# RELIABILITY FORECAST FOR A PARALLEL REDUNDANT COMPLEX SYSTEM WITH THE CONCEPT OF WAITING

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**Abstract- In this paper, the authors have considered a complex system having two sub-systems namely '*A*' and '*B*' connected in series. Sub-system '*A*' consists of *n* non-identical units in series, while sub-system '*B*' consists of *m* identical units series. The complex system is subjected to minor failure, major failure critical human failure and common cause failure.**

**Keywords – Multi component system, Multiple failures, Repairable system, Laplace transform, cost function**

## I. INTRODUCTION

A complex system having two sub-systems namely '*A*' and '*B*' connected in series. Sub-system '*A*' consists of $n$ non-identical units connected in series, while sub-system '*B*' consists of $m$ identical units in series. The complex system is subjected to minor failure, major failure, critical human failure and common cause failure. Failures follow exponential time distribution where as repairs follow general time distribution. The system is repaired immediately when it is in the state $S_1...S_6...S_7...S_8$. However in the states $S_2$ & $S_4$ the system has to wait with a constant rate $u_i$ till the adequate facilities are made available to repair the system. System goes to reduced efficiency state if $i^{th}$ unit of sub-system '*A*' is failed while system goes to complete breakdown if more than one unit of subsystem '*A*' failed or $j^{th}$ unit of sub-system '*B*' failed.

Using the supplementary variable technique, Laplace transforms of various state probabilities have been evaluated and by using Abel's Lemma the steady state behaviour have also been examined. Some particular cases, *M.T.T.F.*, availability and cost incurred during the operation of system have also been appended at the end to highlight important results. Also, numerical illustrations with various graphs have been given at the end to connect the model with physical situations.

## II. ASSUMPTION AND NOTATIONS

### *A. Assumptions:*

1. Initially at time $t = 0$, the system is in good state.
2. The system consists of two sub-systems namely; A and B connecting inseries.
3. Sub-system A consists of $n$ –non identical units in series. While subsystem 'B' consists of m identical in series.
4. The system has three states as good, degraded and failed.
5. Each unit of the system has a constant failure rate.
6. All the failures are statistically independent.
7. Common cause failure and human error rates are constant.
8. Repair rates follow general time distribution.
9. A common cause failure or a critical human error or a major failure leads to complete breakdown.
10. The system has to wait for repair when two units of sub-system 'A' fail or more than two units fail due to major failure.
11. Repair is giving only when the system is in either degraded or in failed state.
12. After repair, the system works like new one and never damages anything.

### *B. Notations:*

The following notations have been used in this paper:

| | |
|---|---|
| $\overline{F}(s)$ | Laplace transform of function $F(t)$ |
| $\int$ | Integration in the range 0 to $\infty$ |
| $\alpha_1 / \alpha_2 / \alpha_3 / \alpha_B$ | Constant failure rates of state $S_0$ to $S_1 / S_0$ to $S_2 / S_0$ to $S_4 / S_0$ to $S_8$ |
| $\alpha_C / \alpha_{C_i} / \alpha_h / \alpha_{1\eta}$ | Constant failure rate of state $S_0$ to $S_6 / S_1$ to $S_6 / S_0$ to $S_7 / S_1$ to $S_7$ |
| $\beta_i(x)\,\Delta / \eta_i(\mathrm{y})\,\Delta$ | The first order probability that the system will be repaired in the time interval $(x, x+\Delta)/(\mathrm{y}, \mathrm{y}+\Delta)$ conditioned that it was not repaired up the time $x/\mathrm{y}$ |
| $\lambda_{\mathrm{j}}(u)$ | General repair rates for sub-system 'B' from state $S_8$ to $S_0$ |
| $r(w)/\mu(z)$ | General repair rates for common cause failure and critical human error |
| $i,\ j$ | Subscript denotes the serial number of A-unit and B-unit $\left[i = 1, 2 \,.....\, n\right], \left[\,j = 1, 2 \,.....\, m\right]$ |
| $P_0(t)$ | The probability that at time '$t$' the system is in good state |
| $P_1(x, t)\,\Delta$ | The probability that at time '$t$' the system is in degraded state due to the failure of $i^{th}$ unit of sub-system '$A$'. The elapsed repair time lies in the |

| | |
|---|---|
| | interval $(x, x+\Delta)$ |
| $P_2(t)$ | The probability that at time '$t$' the system is in failed state $S_2$. |
| $P_3(\mathrm{y}, t)\,\Delta$ | The probability that at time '$t$' the system is in failed state $S_3$ and the elapsed repair time lies in the interval $(\mathrm{y}, \mathrm{y}+\Delta)$ |
| $P_4(t)$ | The probability that at time '$t$' the system is in failed state $S_4$ |
| $P_5(\mathrm{y}, t)\,\Delta$ | The probability that at time '$t$' the system is in failed state $S_5$ and the elapsed repair time lies in the interval $(\mathrm{y}, \mathrm{y}+\Delta)$ |

**FLOW CHART**

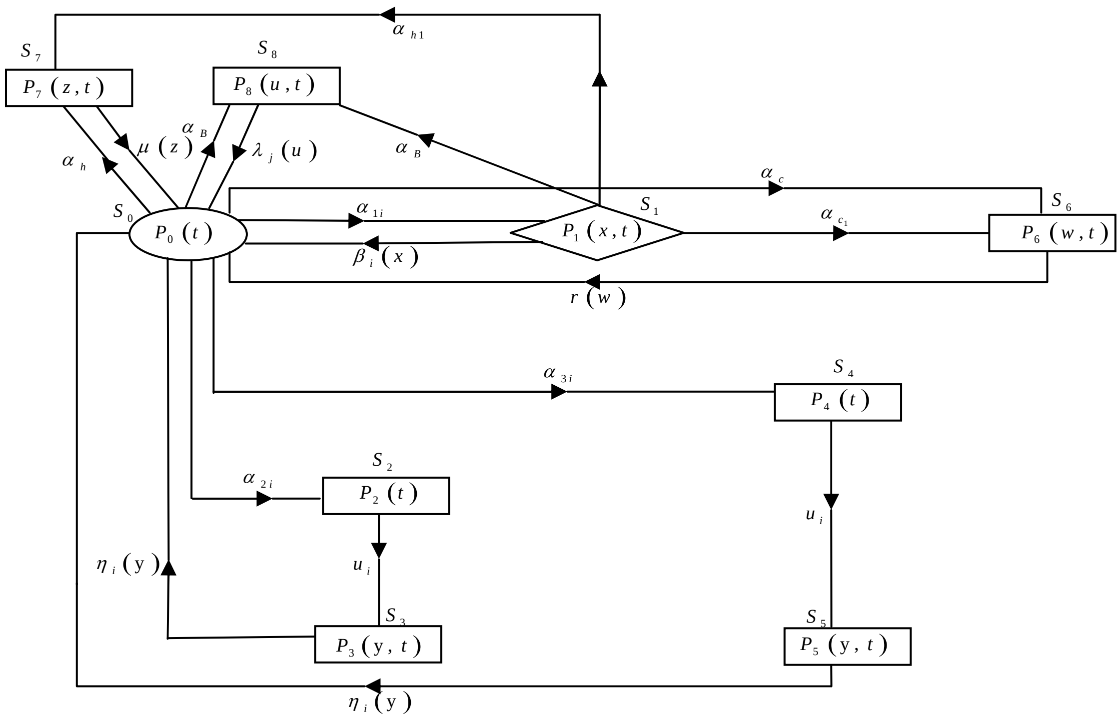


**Fig. 1**

## III. FORMULATION OF THE MATHEMATICAL MODEL

### *A. Formulation:*

By elementary probability consideration and continuity arguments, the following set of difference-differential equations governing the behaviour of the system may be seen to hold good.

$$\left[\frac{\partial}{\partial t}+\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B\right]P_0(t)=\sum_i\int P_1(x,t)\,\beta_i(x)\,dx+\sum_i\int P_3(\mathrm{y},t)\,\eta_i(\mathrm{y})\,d\mathrm{y}$$

$$+ \sum_i \int P_5(\mathrm{y}, t)\, \eta_i(\mathrm{y})\, d\mathrm{y} + \int P_6(w, t)\, r(w)\, dw + \int P_7(z, t)\, \mu(z)\, dz$$
$$+ \sum_j \int P_8(u, t)\, \psi_j(u)\, du \qquad \text{…. (1)}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \alpha_{c_1} + \alpha_{i1_\eta} + \alpha_B + \beta_i(x)\right] P_1(x, t) = 0 \qquad \text{…. (2)}$$

$$\left[\frac{\partial}{\partial t} + u_i\right] P_2(t) = \alpha_{2i}\, P_0(t) \qquad \text{….(3)}$$

$$\left[\frac{\partial}{\partial \mathrm{y}} + \frac{\partial}{\partial t} + \eta_i(\mathrm{y})\right] P_3(\mathrm{y}, t) = 0 \qquad \text{…. (4)}$$

$$\left[\frac{\partial}{\partial t} + u_i\right] P_4(t) = \alpha_{3i}\, P_0(t) \qquad \text{…. (5)}$$

$$\left[\frac{\partial}{\partial \mathrm{y}} + \frac{\partial}{\partial t} + \eta_i(\mathrm{y})\right] P_5(\mathrm{y}, t) = 0 \qquad \text{…. (6)}$$

$$\left[\frac{\partial}{\partial w} + \frac{\partial}{\partial t} + r(w)\right] P_6(w, t) = 0 \qquad \text{…. (7)}$$

$$\left[\frac{\partial}{\partial \mathrm{z}} + \frac{\partial}{\partial t} + \mu(\mathrm{z})\right] P_7(z, t) = 0 \qquad \text{…. (8)}$$

$$\left[\frac{\partial}{\partial u} + \frac{\partial}{\partial t} + \psi_j(\mathrm{z})\right] P_8(z, t) = 0$$

***B Boundary Conditions:***

$P_1(0, t) = \alpha_{1i}\, P_0(t)$ …. (10)

$P_3(0, t) = u_i\, P_2(t)$ …. (11)

$P_5(0, t) = u_i\, P_4(t)$ …. (12)

$P_6(0, t) = \alpha_c\, P_0(t) + \alpha_{c_1}\, P_1(t)$ …. (13)

$P_7(0, t) = \alpha_h\, P_0(t) + \alpha_{1_\eta}\, P_1(t)$ …. (14)

$P_8(0, t) = \alpha_j\left[P_0(t) + P_1(t)\right]$ …. (15)

***C Initial Conditions:***

$P_0(0) = 1$, otherwise zero …. (16)

## IV. SOLUTION OF THE MODEL

Taking Laplace transform of equations (1) through (15) by making use of (16), one can obtain:

$$\left[s + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_c + \alpha_h + \alpha_B\right] \overline{P}_0(s) = 1 + \sum_i \int \overline{P_1}(x, s)\, \beta_i(x)\, dx + \sum_i \int \overline{P_3}(\mathrm{y}, s)\, \eta_i(\mathrm{y})\, d\mathrm{y}$$
$$+ \sum_i \int \overline{P_5}(\mathrm{y}, s)\, \eta_i(\mathrm{y})\, d\mathrm{y} + \int \overline{P_6}(w, s)\, r(w)\, dw + \int \overline{P_7}(\mathrm{z}, s)\, \mu(z)\, dz$$
$$+ \sum_j \int \overline{P_8}(u, s)\, \psi_j(u)\, du \qquad \text{…. (17)}$$

$$\overline{P}_1(0, s) = \alpha_{1i}\, \overline{P}_0(s) \qquad \text{…. (18)}$$
$$\overline{P}_3(0, s) = \mu_i\, \overline{P}_2(s) \qquad \text{…. (19)}$$
$$\overline{P}_5(0, s) = \mu_i\, \overline{P}_4(s) \qquad \text{…. (20)}$$

After solving the above equations, we get finally

$$\overline{P}_0(s) = \frac{1}{A(s)} \quad \text{.... (21)} \qquad \overline{P}_1(s) = \frac{\alpha_{1i}}{A(s)} D_{\beta i}\,(s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B) \quad \text{.... (22)}$$

$$\overline{P}_2(s) = \frac{\alpha_{2i}}{(s+u_i)\,A(s)} \quad \text{.... (23)} \qquad \overline{P}_3(s) = \frac{u_i\,\alpha_{2i}}{(s+u_i)\,A(s)} D_{\eta_i}(s) \quad \text{.... (24)}$$

$$\overline{P}_4(s) = \frac{\alpha_{3i}}{(s+u_i)\,A(s)} \quad \text{.... (25)} \qquad \overline{P}_5(s) = \frac{u_i\,\alpha_{3i}}{(s+u_i)\,A(s)} D_{\eta_i}(s) \quad \text{.... (26)}$$

Where,

$$\begin{aligned} A(s) = {} & s+\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B-\sum_i \alpha_{1i}\,\overline{S}_{\beta i}\,(s+\alpha_{\eta_i}+\alpha_{c_1}+\alpha_B)-\sum_i \frac{u_i}{s+u_i}(\alpha_{2i}+\alpha_{3i})\,\overline{S}_{\eta i}(s) \\ & -\left[\alpha_c+\alpha_{c_1}\sum_i \alpha_{1i} D_{\beta i}\,(s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B)\right]\overline{S}_r(s)-\left[\alpha_h+\alpha_{1_\eta}\sum_i \alpha_{1i} D_{\beta i}\,(s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B)\right]\overline{S}_\mu(s) \\ & -\left[1+\sum_i \alpha_{1i} D_{\beta i}\,(s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B)\right]\sum_j \alpha_j\,\overline{S}_{\psi_j}(s) \end{aligned} \quad \text{.... (27)}$$

## V. EVALUATION OF UP AND DOWN STATE PROBABILITIES

$$\overline{P}_{up} = \overline{P}_0(s)+\overline{P}_1(s) = \frac{1}{s+\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B}\left[1+\frac{\alpha_1}{s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B}\right]$$

Taking inverse Laplace transform, we get

$$\begin{aligned} P_{up}(t) = {} & \left[\frac{\alpha_{c_1}+\alpha_{\eta_1}-\alpha_2-\alpha_3-\alpha_c-\alpha_h}{\alpha_{c_1}+\alpha_{\eta_1}-\alpha_1-\alpha_2-\alpha_3-\alpha_c-\alpha_h}\right]\exp\left[-(\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B)t\right] \\ & +\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h-\alpha_{\eta_1}-\alpha_{c_1}}\exp\left[-(\alpha_{c_1}+\alpha_{1\eta}+\alpha_B)t\right] \end{aligned} \quad \text{.... (28)}$$

Also, $P_{down}(t) = 1-P_{up}(t)$ .... (29)

Again,

$$\overline{R}(s) = \frac{1}{s+\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B}$$

On inverting, we get

$$R(t) = \exp\left[-(\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B)t\right] \quad \text{.... (30)}$$

Again,

$$\text{M.T.T.F.} = \lim_{s\to 0}\overline{R}(s) = \frac{1}{\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B} \quad \text{.... (31)}$$

Cost function of the system is given by

$$G(t) = C_1\int_0^t P_{up}(t)\,dt - C_2\,t$$

Where, $C_1$ and $C_2$ are revenue cost per unit up time and repair cost per unit time respectively.

## VI. NUMERICAL COMPUTATION

$$P_{up}(t) = \left[\frac{\alpha_{c_1} + \alpha_{\eta_1} - \alpha_2 - \alpha_3 - \alpha_c - \alpha_h}{\alpha_{c_1} + \alpha_{\eta_1} - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_c - \alpha_h}\right] \exp\left[-(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_c + \alpha_h + \alpha_B)t\right]$$
$$+ \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_c + \alpha_h - \alpha_{\eta_1} - \alpha_{c_1}} \exp\left[-(\alpha_{c_1} + \alpha_{1\eta} + \alpha_B)t\right]$$

$$R(t) = \exp\left[-(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_c + \alpha_h + \alpha_B)t\right]$$

$$G(t) = C_1 \int_0^t P_{up}(t)\,dt - C_2\,t$$

For a numerical example, let us consider

$\alpha_1 = \alpha_h = 0.01,\ \alpha_2 = \alpha_B = 0.02,\ \alpha_3 = 0.03,\ \alpha_c = \alpha_{\eta_i} = \alpha_{1\eta} = 0.04,\ \alpha_{c_1} = 0.02$ and $C_1 = 2$, $C_2 = 1$, we get

$$P_{up}(t) = 0.08\exp(-0.13t) + 0.2\exp(-0.08t)$$

$$R(t) = \exp(-0.13t)$$

$$G(t) = 1.6\left[\frac{1 - e(-0.13t)}{0.13}\right] + 0.4\left[\frac{1 - e(-0.08t)}{0.08}\right] - t$$

## VII. EXPERIMENTAL RESULT IN TABULATION AND FIGURE

### A. *Table for $P_{up}(t)$ and Curve:*

**Table-1**

| S.No. | *t* | $P_{up}(t)$ |
|---|---|---|
| 1 | 0 | 1 |
| 2 | 1 | 0.8871 |
| 3 | 2 | 0.78727 |
| 4 | 3 | 0.698971 |
| 5 | 4 | 0.620846 |
| 6 | 5 | 0.551701 |
| 7 | 6 | 0.490481 |
| 8 | 7 | 0.436261 |
| 9 | 8 | 0.388222 |
| 10 | 9 | 0.345644 |
| 11 | 10 | 0.307891 |
| 12 | 11 | 0.274404 |
| 13 | 12 | 0.244687 |
| 14 | 13 | 0.218307 |

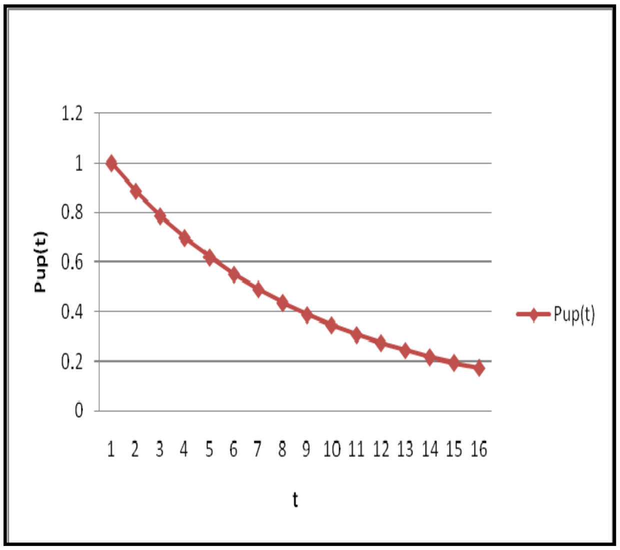


**Figure-2**

### *B. Table for R(t) and Curve:*

**Table-2**

| S.No. | *t* | *R*(*t*) |
|---|---|---|
| 1 | 0 | 1 |
| 2 | 1 | 0.878095 |
| 3 | 2 | 0.771052 |
| 4 | 3 | 0.677057 |
| 5 | 4 | 0.594521 |
| 6 | 5 | 0.522046 |
| 7 | 6 | 0.458406 |
| 8 | 7 | 0.402524 |
| 9 | 8 | 0.353455 |
| 10 | 9 | 0.310367 |
| 11 | 10 | 0.272532 |
| 12 | 11 | 0.239309 |
| 13 | 12 | 0.210136 |
| 14 | 13 | 0.18452 |
| 15 | 14 | 0.162026 |

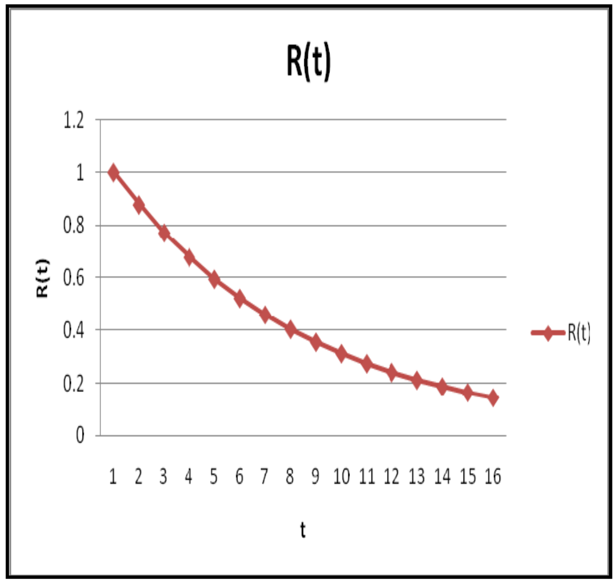


**Figure-3**

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