Discrete Wavelet Transform: A Technique for Image Compression & Decompression

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Abstract: This paper applies wavelet analysis to image compression. A mother or basis wavelet is first chosen for the compression. The image is then decomposed to a set of scaled and translated versions of the mother wavelet. The resulting wavelet coefficients that are insignificant or close to zero are truncated achieving image compression. Analysis of the compression process was performed by comparing the compressed-decompressed image against the original. This was conducted to determine the effect of the choice of mother wavelet on the image compression. The results however showed that regardless of bases wavelet used the compression ratio is relatively close to one another.

Keywords: Compression, Lossless & Lossy Compression, Image Compression, Wavelet Transform, 2-D DWT.

I. HISTORY OF WAVELETS

From an historical point of view, wavelet analysis is a new method, though its mathematical underpinnings date back to the work of Joseph Fourier in the nineteenth century. The first recorded mention of what we now call a "wavelet" seems to be in 1909, in a thesis by Alfred Haar.

The concept of wavelets in its present theoretical form was first proposed by Jean Morlet and the team at the Marseille Theoretical Physics Center working under Alex Grossmann in France. The methods of wavelet analysis have been developed mainly by Y. Meyer and his colleagues, who have ensured the methods' dissemination.

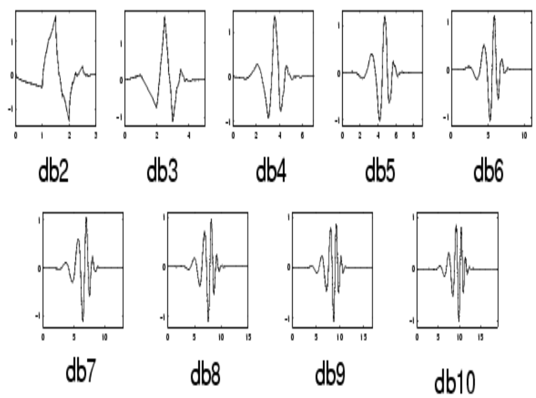
The main algorithm dates back to the work of Stephane Mallat in 1988. Since then, research on wavelets has become international. Such research is particularly active in the United States, where it is spearheaded by the work of scientists such as Ingrid Daubechies, Ronald Coifman, and Victor Wickerhauser.

Barbara Burke Hubbard describes the birth, the history, and the seminal concepts in a very clear text. See "The World According to Wavelets," A.K. Peters, Wellesley, 1996.

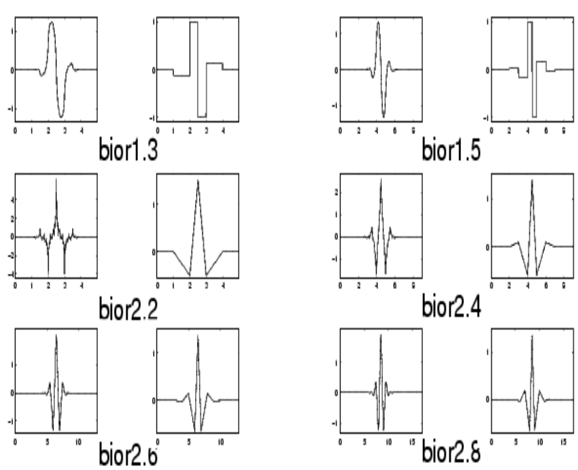
II. WAVELET FAMILIES

A. Haar: Haar wavelet is discontinuous, and resembles a step function. It represents the same wavelet as Daubechies db1.

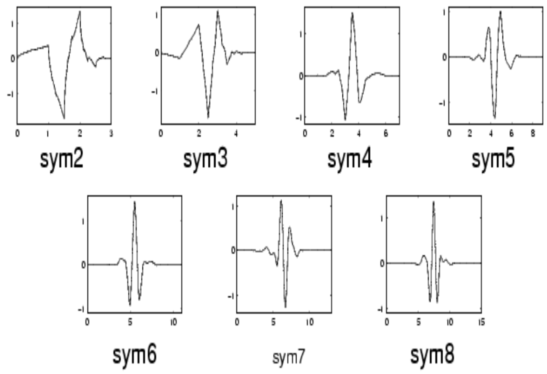
B. Daubechies: The names of the Daubechies family wavelets are written dbN, where N is the order, and db the "surname" of the wavelet.



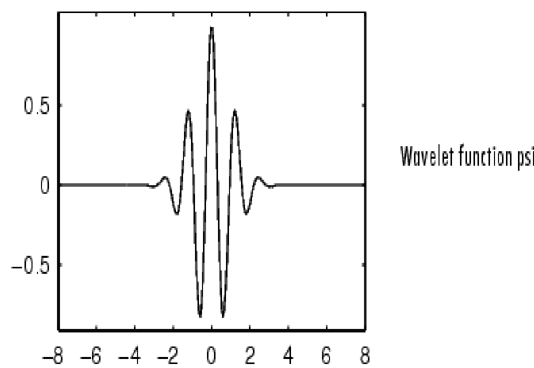
C. Biorthogonal: This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition (on the left side) and the other for reconstruction (on the right side) instead of the same single one, interesting properties are derived.



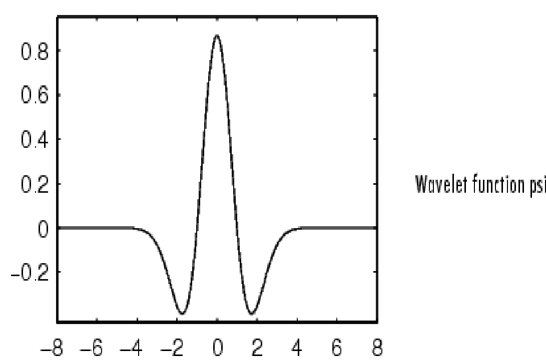
D. Symlets: The symlets are nearly symmetrical wavelets proposed by Daubechies as modifications to the db family. The properties of the two wavelet families are similar. Here are the wavelet functions psi.



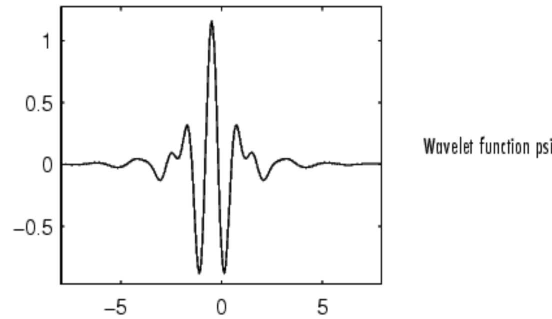
E. Morlet: This wavelet has no scaling function, but is explicit.



F. Mexican Hat: This wavelet has no scaling function and is derived from a function that is proportional to the second derivative function of the Gaussian probability density function.

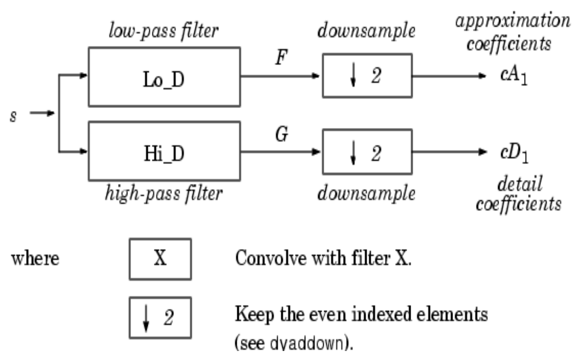


G. Meyer: The Meyer wavelet and scaling function are defined in the frequency domain.



III. ALGORITHM

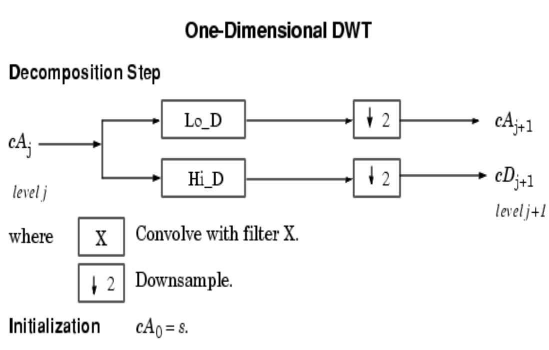
Given a signal s of length N, the DWT consists of log2N stages at most. Starting from s, the first step produces two sets of coefficients: approximation coefficients cA1, and detail coefficients cD1. These vectors are obtained by convolving s with the low-pass filter Lo_D for approximation, and with the high-pass filter Hi_D for detail, followed by dyadic decimation.

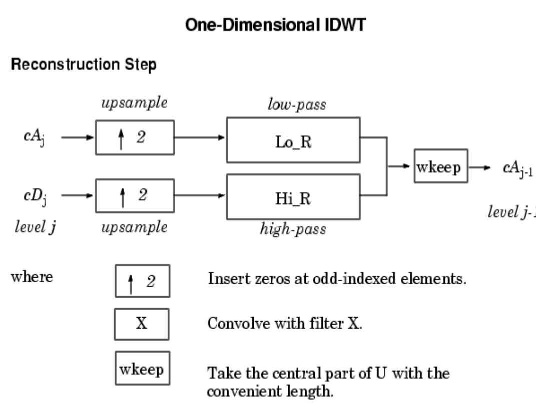


The length of each filter is equal to 2N. If n = length (s), the signals F and G are of length n + 2N - 1, and then the coefficients cA1 and cD1 are of length

$$\text{floor}\left(\frac{n-1}{2}\right) + N$$

The next step splits the approximation coefficients cA1 in two parts using the same scheme, replacing s by cA1 and producing cA2 and cD2, and so on.





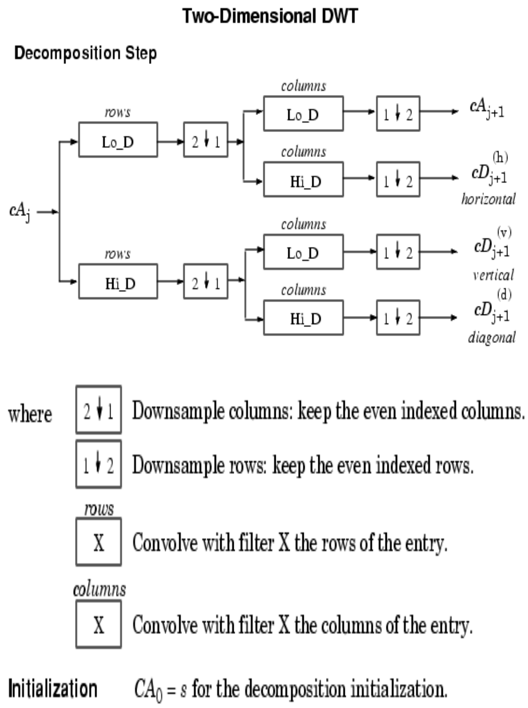
So the wavelet decomposition of the signal s analyzed at level j has the following structure: [cAj, cDj, ..., cD1].

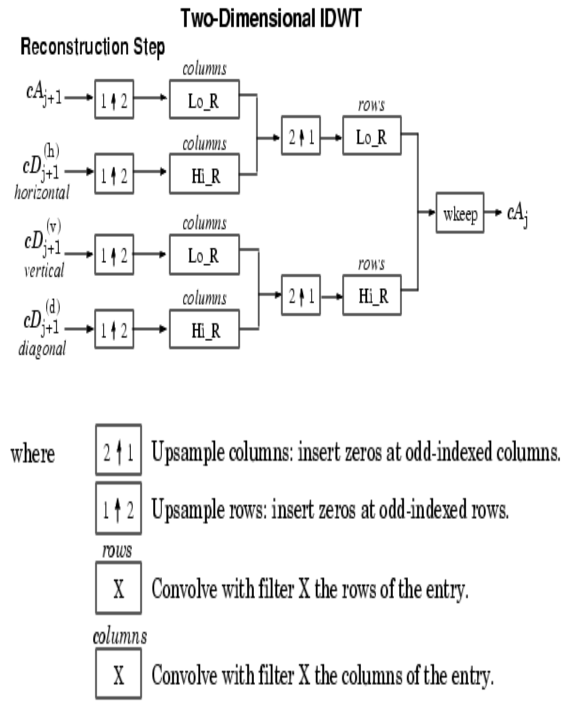
Conversely, starting from cAj and cDj, the IDWT reconstructs cAj-1, inverting the decomposition step by inserting zeros and convolving the results with the reconstruction filters.

For images, a similar algorithm is possible for two-dimensional wavelets and scaling functions obtained from one-dimensional wavelets by tensorial product.

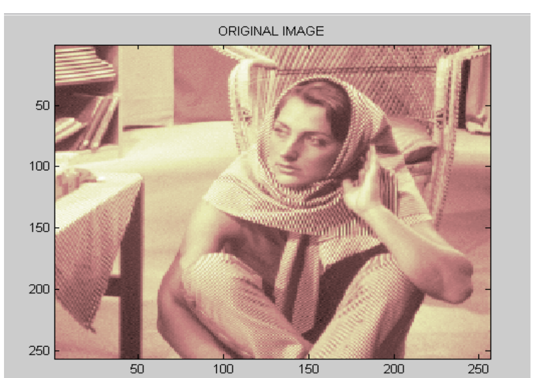
This kind of two-dimensional DWT leads to a decomposition of approximation coefficients at level j in four components: the approximation at level j + 1 and the details in three orientations (horizontal, vertical, and diagonal).

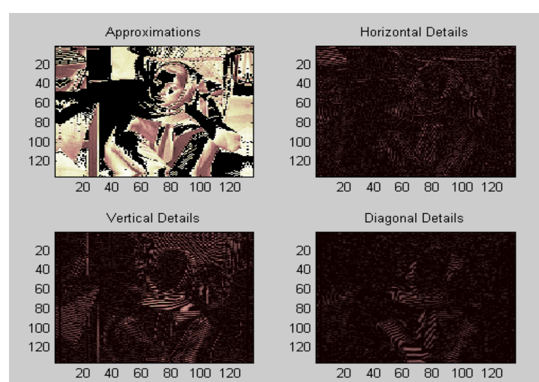
The following charts describe the basic decomposition and reconstruction steps for images.

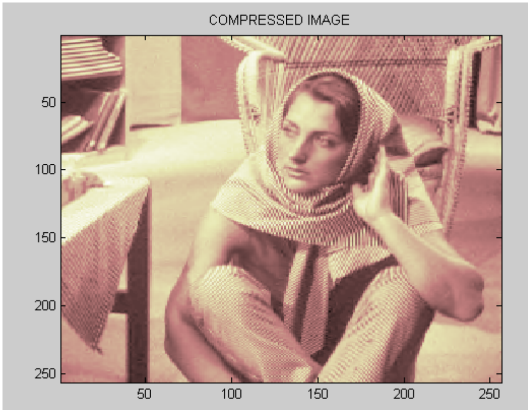


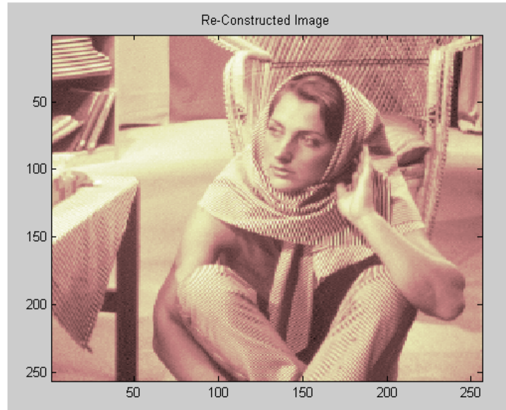


IV. RESULTS









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