Order Reduction using Mihailov Criterion and Pade Approximations

J.S. Rana
Research Scholar, Department of Electronics
Shobhit University, Meerut
jasvir_davv@yahoo.co.in

Abstract - In this paper author presents a mixed method for reducing the order of the large-scale dynamic systems using the Mihailov Criterion and Pade Approximation. The denominator coefficient of the reduced order model is obtained by using the Mihailov Criterion while the coefficients of the numerator are obtained by Pade approximations. The mixed method is conceptually simple and always generates stable reduced models if the original high-order system is stable. The proposed method is illustrated with the help of the numerical examples taken from the literature.

Keywords - Mihailov Criterion, Order reduction, Pade approximation, Stability, Transfer function.

I INTRODUCTION

The analysis of high order systems is both tedious and costly as high order systems are often too complicated to be used in real problem. Using mathematical approaches are generally employed to realize simple models for the original high order systems. Reducing a high order system into its lower order system is considered important in analysis, synthesis and simulation of practical systems. Numerical methods are available in the literature for order reduction of large scale systems. A wide variety of model reduction methods [1-5] have been proposed by several authors in frequency domain. In spite of these reduction methods, no one always gives the best results. The Mihailov criterion [6] is one of the most popular reduction techniques available in the literature. It avoids calculating the time-moments and Markov parameters of the original systems. This method also ensures the stability of reduced model if the original high order system is stable. The Pade approximation method was originally introduced by Pade [7]. This method is computationally simple and fits initial time moments and matches the steady state values. The disadvantage of this Method is that the reduced model may be unstable even though the original system is stable. To overcome this problem, Shamash [8-9] introduced a method of reduction based on retention of poles of the high order system in the reduced model and Pade approximation about more than one point. In the proposed method, the denominator polynomial of the reduced order model is determined by using Mihailov Criterion while the coefficients of the numerator are obtained by Pade approximations. The proposed method is compared with the other well-known order reduction techniques available in the literature.

The rest of the paper is organized as follows. Proposed method is explained in section II. Numerical and results are presented in section III. Concluding remarks are given in section IV.

II. PROPOSED METHOD

Let the transfer function of high order original system of the order ‘n’ be

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \cdots b_n s^n} \tag{1}$$

Where; a_i $0 \le i \le n-1$ and b_i $0 \le i \le n$ are known scalar constants.

$$R_k(s) = \frac{N(s)}{D(s)} \quad \frac{c_0+c_1s+c_2s^2+\cdots c_{k-1}s^{k-1}}{d_0+d_1s+d_2s^2+\cdots d_ks^k} \tag{2}$$

Where; c_i $0 \le i \le k-1$ and d_i $0 \le i \le k$ are known scalar constants.

The objective of this paper is to realize the k^{th} order reduced model in the form of (2) from the original system (1) such that it retains the important features of the original high order system.

The reduction procedure consists of the following two steps:

Step-1: Determination of the denominator polynomial $D_k(s)$ for the k^{th} order reduced model by using Mihailov criterion [4]: The brief procedure for getting $D_k(s)$ using Mihailov criterion is as follows:

Substituting $s = jw$ in $D(s)$ and separating into real and imaginary parts as

$$\begin{aligned} D(jw) &= b_0+b_1(jw)+b_2(jw)^2+\dots\dots+b_n(jw)^n \\ &= (b_0-b_2w^2+b_4w^4-\dots)+j\,(b_1w-b_3w^3+b_5w^5-\dots) \\ &= \Phi(w)+j\psi(w) \end{aligned} \tag{3}$$

where w is the angular frequency in rad/sec.

Setting $\varphi(w)=0$ and $\psi(w)=0$, the intersecting frequencies $w_i=0, \pm w_1, \pm w_{2\dots\dots\pm w_{n-1}}$ are obtain

where $|w_1|<|w_2|<|w3|<\dots\dots\dots < |w_{n-1}|$

Similarly, substitute $s = jw$ in $D_k(s)$, which results

$$D_k(jw) = \xi(w) + j\eta(w) \tag{4}$$

Where $\xi(w) = d_o-d_2w^2+d_4w^4+\dots\dots$

$\eta(w) = d_1w-d_3w+d_5w-\dots\dots$

If the reduced model is stable, its Mihailov frequency characteristic must intersect k times with abscissa and ordinate alternatively in the same manner as that of the original system. In other words, k roots of $\xi(w)=0$ and $\eta(w)=0$ must be real and positive and alternately distributed along the w axis. So, the first k intersecting frequencies $0, w_1, w_2, w_3, \dots w_{k-1}$ are kept unchanged and are set to be then roots of $\xi(w)=0$ and $\eta(w)=0$.Therfore

$$\left.\begin{aligned} \xi(w) &= \lambda_1(w^2-w_1^2)(w^2-w_3^2)(w^2-w_5^2)\dots\dots\dots \\ \eta(w) &= \lambda_2(w^2-w_2^2)(w^2-w_4^2)(w^2-w_6^2)\dots\dots\dots\dots \end{aligned}\right\} \tag{5}$$

where the values of the coefficients λ_1, λ_2 are computed from $\varphi(0)=\xi(0)$ and $\psi(w_1)=\eta(w_1)$ respectively. By putting the values of λ_1 and λ_2 in (5) and (6) respectively, we get

$$D_k(jw) = \xi(w) + j\eta(w) \tag{6}$$

Now replacing jw by s, the denominator $D_k(s)$ is obtained as

$$D_k(s)=d_o+d_1s+d_2s^2+d_3s^3+\dots\dots\dots d_ks^k \tag{7}$$

Step-2: Determination of the numerator coefficients of the reduced model by using the pade approximation

The original n^{th} -order system can be expanded in power series about s = 0 as

$$G_n(s) = \frac{\sum_0^{n-1} a_i s}{\sum_0^{n} b_i s} = e_0 + e_1 s + e_2 s^2 + \ldots\ldots\ldots\ldots..$$

The coefficients of the power series expansion can also be calculated as follows:

$$e_0 = \frac{a_0}{b_0}$$

$$e_i = \frac{1}{b_0}\left[a_i - \sum_{j=1}^{i} a_j \, e_{i-j}\right] \qquad i>0 \qquad (8)$$

$a_i=0, \qquad i>n-1$

The k^{th} - order reduced model is taken as

$$R_k(s) = \frac{N_k(S)}{D_k(S)} = \frac{\sum_{i=0}^{k-1} c_i s^i}{\sum_{i=0}^{k} d_i s^i} \qquad (9)$$

here, $D_k(s)$ is known through equations (3-7)
For $N_k(s)$ of eqn (9) to be Pade approximants of $G_n(s)$ of equation (8), we have

$$\left.\begin{array}{l} c_0=d_0e_0 \\ c_1=d_0e_1+d_1c_0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ c_{k-1}=d_0e_{k-1}+d_1e_{k-2}+\ldots\ldots+d_{k-2}e_1+b_{k-1}e_0 \end{array}\right\} \qquad (10)$$

the coefficients c_j ; j=0,1, 2.........k-1 can be found by solving the above k linear equations.
hence, the numerator $N_k(s)$ is obtained as
$$N_k(s)=c_0+c_1s+c_2s^2+c_3s^3+\ldots\ldots..c_{k-1}s^{k-1} \qquad (11)$$

III. NUMERICAL EXAMPLE AND RESULTS

A numerical example is taken from the literature to illustrate the algorithm of the proposed method. The example is solved to get second order reduced model An integral square error (ISE) [1] in between the transient parts of the original and reduced model is calculated using Matlab to measure the goodness of the reduced order model i.e. lower the ISE, closer the $R_k(s)$ to $G_n(s)$, which is given by

$$ISE=\int_0^{\infty} |y(t) - y_k(t)|^2 dt$$

Where, $y(t)$ and $y_k(t)$ are the unit step responses of original and reduced system respectively.

Example-: Consider a 4^{th}-order system from the literature [5].

$$G_4(s) = \frac{N(s)}{D(s)} = \frac{24+24s+7s^2+s^3}{24+50s+35s^2+10s^3+s^4}$$

Putting $s = jw$ in the denominator $D(s)$

$D(jw)=(24-35w^2+w^4)+jw(50-10w^2)$

The intersecting frequencies are

$w_i=0,0.8365,2.2361,5.8566(i=0,1,2,3)$

using the procedure given in step -1, the denominator of the second order model is taken as

$D_2(jw)= \lambda_1 (w^2 -1.190362)+j\lambda_2 w,$

$\lambda_1=-34.2988$

$\lambda_2= 43.0027$

Hence the denominator $D_2(s)$ is obtained as

$D_2(s) =s^2+1.25377s+0.6997009$

using the eqns (8-10), the following coefficients are calculated

$e_0=1$ $a_0=24$

$e_1=-1.0833$ $a_1=17.0035$

Therefore, finally 2nd –order reduced model is obtained as

Thus

$$R_2(S) = \frac{24+17.0035s}{24+43.0027s+34.2988s^2}$$

$$= \frac{0.6994+04957s}{0.6994+1.2537s+s^2}$$

The step responses of the reduced order model and the original system are compared in the Fig.1 and the bode plot comparison is shown in the Fig. 2. The proposed method is compared with the well-known reduction methods and shown in the Table-1, from which it is concluded that this method is comparable in quality.

TABLE I: COMPARISON OF THE REDUCTION METHODS

Reduction Methods	Reduction Models	ISE
Proposed Method	$R_2(s)=\frac{0.6994+0.4957s}{0.6994+1.2537s+s^2}$	0.03300
G.Parmar et al.[1]	$R_2(s)=\frac{8+24.11429s}{8+9s+s^2}$	0.04809
Mukherjee et al.[9]	$R_2(s)=\frac{4.457+11.3909s}{4.4357+4.2122s+s^2}$	0.05697
Mittal et al.[10]	$R_2(s)=\frac{1.9906+7.0908s}{2+3s+s^2}$	0.2689

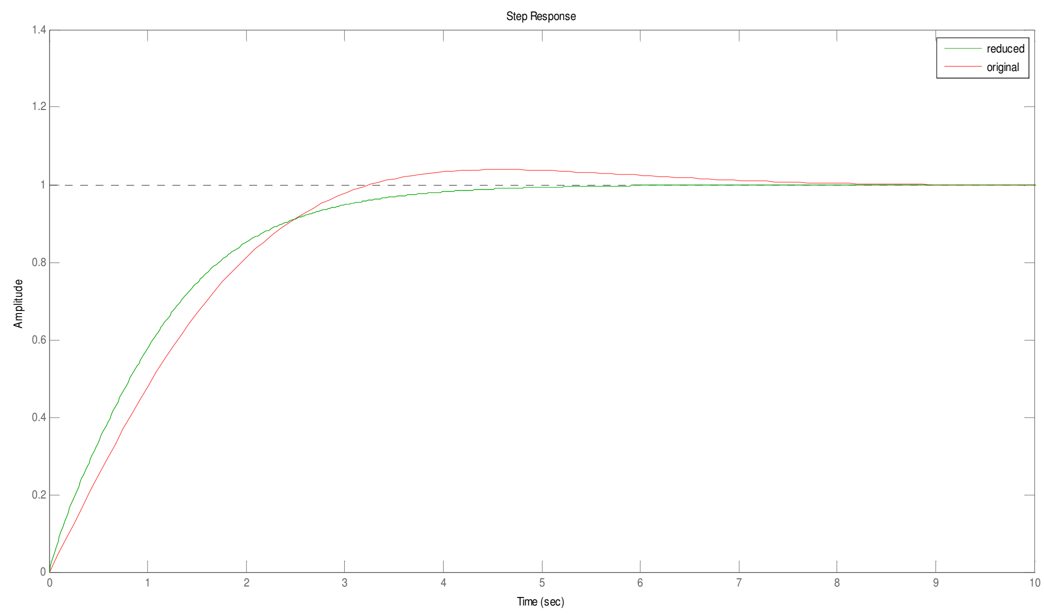
Step Response
reduced
original
Amplitude
Time (sec)

Fig.1

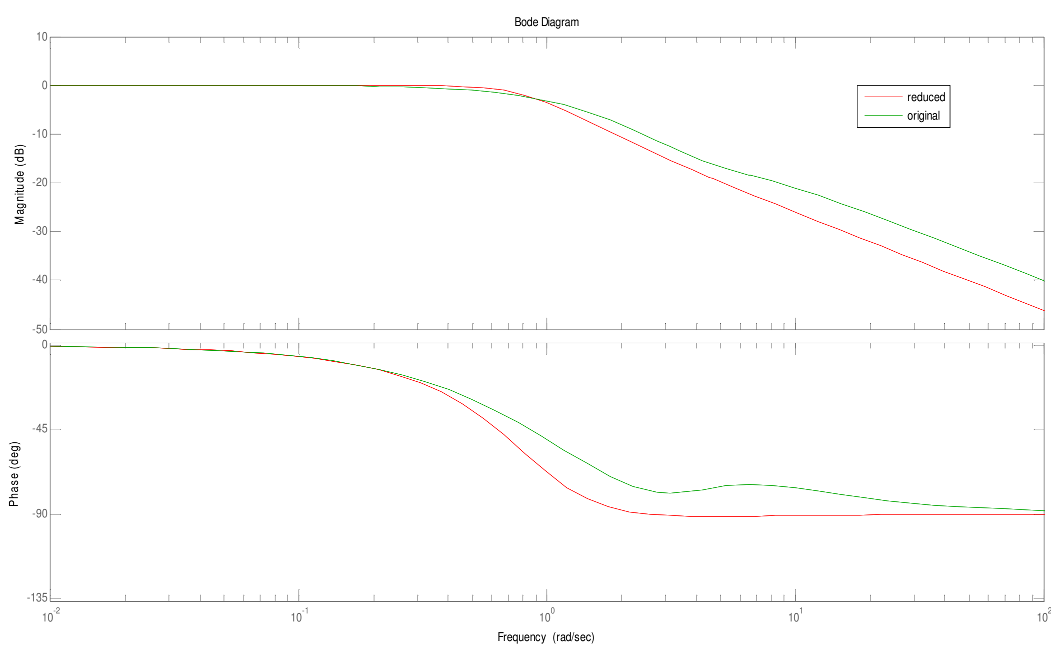
Bode Diagram
reduced
original
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)

Fig.2

TABLE-II: QUALITATIVE COMPARISON WITH ORIGINAL SYSTEM

System	Rise Time	Peak time	Settling Time	Peak
4^{th} order(Original)	2.2602	6.9770	3.9307	0.9991
2^{nd} order(Reduced)	2.2243	4.6025	6.2757	1.0389

IV. CONCLUSIONS

The author presented a mixed method for reducing the order of the large-scale single-input-single-output system. In this method, the denominator polynomial is determined by using the Mihailov Criterion while the coefficients of the numerator are obtained by Pade approximations. This method has been tested on the numerical example chosen from the literature and time and frequency responses of the original and reduced system are compared graphically and shown in the Fig.1, and 2 and error (ISE) and qualitative properties comparison are shown in the Table-1 and Table-2 respectively. From these comparisons, it has been concluded that the proposed method is simple, computer oriented and comparable in quality. This method guarantees stability of the reduced model if the original high-order system is stable and exactly matches the steady-state value of the original system.

REFERENCES

[1] G. Parmar, S. Mukherjee and R. Prasad, "System reduction using factor division algorithm and eigen spectrum analysis", Applied Mathematical Modelling, Elsevier, Vol.31, pp. 2542-2552, 2007.

[2] R. Prasad, S.P. Sharma and A.K. Mittal, "Improved Pade approximation for multivariable systems using stability equation method", Journal of Institution of Engineers IE (I) Journal-EL,Vol.84, pp. 161-165, 2003.

[3] S.K Bhagat et. al, "Some mixed methods for the simplification of high-order single-input-single-output systems", Journal of Institution of Engineers IE(I) Journal-EL,Vol.85, pp.120-123, 2004.

[4] R. Prasad, S.P Sharma and A.K. Mittal, "Linear model reduction using the advantages of Mihailov criterion and factor division", Journal of Institution of Engineers IE (I) Journal-EL, Vol.84, pp. 7-10, 2003.

[5] S.K Bhagat, J. P Tewari and T Srinivasan, "Some mixed method for the simplification of high-order single-input single-output systems''Journal of Institution of Engineers IE (I) Journal-EL, Vol.85, pp. 120-123, 2004.

[6] Mittal A.K, Sharma S.P and Prasad R, "Reduction of multivariable systems using the advantages of Mihailov criterion and factor division", Proc. Int. conf. CERA at IIT Roorkee, India, pp. 477-482, 2002.

[7] Shamash Y, "Stable reduced order models using Pade type approximants", IEEE Trans. Autom. Control, Vol.AC-19, No.5, npp.615-616, October 1974.

[8] Shamash Y, "Linear system reduction using Pade approximation to allow retention of dominant modes", Int. J. Control, Vol.21, No. 2, pp.257-272, 1975. World Academy of Science, Engineering and Technology 80 2011.

[9] S. Mukherjee, Satakshi, R.C Mittal, "Model order reduction using response matching", Journal of Franklin Inst., Vol.342, pp.503-519, 2005.

[10] A.K Mittal, R. Prasad, S.P Sharma, "Reduction of linear dynamic systems using an error minimization technique", Journal of Institution of Engineers IE(I) Journal-EL, Vol.84, pp.201-206, March 2004.