Minimization Of Makespan in Three Stage Flowshop Scheduling Problem Including Transportation Time And Job Block Criteria With Application Of Idle/Waiting Time Operator $O_{i,w}$

. Deepak Gupta
Prof. & Head, Department of Mathematics
Maharishi Markandeshwar University, Mullana, Ambala, India
guptadeepak2003@yahoo.co.in

Pooja Sharma, Hardeep Singh, Harminder Singh
Research Scholar, Department of Mathematics
Maharishi Markandeshwar University, Mullana, Ambala, India
sharmapooja4988@gmail.com, hardeep10maan@gmail.com, harminder.cheema85@gmail.com

Abstract: This paper is an attempt to study n-job,3-machine flow shop scheduling problem in which processing times are associated with their respective probabilities and the transportation time is also considered .The idle/waiting time operator is being used for the equivalent job for a job block criteria.The objective of the study is to get an optimal sequence of jobs in order to minimize the total elapsed time. Algorithm is made clear with the help of numerical illustration.

Keywords:Flow shop, Equivalent Job block, Transportation Time, Elapsed Time, idle/waiting time operator $O_{i,w}$

I. INTRODUCTION

The basic study in the field of scheduling was made by Johnson(1954) ,who developed an algorithm to minimize the total elapsed time in two,three stage flow shop , Smith(1967) considered minimization of mean flow time and maximum tardiness.Yashida &hitomi (1979) further considered the problem with set up time.The work was developed by Sen and Gupta (1983),Chandasekharan (1992).Bagga & Bhambani (1997) and Gupta Deepak(2011) by considering various parameters.Maggu & Dass(1977) established an equivalent theorem.Singh T.P & Gupta Deepak(2004) was made an attempt to study the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria.

Further we have made an attempt to extend the study done by Gupta Deepak(2012) for n-job,3-machine flow shop scheduling to minimize the total elapsed time with the application of idle/waiting time operator for an equivalent job for a job block.The processing times are associated with their respective probabilities and the concept of transportation time is also included . The operator theorem is very useful in economical and computational point of view and gives optimal schedule in order to optimize total elapsed time . Also the concept of equivalent job block has many applications in different fields where priority of one job over the other becomes significant.

II. PRACTICAL SITUTATION

Various problems occur in real life when priority of one job over other becomes significant.It usually occurs in so many fields such as hospitals,factories etc.An extra price is paid to get the production in desired sequence.And to do so in production, we can use the job block criteria.The different applications are given by researchers for the job block criteria.Maggu and dass gave an algorithm for the job block criteria,which is very useful.Also an Idle/Waiting time operator is used for finding an equivalent job for a given job block.Operator theorem becomes significant for the application of idle/waiting time operator.

III. NOTATIONS

S = sequence of jobs 1,2,3,4,….n.

A,B,C: Three different machines.

A_i=Processing times of i^{th} job on i^{st} machine A.

B_i=Processing times of i^{th} job on second machine B.

C_i=Processing times of i^{th} job on third machine C.

t_i=Transportation time of i^{th} job from machine A to machine B.

g_i=Transportation time of i^{th} job from machine B to machine C.

p_i=Probabilities associated with processing times A_i.$0 \leq pi \leq 1$, $0 \leq \sum pi \leq 1$.

q_i=Probabilities associated with processing times B_i.$0 \leq qi \leq 1$, $0 \leq \sum qi \leq 1$.

r_i=Probabilities associated with processing times C_i.$0 \leq r_i \leq 1$, $0 \leq \sum r_i \leq 1$.

IV. PROBLEM FORMULATION

Let n jobs are to be processed on three machines A ,B and C.Let A_i ,B_i ,C_i (i=1,2,3,…n) be the processing times of each job on machines A,B and C respectively.Let t_i and g_i (i=1,2,…n) be the transportation times of job i from machine A to B and from B to C.The mathematical model of the problem is as follows:

TABLE-1

job	Machine A		Transportation time from $A \rightarrow B$	Machine B		Transportation time from $B \rightarrow C$	Machine C	
i	A_i	p_i	t_i	B_i	q_i	g_i	C_i	r_i
1	A_1	p_1	t_1	B_1	q_1	g_1	C_1	r_i
2	A_2	p_2	t_2	B_2	q_2	g_2	C_2	r_2
3	A_3	p_3	t_3	B_3	q_3	g_3	C_3	r_3
4	A_4	p_4	t_4	B_4	q_4	g_4	C_4	r_4
-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-
n	A_n	p_n	t_n	B_n	Q_n	g_n	C_n	r_n

Our intension is to find an optimal sequence so as to find the minimum elapsed time.

V. ALOGRITHM

Step1: Define expected processing times a_i , b_i and c_i on machines A ,B and C respectively as follows:

$a_i = A_i \times p_i$

$b_i = B_i \times q_i$

$c_i = C_i \times r_i$

Step2: Compute processing times by creating two fictitious machines G and H with their processing times G_i

and H_i respectively as follows:

$$G_i = |a_i + b_i + t_i + g_i|$$

And $$H_i = |c_i + b_i + t_i + g_i|$$

if , either $\min(a_i + t_i) \geq \max(b_i + t_i)$

OR $\min(c_i + g_i) \geq \max(b_i + g_i)$

OR both.

Step3: Determine equivalent job for the job block using idle/waiting time operator $O_{i,w}$ as per definition,

$$(G_k, H_k)\ O_{i,w}\ (G_m, H_m) = (G_\beta, H_\beta)$$

$$G_\beta = G_k + \max(G_m - H_k, 0)$$

$$H_\beta = H_m + \max(H_m - G_m, 0)$$

Step 4:Define new reduced problem with two fictitious machines G and H having processing times G_i and H_i respectively,and the job block is replaced by a single job β with processing times G_β and H_β.

Step 5: Apply Johnson's algorithm to find an optimal sequence of jobs ,which gives the minimum elapsed time. This optimal schedule will also be the optimal schedule for the original problem.

VI. DEFINITION AND THEOREM OF IDLE/WAITING TIME OPERATOR $O_{I,W}$

Definition: Let R_+ be the set of non negative numbers.Let $G=R_+ \times R_+$. Then $O_{i,w}$ is defined as a mapping from $G \times G \to G$ given by

$$O_{i,w}[(x_1,y_1),(x_2,y_2)]=(x_1,y_1)O_{i,w}(x_2,y_2)$$

$$=\{x_1 + \max(x_2-y_1,0)\ ,\ y_2+ \max(y_1-x_2\ ,\ 0)\}$$

Where $x_1,x_2,y_1,y_2 \in R$.

THEOREM: Let n jobs 1,2,3,…….n are processed through two machines A&B in order AB with processing times a_i and b_i (i=1,2,3,…n) on machines A &B respectively. If $(a_p,b_p)O_{i,w}(a_q,b_q)=(a_\beta,b_\beta)$ then

$a_\beta = a_p+\max(a_q-b_p\ ,\ 0)$ and $b_\beta = b_q+ \max(b_p-a_q\ ,\ 0)$

where β is an equivalent job for the job block (p,q) and $p,q \in \{1,2,3,\ldots n\}$

PROOF: starting by the equivalent job block criteria theorem for β=(p,q) given by maggu and das,we have:

$a_\beta = a_p + a_q - \min(b_q,a_q)$ ………..(1)

$b_\beta = b_p + bq - \min(b_p,a_q)$ ……….(2)

Now we prove the above theorem by following way:

Case 1 : When $a_q > b_p$

$a_q > b_p > 0$

$\max\{a_q > b_p\ ,0\} = a_q > b_p$ …………(3)

and $b_p > a_q < 0$,

$\max\{b_p > a_q\ ,\ 0\} = 0$ …………(4)

(1) ….. $a_\beta = a_p + a_q - \min(b_p,a_q)$

$=a_p+ a_q - b_p$ as $a_q > b_p$

$=a_p + \max(a_q - b_p , 0)$ using (3) …….. (5)

(2)…..$b_\beta = b_p + b_q – \min(b_p,a_q)$

$=b_p + b_q - b_p$ as $a_q>b_p$

$=b_q + (b_p- b_p)$

$=b_q + 0$

$=b_q + \max (b_p - a_q , 0)$ using (4) ……. (6)

Case 2: when $a_q<b_p$

$a_q-b_p<0$

$\max(a_q-b_p, 0) = 0$ ……. (7)

and $b_p-a_q >0$

$\max (b_p-a_q, 0) = b_p-a_q$ ……….. (8)

(1) $a_\beta = a_p+a_q-\min (b_p,a_q)$

$= a_p+a_q-a_q$ as $a_q>b_p$

$= a_p+0$

$= a_p+\max (a_q-b_q,0)$ using (7) ……….. (9)

(2) $b_\beta = b_p+b_q- \min(b_p,a_q)$

$= b_p+b_q-a_q$ as $a_q <b_p$

$=b_q+(b_p-a_q)$

$=b_q+\max (b_p-a_q,0)$ using (8)……… (10)

Case 3: when $a_q=b_p$, $a_q-b_p =0$

$\max (a_q-b_p,0) =0$ ………… (11)

also

$b_p-a_q = 0$

$\max(b_p-a_q,0) = 0$

(1) $a_\beta =a_p+a_q-\min(b_p,a_q)$ ……….. (12)

$= b_p+a_q-a_p$ as $b_q =a_p$

$= a_p+0$

$= a_p+\max (a_q-b_p,0)$ ……….. (13)

(2) $b_\beta =b_p+b_q-\min(b_p,a_q)$ …………. (14)

$=b_p+b_q-b_p$

$=b_q+(b_p-b_p)$

$= b_q+0$

$= b_q+\max(b_p-a_q,0)$ using (14)……….. (15)

By (5), (6), (9), (10), (13) and (15) we conclude:

$a_\beta =a_p+\max(a_q- b_p,0)$

$b_\beta =b_q+\max (b_p- a_q,0)$ for all possible three cases.

VII. NUMERICAL ILLUSTRATION

Consider 5 jobs and three machines A,B and C.Let A_i ,B_i and C_i be the processing times of the jobs corresponding to machines A,B and C respectively.Also let t_i and g_i be the transportation times of jobs from machine A to B and from machine B to C respectively.

TABLE-2

job	Machine A			Machine B			Machine C	
i	a_i	p_i	t_i	b_i	q_i	g_i	c_i	r_i
1	30	0.3	4	30	0.2	3	20	0.2
2	40	0.2	3	25	0.2	2	30	0.1
3	30	0.2	6	30	0.1	2	35	0.2
4	20	0.2	8	20	0.1	3	20	0.2
5	70	0.1	5	15	0.4	4	10	0.3

Obtain the optimal sequence of jobs so as to minimize the total elapsed time by taking (2,5) as a job block.

Solution:

STEP 1: First of all we find the expected processing times by multiplying the processing times with their respective probabilities.

TABLE-3

jobs	Machine A	t_i	Machine B	g_i	Machine C
i	a_i		b_i		c_i
1	9	4	6	3	4
2	8	3	5	2	3
3	6	6	3	2	7
4	4	8	2	3	4
5	7	5	6	4	3

STEP 2: Now we check the conditions discussed in the algorithm,

And here $\min(a_i+t_i)= \max(b_i+t_i)$ satisfies.So we create two fictitious machines G & H with their processing times G_i &H_i as follows:

$G_i = |a_i + bi + t_i + g_i|$ and $H_i = |c_i + b_i + t_i + g_i|$, as follows

TABLE-4

Jobs	G_i	H_i
1	22	17
2	18	13
3	17	18
4	17	17
5	22	18

STEP 3: Now we find an equivalent job β for a given job block (2,5) by using idle/waiting time operator,

$$(G_k ,H_k)\ O_{i,\omega}\ (G_m ,G_m) =(G_{\beta} , H_{\beta})$$

$$G_{\beta} = G_k + \max (G_m – H_k , 0)$$

& $$H_{\beta} = H_m + \max(H_k – G_m , 0)$$

So G_β =18+ max (22 – 13 , 0)

= 18+ max(9 , 0)

= 18 + 9

= 27

H_β = 18 + max (18 – 22 , 0) = 18

STEP 4: Represent new reduced problem in table form using step 1 and step 2,

TABLE-5

Jobs	G_i	H_i
1	22	17
β	27	18
3	17	18
4	17	17

STEP 5: Now by using Johnson's technique , optimal sequence is

4 , 3 , β ,1

i.e, 4, 3, 2, 5, 1

The above optimal sequence will also be the optimal sequence for the original sequence.

In-Out flow table for sequence S, is as follows:

TABLE-6

Jobs	Machine A	Machine B	Machine C
i	In - Out	In - Out	In - Out
4	0 - 4	12 - 14	17 - 21
3	4 - 10	16 - 19	21- 28
2	10 - 18	21 - 26	28 - 31
5	18 - 25	30 - 36	40 - 43
1	25 - 34	38 - 44	47 - 51

Minimum total elapsed time for the given problem is 51 units.

VIII. CONCLUSION

So clearly we can use idle/waiting time operator to find an equivalent job for a given job block including transportation time and we can minimize total elapsed time using Johnson technique.

IX. REMARK

The study may be extended by introducing different parameters such as set up time, job weights, breakdown interval etc

REFERENCES

[1] Baker K. R. (1974), "Introduction of sequencing and scheduling", John Wiley and sons, New York.

[2] Bansal S. P. (1986), "Resultant job in restricted two machine flow shop problem," IJOMAS, Vol.2 , pp.35-45.

[3] Bellman R. (1956), "Mathematical aspects of scheduling theory," J. Soc. Indust. Appl. Math,Vol. 4, pp.168-205.

[4] Belwal and Mittal (2008), "n jobs machine flow shop scheduling problem with break down of machines, transportation time and equivalent job block, Bulletin of Pure & Applied Sciences-Mathematics," Jan – June, source Vol. 27, Source Issue 1.

[5] Chandramouli A. B. (2005), "Heuristic approach for n-jobs, 3-machines flow-shop scheduling problem involving transportation time, breakdown time and weights of jobs," mathematical and Computational Applications, Vol.10 , pp.301-305.

[6] Gupta Deepak & Sharma Sameer (2011), "Minimizing rental cost under specified rental policy in two stage flow shop , the processing time associated with probabilities including breakdown interval and Job-block criteria, European Journal of Business and Management," Vol. 3 No.2, pp.85-103.

[7] Gupta Deepak and Singla Payal (2012) "Constrained n – job, 3 – machine flow shop scheduling problem with transportation time." Vol. 2, Issue 2.

[8] Heydari (2003), "On flow shop schedule problem with processing of jobs and string of disjoint job blocks fixed and jobs and arbitrary order jobs", JISSOR Vol. XXIV No. pp.1-4.

[9] Jackson J. R. (1956), "An extension of Johnson's results on job scheduling," Nav. Res. Log. Quar.,Vol.3, pp.201-203.

[10] Johnson S.M. (1954), "Optimal two stage and three stage production schedule with set-up times included," Nav. Res. Log. Quar.,1, pp.61-68.

[11] Khodadadi A. (2011), "Solving constrained flow-shop scheduling problem with three machines," International Journal of Academic Research Vol.3 No.1, pp.38-40.

[12] Maggu & Das (1981), "On n x 2 sequencing problem with transportation time of jobs," Pure and Applied Mathematika Sciences, pp.12-16.

[13] Miyazaki S. and Nishiyama N. (1980), "Analysis for minimizing weighted mean flow time in flow shop scheduling," J. O. R. Soc. Of Japan, Vol.32, pp.118-132.

[14] Narain Laxmi and Bagga P.C. (1998), "Two machine flow shop problem with availability constraint on each machine," JISSOR, Vol. XXIV 1-4, pp.17-24.

[15] Nawaz, Ensore & Ham (1983), "A heuristic algorithm for the m-machine, n-job flow-shop sequencing problem," OMEGA, pp.91-95.

[16] Parker R.G. (1995), "Determinstic scheduling theory," Chapman and hall, New York, 1995.

[17] Pandian P. & Rajendran P. (2010), "Solving Constraint flow shop scheduling problems with three machines," Int. J. Contemp. Math. Sciences. Vol.5, no. 19, pp.921-929.

[18] Singh T.P. (1995), "On 2 x n flow-shop problems invoving job-block, transportation time, arbitrary time and break down machine time," PAMS,(1995), Vol XXI, No. 1—2 March.

[19] Singh, T.P., Rajindra K& Gupta Deepak (2005), 'Optimal three stage production schedule the processing time and set up times associated with probabilities including job block criteria', Proceeding of National Conference FACM- (2005), pp 463-470.

[20] Yoshida and Hitomi (1979), **"**Optimal two stage production scheduling with set-up times separated**".** All transactions. Vol. II, pp. 261-263.