical-And-Numerical-Combined Method**"**, Journal of Sound and Vibration ,1998 , Vol. 213(2),PP 317-332.

[5] Thin-Lin Horng, "Analytical Solution of Vibration Analysis on Fixed-Free Single-Walled Carbon Nanotube-Based Mass Sensor", Journal of Surface Engineered Materials and Advanced Technology, 2012, Vol. 2, PP 47-52.