Cost Optimization of Concrete Beam Element – By Direct Exhaustive Search Method

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Abstract – Reinforced Concrete Beam (RCC) is a common flexural load taking element in the widely adopted RCC Framed Structures for buildings & in other structural systems. In design process structural cost reduction & thereby cost of every individual element (Beam, Columns, Slab & Footing) is again one of the most principle objectives in arriving at Techno-Economic-Feasibility of overall structure. Here in this paper it is shown that how the decision variables (b, D, Fck, Fy, Asprov, Mu, Vu etc) participating in design process of beam are interdependent and influences the Overall Objective – i.e. Minimizing Beam Element Total Cost (BE_TC).
There are many mathematical methods for optimization, researches done, with innovations and individual or combinations of optimization techniques utilized specific to Nature & type of optimization problem formulation decision variables, & constraints. (Ref [3], [4]) – However no single method is most appropriate or generalized. Here in this paper the method of optimization used is most primary & conventional that is exhaustive direct search – that is evaluating design and objective at each combination of discrete decision variables & arriving at least cost- BE_TC (but for a feasible design). Effectiveness in these optimization techniques is valid for the present purpose of research - the Type of Design & Objective problem formulation and limitation imposed due to many discontinuities inherent to the RCC Beam Design problem-Codal requirements & discrete nature of many decision variables.

Key Words – Decision Variables (b, D), Design Constraints, Section Design Type, Discrete Sizes, Objective Function (BE_TC), Feasible solution, Optimum Solution.

I. INTRODUCTION

RCC Beam Design involves - based upon the Preliminary Sizing, Elastic properties & subjected loads, - the calculation Design Forces from Analysis (**Mu**, **Vu etc**) & Displacements (**δ**). Thereupon evaluation of various **decision variables** such as formwork cross section sizes – **b**, **D**, grade of concrete & reinforcement steel material respectively - **Fck**, **Fy** respectively, Area of steel - **As** & its length, position & arrangement of reinforcement for various steel such – Longitudinal and Transverse Steel are made such that resultant strength & serviceability requirements are satisfied.

Design is a iterative process in which certain assumption regarding sizing and elastic properties are made, from which after analysis internal resultant forces are evaluated and design performed for strength and more final sizes – leading to further re-analysis and re-design. Iteration are stopped is said to be satisfactory if the Assumption in analysis / provision in design matches or converges- at least with Strength capacity on higher side than subjected internal forces & also satisfying serviceability require-ments. Indian Standards **IS:456** & others illustrates the various procedures for Analysis & Design process to meet Strength & serviceability requirements for satis-factory structural performance of design element.

1.1 Optimization – Scope & Methodology of Work

Here to the standard RCC beam codal design process we are associating further the evaluation of **Objective function Beam Element Total Cost** (**BE_TC**) values at each possibility of different discrete decision variables values (**b**, **D**, **Fck**, **Fy, Asprov**) – i.e. for numerous possible design combination. Therefore it is called **Exhaustive Search Method** of Optim-ization. As per standard codal design procedure if all design constraints are satisfied the design is feasible solution with certain objective function – cost value. Thereupon comparison of only the feasible solution's objective function values is done and the correspondingly the **least cost function design solution** is selected & called the **Optimum Design**.

Here **cost as optimization objective function** is so selected because it is **representation of material resources & labour efforts** that are consumed in construction and making of structural element. As far as engineer's role in design is to produce technically compliant design as well as a design which is accounts for the feasibility aspect , commercial impacts and discrete nature of construction material resources that is commercially available in market.

In this paper the scope is to cover cost optimization of general beam element at individual element level with following considerations:--

- That is a single beam member at a time which may be part of either a larger structural system of many elements (members) or stand alone beam.
- For overall structural optimization / individual frame or continuous beam optimization – here presented optimization techniques for single beam can become a basic elementary step – for each individual member optimization – & its contribution evaluations to overall structure optimization process.
- Overall structure – is assemblage of many beam column elements. Optimum Size evaluated through this optimization approach may be different than the previous size assumed in analysis while calculating Design forces. So change in size lead to change in stiffness and reanalysis / new design force / new optimum size is required to be studied in iterative manner upto convergence. Present scope of paper is limited to single beam design cycle optimization and not to study the effect of sizes alteration on stiffness / Design forces.
- It may be subjected to a given Design Force Envelope of – **Mu** , **Vu** etc (**B.M**.-Bending Moment & **S.F.-** Shear Force from Analysis results) due to several factored design load combinations with varying value of **Mu**, **Vu** along the length of beam. These forces are considered constant during optimization – it becomes preassigned parameter.
- Although there may other forces such as torsion, axial forces, other direction bending moment & shear acting on the beam in some cases – whose effects can be accounted in optimization design formulation but at present it is not considered for simplification(as well as other forces are subjected in lesser no of beam cases).
- **Unitary cost of Concrete - Cc** & Reinf. Steel Material – **Cs** and Formwork - **Cf** is user input, it influences significantly the optimization process. Unitary cost is function of Time, locality & Conventional practices. It becomes preassigned parameter in optimization cycle thereby optimization performed is directly related to Unitary cost values. Any alteration in same may alter the optimization results.
- Concrete Beam Section is considered Prismatic – Uniform **b** (width) & **D** (Depth) throughout length between two supports or one free end in case of cantilever beam, which is most practically & optimally adopted in design and construction.
- For varying **Mu**, **Vu** values along the length of beam – mostly adopted in design / construction is to keep constant **b** & D throughout length but reducing the weight of reinforcing steel by varying / curtailing the amount of Top and Bottom Longitudinal Steel and changing intensity / spacing of Transverse shear steel as per design requirement for Overall cost reduction.

- Although optimum b, D sizes for beam is governed by steel cost contribution as a function of both the following Design Cases:--

Des.Case 1) Design Zone-1No For given b, D the cost contribution of As - steel required as per Maximum Mu – $Mu_{MaxSagg}$ for sagging B.M. & $Mu_{MaxSagg}$ for Hogging B.M. - & Maximum Vu – Vu_{Max} for Shear, in total beam length L & Accordingly uniform Max – steel provision throughout length of beam.

Des.Case 2) Multiple Design Zone- (Normally 3 Nos**-** Near Left Support, Right Support, Middle Zone**)** For given same b, D the cost contribution of As - steel required through possible – feasible reduction in steel provisions for reduced Mu, Vu value along the various design section / zones along the total beam length – i.e. with reduced overall steel required & Steel cost contribution than Des.Case 1).

- Although optimum b, D obtained from Des.Case 2) is more near to the practical design but it evaluation needs performing design at more number of section / design zones- which to be decided by designer. And adding up the steel contribution for each design zone for given b, D (beam sizes) to get overall beam cost (Formwork and concrete cost remaining constant- the same as for Des.Case 1).
- For simplification of design– for evaluation of optimum b, D sizes, we have kept in this paper **single design zone** i.e **Des.Case 1)** and uniform steel provision throughout on basis of single critical max force ie. - $Mu_{MaxHagg}$, $Mu_{MaxSogg}$, Vu_{Max} (i.e. with different steel for top and bottom beam faces- for hogging and sagging moment & uniform shear steel provision).
- Optimum b, D Sizes obtained by procedure for Des.Case1) & Des.Case2) could be compared and b, D sizes through that of Des.Case2) would be marginally more near to practical design i.e. need based steel design provision and variation of steel along beam length. However Optimum b, D Sizes Des.Case1) & Des.Case2) – cost could not be because always Optimum Cost Des.Case1) > = Optimum Cost Des.Case2).

- **Section Design Type** for RCC flexural member can be either **Sec.Des.1) Singly Reinforced Design, Sec.Des.2) Doubly Reinforced.** In this paper Section Design for RCC flexural member with both options has been explored and results plotted & compared.
- Objective is to find Cost optimum **b**, **D**, **Fck**, **Fy** sizes.
- Chart and table are prepared for illustrating variation of optimum cost and feasibility for various b, D for prefixed Fck, Fy values & Design forces & Section Design Type(Singly or Doubly Reinforced).

1.2 Optimization Process of RCC Beam

Process of optimization problem formulation consists of identification & evaluation of numerous decision variables – independent or dependent, deciding its upper and lower bound values, participating Parameters, Variable's governing & binding relation – constraints, objective function evaluations, Min objective function value identification & corresponding Decision Variables for Optimum Design.

II. OPTIMIZATION PROBLEM FORMULATION OF RCC BEAM

As per standard procedure already established in Indian Standard – Plain & Reinforced Concrete Code of practice- **IS 456** and other associated Indian codes. Crucial Design Step in form of basic optimization problem typical design formulation are discussed in following paras.

Indices & Notations:

Wherever following indices are used in addition to the main notation it should mean as follows:--

- **Min** / **Max** : Minimum & Maximum values respectively.
- **top** / **bot**. : Top & Bottom Beam Face values respectively
- **reqd.** / **pr** (or **prov**) : required and provided values
- **Hogg** / **Sagg** : Corresponding to Hogging & Sagging Moment repectively.
- **Eff :** Effective, **Cov :** cover, **SF**: Side Face, **SFR** : side face reinf., d_b**:**Dia of Bar, **A** : Area.
- **B.M.:** Bending moment, **S.F.**: Shear Force
- **S.R.Des. / D.R. Des. :** Singly & Doubly reinforced section design.

a. Independent Decision Variables:

These are variables which need to finalized for optimum design and fixed to single value for current design evaluation in optimization process iteration cycle. In practical situation this takes discrete value. Ex- **b** , **D** takes discrete values based on limitation of commercially available formwork sizes, **Fck** , **Fy** - Takes discrete value based on commercial available material grades.

- **b** : Discr. Beam width (mm)
  (ex-200, 250, 300, 400 ..mm etc)
  (b_{LL} < = **b** < = b_{UL}, i.e **b**- Lower & Upper Bound Value)
- **D** : Discr. Beam Overall Depth (mm)
  (ex-300, 450, 600, 750 ..mm etc)
- (D_{LL} < = **D** < = D_{UL}, i.e **D**- Lower & Upper Bound Value)
- **Fck** : Material Grade of Concrete (N/mm^2).
  (ex- **Fck** 20, 25, 30 … N/mm^2 etc)
- **Fy** : Material Grade of Reinforcement Steel (N/mm^2).
  (ex- Fy 250, 415, 500 … N/mm^2 etc).

b. Preassigned Design Decision Parameters:

This are the parameter in individual optimization cycle which are relation wise dependent on current independent decision variables or otherwise independently Pre-assigned the values on the onset of each optimization cycle and are involved in design process along with decision variables in developing governing design relations – constraints & Objective function Evaluation.

i. Independent Preassigned Design Decision Parameters:

- L_{clr} :Clear Length of the beam between supports.
- **Design Loads From Analysis Results** :
  - ✓ $M_{uMaxHogg}$: Maximum Fact. Hogging. Bending Moment (KN.M) in total unsupported beam Length.
  - ✓ $M_{uMaxSagg}$: Maximum Fact. Sagging. Bending Moment (KN.M) in total Beam Length.
  - ✓ V_{uMax} : Maximum Factored Shear force (KN) in total unsupported beam Length.

  Refer Fig.1) - B.M. & S.F. Envelope
- **Section Design Type :** Singly Reinforced Or Doubly Reinforced
- **Design No Of Zone :** 1 (or 3), For present purpose uniform Design Provision for maximum forces done throughout the beam length, **Design No. of Zone :1**
- **Es**, **ρs** : Elastic Modulus & Density of Steel.
- pA_{smax} : 4% or Even lesser User Defined (Max. Percentage Steel)
- **Based on Engineering Judgement :**

- ✓ Min & Max.dia of Stirrup, Longitudinal Bars –Designer defined as well codal restriction whichever is strictest.
- ✓ Permitted **d_{eff} / D_min** ratio (=0.8), Permitted **d'c/D_max** ratio (=0.2) for effective utilization of provided steel.
- ✓ Exposure Type : Mild, Severe etc for Nominal Clear Cover fixing.

ii. Dependent Preassigned Design Decision Parameters:

- **C_C, C_S, C_F :** Unitary Rate of Concrete, Formwork, Reinforcement Steel respectively – with Material supply, Labour, Fixing & placement all inclusive.

Table -1 Unitary Rates

Concrete Unit Rates		**Reinf. Steel Unit Rates**	
f_{ck}	**C_C (f_{ck})**	**f_y**	**C_S (f_y)**
(N/mm^2)	(Rs. / m^3)	(N/mm^2)	(Rs. / ton)
20	5450	250	57000
25	5800	415	62500
30	6000	500	64000
35	6150	**Formwork Unit Rates**	
40	6300	**(Beam)**	(Rs. / m^2)
		C_F =	315

- **X_{umax}/d_{eff}** : Neutral Axis ratio as per steel grade.
- **Q** : Limiting Moment of resistance Factor.
  =0.36* (X_{umax} /d_{eff})*(1-0.42*Xumax/d_{eff})*Fck
- **pA_{smin}** : 0.85/ Fy *100 (Min. Percentage Steel)
- **NC** : Nominal Cover depending on exposure.

c.Dependent Design Decision Variables

These are strict equality constraint (relations) or at time based on combination of Strict Equality and and inequality relation as prescribed in standard flexural design by IS 456. General Notation are presented here however evaluation of both Face (top and bottom) and both type of moment (hogging & sagging need to be done).

- **Eff_{covtop} or $d'c_{Sagg}$: D - NC – $d_b$stirrup –** C.G. dist of current assumed provided area of Top Steel to stirrup inner face.
- **Eff_{covbot} or $d'c_{Hogg}$: D - NC – $d_b$stirrup –** Centroid distance of current assumed provided area of Bot. Steel Layers o stirrup inner face. However above value restricted max to **0.2*D** and min value as per min effective cover.
- **$d_{eff\ hogg\ /\ sagg}$ = D- $Eff_{cov\ top\ /\ bot}$** for Hogging & Sagging moment respectively. However above value restricted min. to **0.8*D** & max. value as per min effective cover.
- **A_{smin}** : **pA_{smin}** * **b** ***d_{eff}** /100 (Min. Area of Steel)
- **A_{smax}** : **pA_{smax}** * **b** ***D** /100 (Max. Area of Steel)
- **M_{ulim}** : **Q** * **b** * **d_{eff}^2** (Limiting Moment)

➢ **Singly reinforced design** :

- If **M_{uMax} <= M_{ulim} ,** Then Tension Steel Area reqd
  A_{streqd} = 0.5 *Fck/Fy*[1 – Sqrt (1-4.6*M_{uMax} / (Fck*b*d_{eff}^2))] *b*d_{eff}

Otherwise, Failed in singly reinforced design. Go For

➢ **Doubly reinforced design :**

- If **M_{uMax} > M_{ulim} ,** Then Tension Steel Area reqd
- **A_{streqd} = A_{stlim} (i.e A_{st1}) + A_{st2}**
- **A_{stlim} = 0.5 *Fck/Fy*[1 – Sqrt (1-4.6*M_{ulim} / (Fck*b*d_{eff}^2))] *b*d_{eff}**
- **A_{st2} = A_{sc} * F_{sc} / (0.87 * Fy)**
- **A_{sc} = (M_{uMax} - M_{ulim}) / [F_{sc} * (d_{eff} – d'c)]**
- **Fsc =** Compression steel stress corresponding to strain – 0.0035 *(Xumax – d'c) / Xumax.

➢ **Shear Design :**

- **τ_v = V_{uMax} / b d_{eff}** (Nominal shear Stress)
- **τ_c** = Depending upon **A_{stprov}** (Area of Steel Tension provided) & Grade of Concrete Fck.

- **$Vus = V_{uMax} - \tau_c * b * d_{eff}$** Transverse Steel Shear resistance Required.
- **$Vus\_pr = 0.87 * Fy * A_{sv\_pr} * d_{eff} / S_{v\_pr}$**
- **$(A_{sv\_reqd} / S_{v\_reqd})_{min} >= 0.4 * / (0.87*Fy)$**

  Where A_{sv} & S_v are shear steel area & spacing.
- **$Vuc = \tau_c * b * d_{eff}$** (Concrete Shear Resistance)
- **$V_{dmax} = Min\ of\ (V_{umaxconc}, Vus\_pr + Vuc)$,** Total Shear Resistance.
- **$V_{umaxconc} = \tau_{cmax} * b * d_{eff}$**, where τ_{cmax} = Max concrete shear stress, depends on Fck.

➢ **Optimum Discrete Steel provision :--**

For evaluation of steel to be provided in terms of discrete integer no of commercially available dia of Rebars, preferred spacing & no of layers etc for longitudinal as well as transverse steel – a elaborate list of steel rebar combination with same above details is prepared covering all range of areas, spacing, dias of rebars,. Of this rebar combination list exhaustive search. – i.e. checks are made for each combinations for their suitability compared to **As** required and the one with minimum weight satisfying all design strength, spacing, effective cover requirement is selected as **As** provided .

d. Design Constraints:

These inequality constraints are evaluated in terms of relation between previous dependent, independent decision variables & Preassigned parameters. Any violation in constraints make that set of independent decision variable as a infeasible solution otherwise objective function value is evaluated & corresponding set of independent and dependent decision variable forms feasible solution. Inequality constraints list imposed in present design problem is as below:--

A) Bending Strength Related Constraints :

1) $M_{uMaxHogg} <= MOR_{Hogg\_pr}$
2) $M_{uMaxSagg} <= MOR_{Sagg\_pr}$
3) $V_{uMax} <= V_{dmax}$

Where MOR_{Hogg_pr} / MOR_{Sagg_pr} are hogging and sagging Moment of resistance evalauted on the basis of provided steel area As_{prov} and corresponding effective depth and cover.

B) Top Steel Constraints :

4) $As_{topreqd} <= As_{topprov}$
5) $As_{topprov} <= As_{max}$
6) $As_{min} <= As_{topprov}$
7) $d_{effhogg} <= d_{effhogg\_pr}$

C) Bottom Steel Constraints :

8) $As_{botreqd} <= As_{botprov}$
9) $As_{botprov} <= As_{max}$
10) $As_{min} <= As_{botprov}$
11) $d_{effsagg} <= d_{effsagg\_pr}$

D) Side Face Steel Constraints :

12) $A_{S\_SF\_reqd} <= A_{S\_SF\_prov}$
13) $SFR_{distprov} <= SFR_{distmax}$

E) Transverse Steel Constraints :

14) $A_{sv/sv\_reqd} <= A_{sv/sv\_prov}$
15) $A_{sv/sv\_minreqd} <= A_{sv/sv\_prov}$

F) Upper & Lower Bound Constr. on Beam Sizes:

16) $b <= b_{UL}$
17) $b_{LL} <= b$
18) $D <= D_{UL}$
19) $D_{LL} <= D$

e.Objective Function:

It can be stated as Total Beam Element Cost (**BE_TC**) i.e. Total of all cost components (Concrete Reinf.Steel & Formwork) :--

- **BE_TC = BE_CC + BE_FC + BE_RC**

➢ Concrete Cost
- **$BE\_CC : BE\_C_{VOL} * C_C$,**
- **$C_{VOL} = b * D * L_{clr}$** (Concrete Volume)

➢ Formwork Cost
- **$BE\_FC : BE\_FA * C_F$,**
- **$BE\_FA = [\ 2 * D + B\ ] * L_{clr}$** (Formwork Area)

➢ Reinforcement Cost
- **$BE\_RC : BE\_Rwt * C_S$**
- **BE_Rwt : LRwt + Tr.Rwt** (Reinf Weight), where **LRwt & Tr.Rwt** Longitudinal & Transverse Reinf Steel Weight.
- **LRwt : TLRwt + BLRwt + SFLRwt,**

  Where **TLRwt, BLRwt, SFLRwt** are Top, Bottom & Side Face Long. Reinf. Steel wt. respectively.

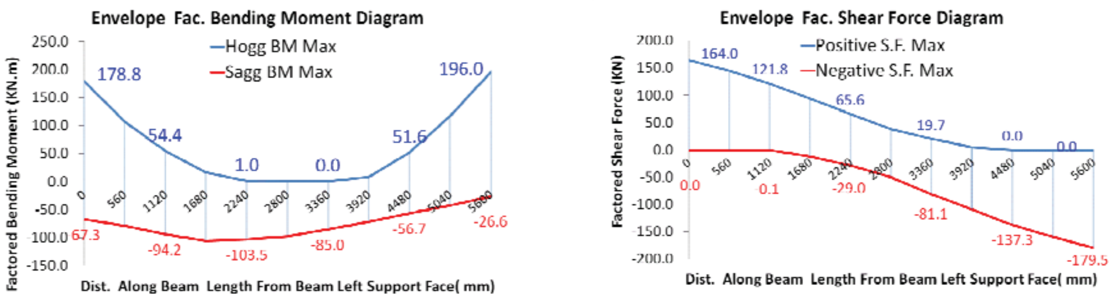
Where reinforcement weight (**Rwt**) for each category of steel is obtained by multiplying Length of Reinf. Steel x Area of Bar x **ρs** on the basis of steel provided.

Item discussed in Para 2.1 to 2.5 cover single basic design cycle level, where discrete nature steel requirements by exhaustive search are evaluated for feasible optimum (Minimum Weight). For optimization we have extended it for exhaustive direct search method of optimization for different value of Major Independent Decision Variables (b, D, fck, Fy) - combination sets, by performing each variable set single basic iteration complete beam design for optimum reinf. steel. Then comparing for overall economical objective function value (BE_TC) and thereby arriving at its corresponding Independent Design Decision Variables value for optimum design.

III. OPTIMIZATION EXAMPLE PROBLEM

Below is a B.M. And S.F.Analysis result of a beam design problem for which optimization is done by exhaustive search method.

Fig.1):Factored B.M. & S.F. Force Envelope Diag



$Mu_{MaxHogg}$: 196 KN.m, **$Mu_{MaxSagg}$** : 103.5 KN.m , V_{umax}: 179.5KN, Lclr :5.6 m

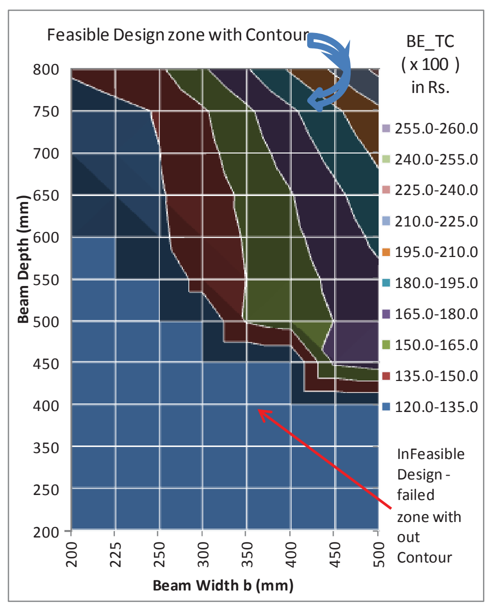
➢

➢Results of Singly Reinforced Design Optimization Example Problem

Table.2 : Cost of Singly reinforced Design for Optimization Example Problem.

Discr. D (mm) ↓	Beam Element Total Cost (BE_TC) in (x 100) Rs :-- Shaded Value are Failed in Design (Infeas.) Sizes.								Corr. Optm. b ↓
800	137.8	144.0	147.5	163.0	178.8	197.1	212.9	229.2	200
750	124.4	130.4	136.1	148.1	162.3	177.6	192.5	207.2	200
700	124.3	129.8	134.8	145.3	157.7	171.0	184.9	198.2	200
650	122.8	128.2	133.4	142.3	152.9	164.2	176.8	189.9	225
600	119.9	127.7	132.3	142.7	151.1	161.8	172.1	183.5	250
550	113.8	122.1	129.1	141.5	150.1	158.9	168.1	176.7	300
500	111.0	118.9	124.8	137.5	150.9	157.6	165.3	173.9	350
450	103.4	111.4	119.9	132.1	144.4	155.9	168.1	173.4	450
400	94.9	102.9	111.3	127.2	138.6	151.1	159.0	172.8	Infeas.
350	87.9	94.3	100.9	115.9	128.6	144.5	154.5	162.4	Infeas.
250	71.0	77.0	83.6	91.4	103.0	112.1	123.3	134.4	Infeas.
200	58.4	65.2	68.7	78.9	89.1	99.3	109.5	119.0	Infeas.
b ⇨	200	225	250	300	350	400	450	500	
Optm.D (mm) ⇨	700	650	600	550	550	500	500	450	

Fig.2 : Contour Plot of Cost for Singly Reinf. Optimization Example Problem.



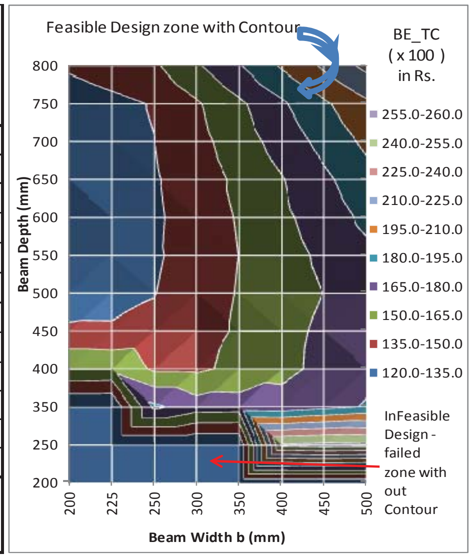
Above Table 2) Result Summary : **Fck** : 20 N/mm^2, **Fy** = 415 N/mm^2, **b optm.** = 200 mm, **D optm.** = 700 mm

➢**Results of Doubly Reinforced Design : Optimization Example Problem**

Table.3 : Cost of Doubly reinforced Design for Optimization Example Problem.

Discr. D (mm) ↓	Beam Element Total Cost (BE_TC) in (x 100) Rs								Corr. Optm. b ↓
	[Dotted Cell] :-- Dotted Cell - S. R. Des. Governs, For Others Bold Font Below Seperator Line - D. R. Des. Governs.								
	[Shaded Cell] :-- Shaded Value are Cost of Failed in Design Sizes.								
800	137.8	144.0	147.5	163.0	178.8	197.1	212.9	229.2	200
750	124.4	130.4	136.1	148.1	162.3	177.6	192.5	207.2	200
700	124.3	129.8	134.8	145.3	157.7	171.0	184.9	198.2	200
650	125.2	128.2	133.4	142.3	152.9	164.2	176.8	189.9	200
600	123.7	129.1	132.3	142.7	151.1	161.8	172.1	183.5	200
550	124.9	129.6	132.9	141.5	150.1	158.9	168.1	176.7	200
500	128.9	130.4	134.4	142.1	150.9	157.6	165.3	173.9	200
450	136.9	136.7	139.0	142.1	152.3	159.9	168.1	173.4	250
400	164.1	162.5	147.6	147.4	153.9	161.3	168.5	176.7	300
350	*198.2*	*196.7*	183.0	171.8	171.3	170.0	175.8	181.0	400
250	*159.4*	*176.6*	*190.3*	*225.9*	*258.1*	257.7	261.7	258.7	400
200	*129.6*	*143.6*	*157.6*	*185.6*	*214.3*	*238.2*	*268.8*	*291.3*	Infeas.
b ˇ	200	225	250	300	350	400	450	500	
Optm.D (mm) ˇ	700	650	600	550	550	500	500	450	

Fig.3 : Contour Plot of Cost for Doubly Reinf.Optimization Example Problem.



Above Table 3) Result Summary : **Fck** : 20 N/mm², **Fy** = 415 N/mm², **b optm.** = 200 mm, **D optm.** = 600 mm

It can be observed that more portion of beam sizes become feasible in Doubly Reinforcement design [Table-3), Fig.3)] more sizes are however optimum cost remain nearly same

Table-4) Results Tabulation For Cost Optimum b, D - For Variation in other Independent Decision Variable - Fy, Fck , Section Design Type - S.R.Des/D.R.Des. (For Mumaxhogg = 196Kn.m , Mumaxsagg = 103.5 Kn.m, Vumax = 179.5 Kn)

Section Design Type	Fy (N/ mm²)	Fck (N/ mm²)	Optim. Design Case No.	Optimum Discrete sizes b (mm)	Optimum Discrete sizes D (mm)	Cross Sec. Area (x 10⁴) mm²	Lim. Mom. & As Mulim Kn.m	Lim. Mom. & As ptlim %	Optm Des- (S.R / D.R.)	Prov.Reinf. % Area pAstop %	Prov.Reinf. % Area pAsbot %	Perc. Cost Conc. %	Perc. Cost F/W %	Perc. Cost Reinf %	Total Cost BE_TC x 10² Rs.
Allowed Singly. Reinf. Design Only	250	20	A.1.1.1	***200***	***750***	1500	282	1.75	S.R.Des.	1.12	0.55	30.7	16.6	52.7	**148.9**
		30	A.1.1.2	***200***	***750***	1500	409	2.63	S.R.Des.	1.07	0.53	32.8	16.1	51.1	**153.6**
		40	A.1.1.3	***200***	***750***	1500	552	3.51	S.R.Des.	1.04	0.52	33.9	15.8	50.2	**156.0**
	415	20	A.1.2.1	***200***	***700***	1400	224	0.95	S.R.Des.	0.80	0.38	34.4	18.4	47.2	**124.3**
		30	A.1.2.2	***200***	***600***	1200	241	1.43	S.R.Des.	1.12	0.52	32.0	15.4	52.5	**125.8**
		40	A.1.2.3	***200***	***650***	1300	389	1.90	S.R.Des.	0.85	0.42	36.1	16.7	47.2	**127.0**
	500	20	A.1.3.1	***200***	***700***	1400	223	0.75	S.R.Des.	0.64	0.30	37.0	19.9	43.2	**115.5**
		30	A.1.3.2	***200***	***650***	1300	278	1.13	S.R.Des.	0.74	0.37	37.3	18.1	44.6	**117.1**
		40	A.1.3.3	***200***	***600***	1200	306	1.50	S.R.Des.	0.89	0.42	36.0	16.5	47.4	**117.4**
Allowed Singly. Or Doubly Reinf. Des. as Reqd. for Cost optim.- b,D	250	20	A.2.1.1	***200***	***750***	1500	282	1.75	S.R.Des.	1.12	0.55	30.7	16.6	52.7	**148.9**
		30	A.2.1.2	***200***	***750***	1500	409	2.63	S.R.Des.	1.07	0.53	32.8	16.1	51.1	**153.6**
		40	A.2.1.3	***200***	***750***	1500	552	3.51	S.R.Des.	1.04	0.52	33.9	15.8	50.2	**156.0**
	415	20	A.2.2.1	***200***	***600***	1200	158	0.95	D.R.Des.	1.15	0.54	29.6	15.7	54.7	**123.7**
		30	A.2.2.2	***200***	***600***	1200	237	1.43	S.R.Des.	1.12	0.52	32.0	15.4	52.5	**125.8**
		40	A.2.2.3	***200***	***650***	1300	389	1.90	S.R.Des.	0.85	0.42	36.1	16.7	47.2	**127.0**
	500	20	A.2.3.1	***200***	***550***	1100	128	0.75	D.R.Des.	1.14	0.56	29.6	15.6	54.8	**113.4**
		30	A.2.3.2	***200***	***650***	1300	278	1.13	S.R.Des.	0.74	0.37	37.3	18.1	44.6	**117.1**
		40	A.2.3.3	***200***	***600***	1200	306	1.50	S.R.Des.	0.89	0.42	36.0	16.5	47.4	**117.4**

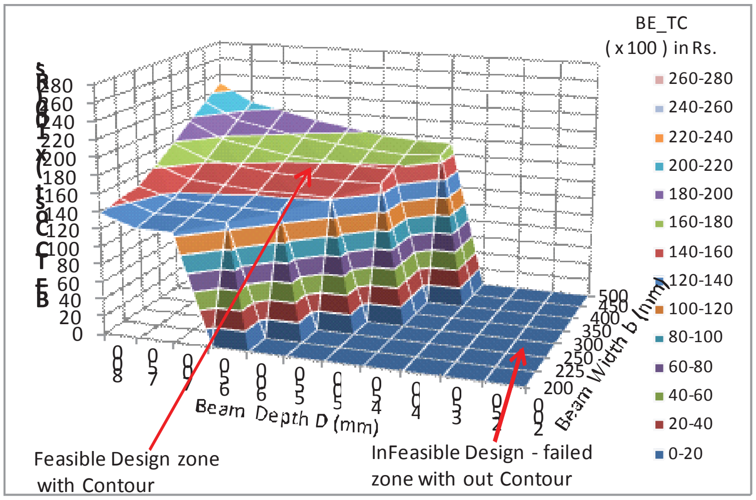


Fig. 4) 3D Surface Countour plot of Cost versus – b & D For Singly reinforced section

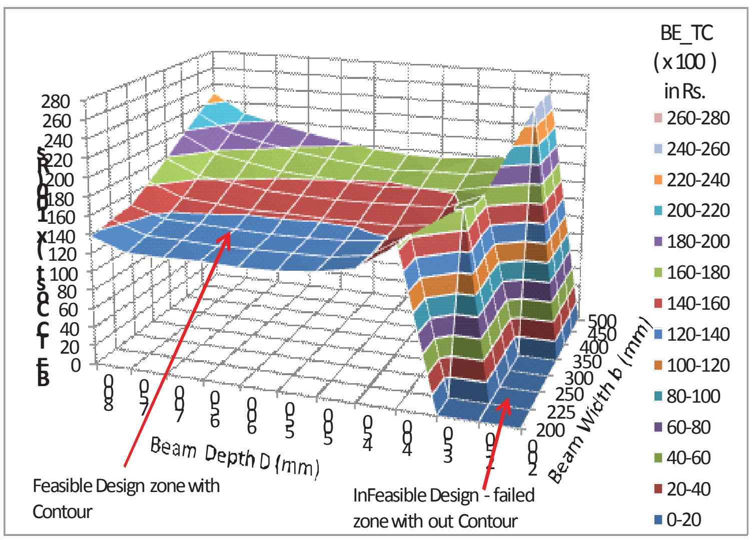


Fig. 5) 3D Surface Countour plot of Cost versus – b & D For Doubly reinforced section

IV.CONCLUSION

Referring Table 4):

➢ **Influence of Reinforcement Grade on optimum cost**. Least cost feasible optimum design is achieved best in Grade Fy500 as compared to Fy 415 and similarly Fy415 designs more economical than Fy 250. reinf steel.- which is irrespective of type of design whether it is doubly reinforced or singly reinforced. Reason being high strength increase (with increase in Fy Grade-reduced steel wt requirements) compared to relatively lesser increase in corresponding unitary cost of steel (Rs /ton) of that reinforcing steel grade.

➢ **Influence of Concrete Grade**:- As the concrete grade increase within a prefixed Reinf grade than the optimum cost design is becoming marginally costly irrespective of S.R. or D.R. Design.

➢ **Influence of Singly or Doubly Reinforced Design**:- For similar concrete grade and similar reinf grade the optimum cost by either singly or doubly allowed design type – no changes in respective optimum cost is observed. Mostly the optimum section size is the size that is governed by singly reinforced design (provided if their no size restrictions). If at all the optimum cost is governed by Doubly reinforced design still the optimum cost of singly and that of doubly reinforced section is nearly matching- only variation is in corresponding optimum b,D sizes.

- **Concrete / Formwork/ Reinforcement Ratio for optimum Design :--**

It varies as Approx. 30 to 35% for concrete, 15 to 20 % for Formwork and 45 to 55% for Reinforcement cost of the Total cost unless there is size restriction

- **Sec.Des.1) Singly Reinforced Design:** It lead to Higher b, D requirement reflected in increased cost contribution in concrete & Formwork cost & lesser reinf. Steel cost because tension steel restricted to $\mathbf{A_{slim}}$ however with no utilization in design of opposite face steel as compression steel.
- **Sec.Des.2) Doubly Reinforced Design** : It leads to lesser, b, D requirement, more percentage of tension steel required than $\mathbf{A_{slim}}$ – If section doesn't passes in singly reinforced design in current b, D considered & accordingly same reflected in Total beam cost as reduced cost contribution in concrete & Formwork cost & increased Tension reinf. steel cost however with Utilization of opposite face steel in compression.

- Optimum Cost obtained by procedure for **Sec.Des.1)** & **Sec.Des.2)** cases (Singly /Doubly Reinf Design) could be compared and cost through that of Des.Case2) would be marginally more near to practical design i.e. Doubly Reinf design provision wherever required due to size limitation and utilizing compression steel. However b, D sizes corresponding to the Optimum Cost in Sec.Des.Case1) & Sec.Des.Case2) – could not be compared because always Optimum b, D Through SecDes.1) > Optimum b, D SecDes.2) case.

- Similar Study could extended to see the effects of optimum design for ductile beam element design with applicable design constraints and its effect in optimum b,D or Fck, Fy.

Referring Fig2)/ Fig3) & Table 2) / 3):

- **No. of Optimum in the problem** Since we have plotted over the entire range of allowable discrete sizes b, D –we are able to know the global optimum as well as Local optimum values (Local optimum in case when –we fixed any one of the b or D value and find the other optimum size (i.e D or b respectively)- refer Table 2) / 3).

- Looking to contour plots (Fig.-2,3) we are able to see that there in well defined patterns of cost contours and least cost contour is getting confined to small range of discrete sizes –majority of times a single b,D value- i.e. Global optimum size. Its means majority of beam optimization problem even in discrete nature of variables is a convex problem with single local optima – which is even the global optima.

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