Minimization of Elapsed Time in N×3 Flow Shop Scheduling Problem, The Processing Time Associated With Probabilities Including Transportation Time

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Abstract: This paper is an attempt to study 3-stage flow shop scheduling problem in which the processing time are associated with their respective probabilities including the transportation time. The objective of the study is to obtain an optimal sequence of jobs in order to minimize the total elapsed time. The method is made clear with the help of numerical illustration.

Keyword: Flow Shop Scheduling, Processing Time, Transportation Time, Optimal Sequence.

I. Introduction

Scheduling is an important process widely used in manufacturing, management, computer science etc. Finding good scheduling for given set of jobs can help factory supervisors effectively to control job flows and provide solution for job sequencing. A flow shop scheduling problem consists of determine the processing sequence for n jobs on M machines, where each job is processed on all the machines in the same order and the objective is to minimize the time required to process all jobs. The basic study in flow shop production scheduling has been made by Johnson [1954], The scheduling problem practically depends upon the important factors namely transportation time, relative importance of a job over another job. In a practical situation the processing times may always not be exact as has been taken by most of the researchers. Hence we associated probabilities with processing times. The concept of transportation time was separately studied by various researchers Jaksen (1956), Belman (1956), Maggu and Dass (1981), Nawaz et al(1983) Bansal, (1986), Parker (1995), Singh T.P (1995), Narain and Bagga (1998), Heydari (2003), chandramouli (2005), Belwal and Mittal (2008), Pandian and Rajendran (2010), Khodadadi (2011), Gupta Deepak and Sharma Sameer (2011), Singh T.P (2005), and khodadadi (2011), solved the flow shop three stage problem with different structural condition.

Gupta Deepak and Singla Payal (2012) studied a three stage flow shop scheduling problem with the concept of transportation time with some advanced structured condition to minimize the elapsed time. Here we have extended the work Gupta Deepak and Singla payal (2012) by associated probabilities with the processing time. Thus this paper is more wider and practically more applicable and has significant results in the process industry.

II. Practical Situation:

In manufacturing companies different jobs are processed on various machine . These jobs are required to be processed in a machine shop M_1, M_2, M_3,...... etc in specified order. In real life we always want to minimize the elapsed time in the manufacturing of product. The processing time are always not exact as has been taken by most of the researcher. So we associate probabilities with processing times. By using the concept of probabilities we reach nearer to the optimal solution.

III. Notation

S : Sequence of jobs 1, 2, 3,…n.

M_j : Machine j, j = 1,2,…….

a_i : Processing time of i^{th} job on machine M_1.

b_i : Processing time of i^{th} job on machine M_2.

c_i : Processing time of i^{th} job on machine M_3.

p_i : Probability associated to the processing time a_i of i^{th} job on machine M_1

q_i : Probability associated to the processing time b_i of i^{th} job on machine M_2.

s_i : Probability associated to the processing time c_i of i^{th} job on machine M_3

t_i : Transportation time of i^{th} job from machine M_1 to machine M_2.

g_i : Transportation time of i^{th} job from machine M_2 to machine M_3.

A_i : Expected processing time of i^{th} job on machine M_1.

B_i : Expected processing time of i^{th} job on machine M_2.

C_i : Expected processing time of i^{th} job on machine M_3.

IV. Problem Formulation

Let n job say 1, 2, 3,…, n are processed on three machines M_1, M_2, and M_3 in the order $M_1 M_2 M_3$. a_i, b_i and c_i (i=1, 2, 3, .., n) be the processing time of each job on machine M_1, M_2 and M_3 respective, assuming their respective probabilities p_i, q_i and s_i such that $0 \le p_i \le 1$, $\sum p_i = 1$, $0 \le q_i \le 1$, $\sum q_i = 1$, and $0 \le r_i \le 1$, $\sum r_i = 1$,Let t_i and g_i (i=1,2,3,..,n) be the transportation time of i^{th} job from machine M_1 to machine M_2 and machine M_2 to machine M_3 respectively. The mathematical model of the given problem in matrix form can be stated as :

Table - 1

Jobs	Machine M_1		$t_{i1\to2}$	Machine M_2		$g_{i2\to3}$	Machine M_3	
i	a_i	p_i	t_i	b_i	q_i	g_i	c_i	r_i
1	a_1	P_1	t_1	b_1	q_1	g_1	c_1	r_1
2	a_2	P_2	t_2	b_2	q_2	g_2	c_2	r_2
3	a_3	P_3	t_3	b_3	q_3	g_3	c_3	r_3
	.	.	.	.	.	.	.	.
.								
n	a_n	p_n	t_n	b_n	q_n	g_n	c_n	r_n

Our objective is to obtain the optimal sequence of jobs which minimize the total elapsed time.

V. Algorithm

Step1: Define expected processing time A_i, B_i and C_i on machine M_1, M_2 and M_3 respectively as follows

1. $A_i = a_i \times p_i$
2. $B_i = b_i \times q_i$
3. $C_i = c_i \times r_i$

Step2: Compute processing time by creating two fictitious machines G and H with their processing time G_i and H_i respectively as follows: $G_i = |A_i + B_i + t_i + g_i|$ and $H_i = |C_i + B_i + t_i + g_i|$

If

Either

$$\text{Min }(A_i + t_i) \geq \text{Max }(B_i + t_i)$$

Or

$$\text{Min }(C_i + g_i) \geq \text{Max }(B_i + g_i)$$

Step3: Define new reduced problem with processing time G_i and H_i as defined in step2.

Step4: Find the optimal sequence by using Johnson's technique (1954) for two machines G and H with processing time G_i and H_i obtained in step2.

Step5: Compute the In-Out table for the sequence obtained in step 4.

VI. Numerical Illustration

Consider a five job and three machine flow shop scheduling problem with processing time with their respective probabilities and transportation time of jobs as follows. Our objective is to obtain optimal or near optimal sequence which minimize the total elapsed time.

Table - 2

Jobs	Machine M_1		$t_{i,1\to2}$	Machine M_2		$g_{i,2\to3}$	Machine M_3	
I	a_i	p_i	t_i	b_i	q_i	g_i	c_i	r_i
1.	45	0.2	4	20	0.3	3	20	0.2
2.	20	0.4	3	50	0.1	2	10	0.3
3.	60	0.1	6	30	0.1	2	70	0.1
4.	20	0.2	8	10	0.2	3	20	0.2
5.	70	0.1	5	20	0.3	4	15	0.2

Solution:

Step 1: Define expected processing time A_i, B_i and C_i on machine M_1, M_2 and M_3 respectively as shown below:

Table - 3

Jobs	Machine M_1	$t_{i,1\to2}$	Machine M_2	$g_{i,2\to3}$	Machine M_3
I	$A_i = a_i \times p_i$	t_i	$B_i = b_i \times q_i$	g_i	$C_i = c_i \times s_i$
1.	9	4	6	3	4
2.	8	3	5	2	3
3.	6	6	3	2	7
4.	4	8	2	3	4
5.	7	5	6	4	3

Step 2: Here $\min(A_i + t_i) \geq \max(B_i + t_i)$ satisfied therefore Let us create two fictitious machines G and H with their processing time G_i and H_i using $G_i = | A_i + B_i + t_i + g_i |$ and $H_i = | C_i + B_i + t_i + g_i |$ as follows:

Table - 4

Jobs	G_i	H_i
1	22	17
2	18	13
3	17	18
4	17	17
5	22	18

Step 3: The optimal sequence with minimum elapsed time using Johnson's technique (1954) is S = 3 -4 -5 -1 -2.

Step 4: In – Out table for sequence S is as follows:

Table - 5

Jobs	Machine M_1	Machine M_2	Machine M_3
i	In – Out	In – Out	In – Out
3.	0 – 6	12 – 15	17 – 24
4.	6 – 10	18 – 20	24 – 28
5.	10 – 17	22 – 28	32 – 35
1.	17 – 26	30 – 36	39 – 43
2.	26 – 34	37 – 42	44 – 47

Thus the minimum total elapsed time for the given problem is 47 units.

VII. Remark

The study may further be extended by introducing different parameters such as set-up time separated from their processing time and job block criteria.

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