Finite Element Analysis of Piezoelectric Cantilever

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Abstract- Energy (or power) harvesting or (scavenging) is a very attractive technique for a wide variety of self-powered micro-systems. By scavenging energy from the environment, miniature sensing/ actuating devices can be self-powered in order to avoid the replacement of finite power sources. One approach to harvest energy is to convert mechanical energy of ambient vibration into electrical energy by electro- magnetic induction or piezoelectric effect. Present work aims at modeling and analysis of piezoelectric-based vibration-extraction devices using Matlab and Ansys software. Static and dynamic analysis of piezoelectric cantilever is carried out in Ansys.

Keywords- Harvesting, Piezoelectric, Vibration, Cantilever.

I.INTRODUCTION

The rapidly decreasing size and power consumption of sensors and electronics has opened up the relatively new research field of energy harvesting. Present working solutions for vibration-to-electricity conversion are based on oscillating mechanical elements that convert kinetic energy into electric energy via capacitive, inductive or piezoelectric methods. Cantilever piezoelectric power generators are being used because of their high strain and high power output even under lower acceleration amplitudes. Piezoelectric ceramics have been used in many applications to convert mechanical energy into electrical energy. The direct piezoelectric effect was early demonstrated by Jacques and Pierre Curie in 1880. They found that when certain ceramic crystals were subjected to mechanical strain, they became electrically polarized and the degree of polarization was proportional to the applied strain. Conversely, these materials deformed when exposed to an electric field as shown in Figure 1.

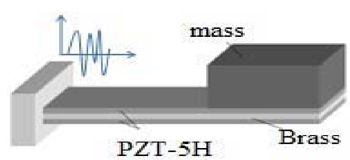


Figure 1. Basic principle of Piezoelectric Generator

Present work aims at, static analysis of piezoelectric bimorph beam to determine deflection, using Ansys, Modal analysis of piezoelectric beam to determine the mode shapes & their corresponding modal frequencies and dynamic analysis (modal and harmonic) of the array of cantilever resonator.

II. STATIC ANALYSIS OF PIEZOELECTRIC BIMORPH BEAM

Piezoelectric bimorph beam is composed of two piezoelectric layers joined together with opposite polarities and is widely used for actuation and sensing. In the actuation mode, on the application of an electric field across the beam thickness, one layer contracts while the other expands. This results in the bending of the entire structure and tip deflection. In the sensing mode, the bimorph is used to measure an external load by monitoring the piezoelectrically induced electrode voltages. Figure 2, shows a 2-D analysis of a bimorph mounted as a cantilever. The top surface has ten identical electrode patches and the bottom surface is grounded.

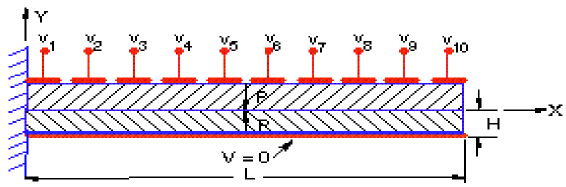


Figure 2.Piezoelectric Bimorph Beam

Specifications:

The bimorph material is Polyvinylidene Fluoride (PVDF) with the following properties:
Young's modulus (E1) = 2.0e9 N/m^2 Poisson's ratio (ν12) = 0.29
Shear modulus (G12) = 0.775e9 N/m^2
Piezoelectric strain coefficients (d_{31}) = 2.2e-11 C/N, (d_{32}) = 0.3e-11 C/N, and (d_{33}) = - 3.0e-11 C/N
Relative permittivity at constant stress $(\varepsilon_{33})^T = 1$
The geometric properties are:
Beam length (L) = 100 mm Layer thickness (H) = 0.5 mm

Actuator mode

For applied voltage of 100 volts along the top surface, deflection is determined. The deflection obtained from the theoretical solution is given by formula,

$$U_y = \frac{-3(d_{31})VL^2}{8H^2}$$

Substituting, d_{31}= 2.2 e-11C/N, V= 100 V, L = 0.1 m, H= 0.5e-3 m, in the above equation,

$$U_y = -33\mu m.$$

Figure 3.shows the Ansys result of piezoelectric bimorph beam for actuator mode.

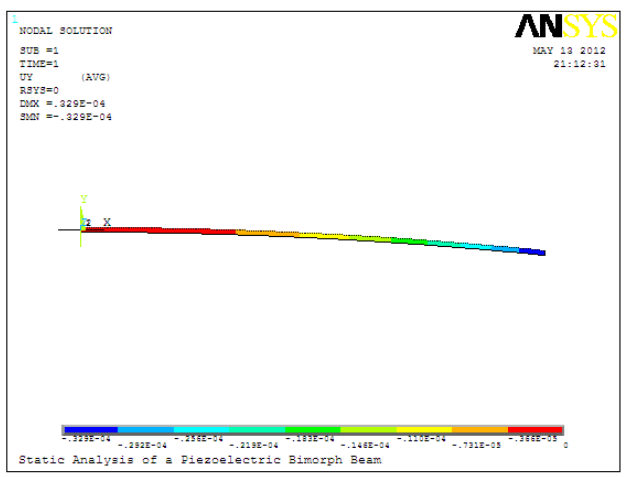


Figure 3.Static analysis of piezoelectric bimorph beam for actuator mode

From Ansys, deflection of -32.9 µm is obtained for 100 Volts.

Sensor mode

For an applied beam tip deflection of 10mm, electrode voltages along the beam are determined. Figure 4.shows the static analysis of piezoelectric bimorph beam for the sensor mode.

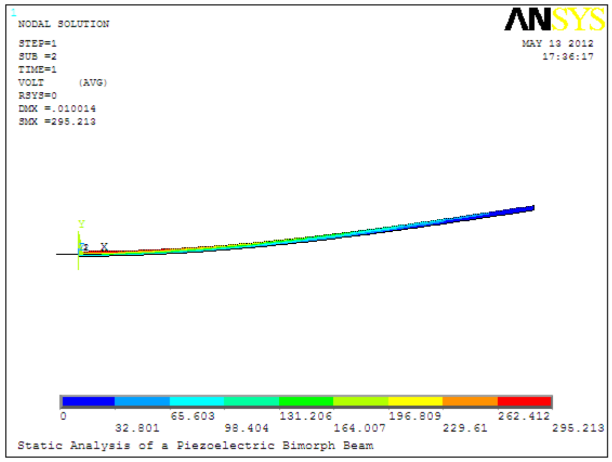


Figure 4.Static analysis of piezoelectric bimorph beam for sensor mode.

From Ansys, the voltages along the piezoelectric bimorph beam are shown in Table 1.

Table 1 Voltage along the piezoelectric bimorph beam at electrodes

Electrode	1	2	3	4	5	6	7	8	9	10
Voltage (Volts)	295.2	262.41	229.61	146.80	164	131.20	98.40	65.60	32.80	0

III. Dynamic Analysis of Piezoelectric Material

Dynamic analysis predicts variation of displacement, velocity, acceleration etc with respect to time/frequency. It is a basic design property for determining the natural frequency of component .Piezoelectric cantilever beam is assumed as continuous system and is modeled accordingly for modal analysis.

Thus the first three natural frequencies are as follows,

First natural frequency is,
$$\omega_1 = (1.875)^2 \sqrt{\frac{EI}{\rho A l^4}} \tag{1}$$

Second natural frequency is,
$$\omega_2 = (4.694)^2 \sqrt{\frac{EI}{\rho A l^4}} \tag{2}$$

Third natural frequency is,
$$\omega_3 = (7.855)^2 \sqrt{\frac{EI}{\rho A l^4}} \tag{3}$$

Specifications:

Young's Modulus E = 2GPa Density of PVDF = 1780kg/m^3

Breadth of beam B = 20mm Thickness T = 5mm, Length of beam, L = 100mm

Area, A= B*T = 100mm^2 = 0.0001m^2

Moment of Inertia, I = $(B*T^3)/12 = (20*5^3)/12 = (2.0833*10^{-10})$ m^4

First, second and third natural frequencies are obtained by substituting the above values in equation (1), (2) & (3) we get

$$\omega_1 = 85.591 \text{ Hz}$$
$$\omega_2 = 536.43 \text{ Hz}$$
$$\omega_3 = 1502.17 \text{ Hz}$$

Table 2 Comparsion of natural frequencies

Mode Shape	Analytical Frequency (Hz)	Frequency (Ansys) Hz	Error (%)
1	85.591	86.663	1.236
2	536.43	537.27	0.156

3	1502.17	1482.2	1.329

Maximum displacement is 15.096 mm for frequency 1502.17 Hz with error of 1.329%.
Figure 5 shows the mode shapes of piezoelectric cantilever beam.

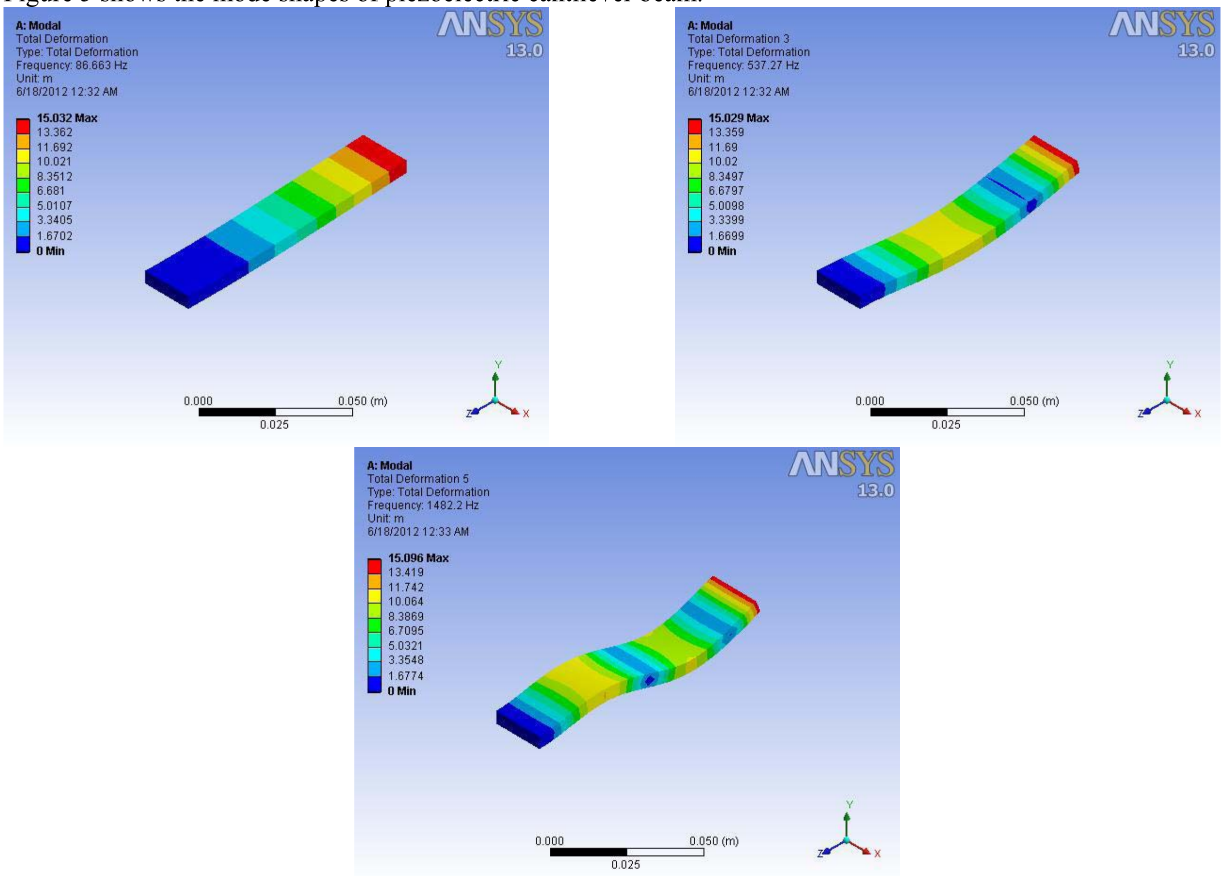


Figure 5 Three Mode shapes of piezoelectric cantilever beam

IV. MODAL ANALYSIS OF ARRAY OF CANTILEVER RESONATOR

For generating power, determination of natural frequency is important. So array of cantilever is arranged in rectangle to determine mode shapes and natural frequency.

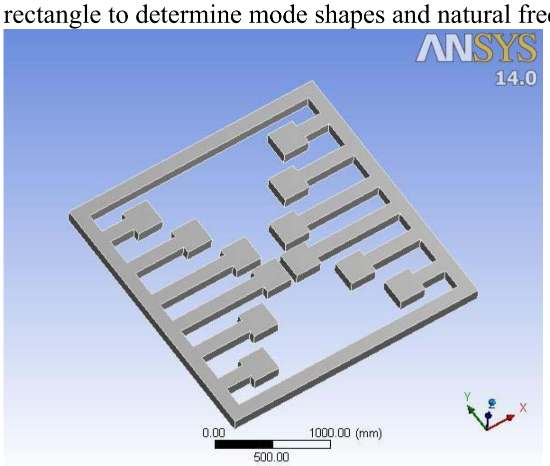


Figure 6 Model of array of cantilever resonator

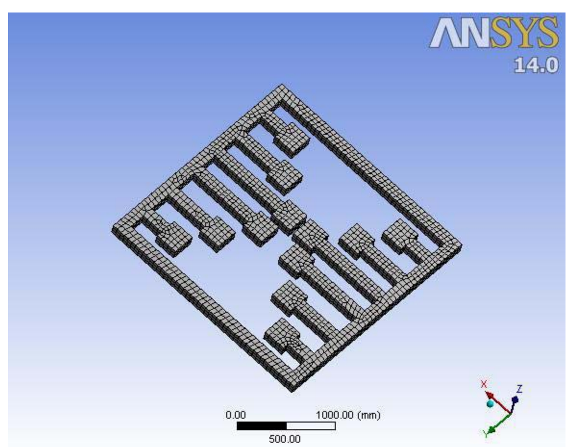
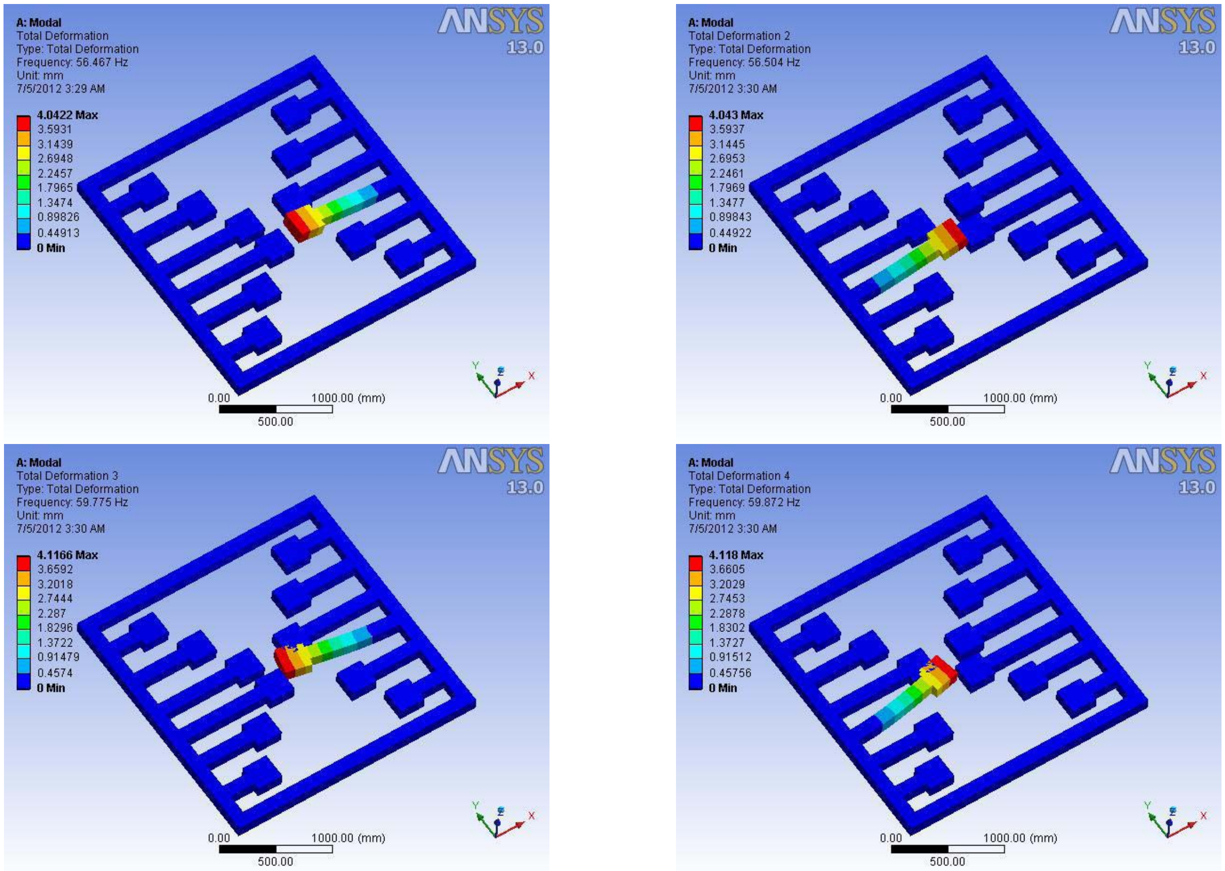


Figure 7 Meshing of array of Cantilever

After meshing, modal analysis is carried out and the first four natural frequencies are obtained.



Further such type of arrangement can be incorporated in vibration devices.

V. HARMONIC ANALYSIS OF ARRAY OF CANTILEVER RESONATOR

Harmonic response analysis is a technique used to determine the steady-state response of a linear structure to loads that vary sinusoidally (harmonically) with time. The idea is to calculate the structure's response at several frequencies and obtain a graph of some response quantity (usually displacements) versus frequency. Peak responses are then identified on the graph and stresses reviewed at those peak frequencies. Figure 8 shows harmonic analysis array of cantilever arranged in rectangle. Maximum displacement obtained is 1.9745mm for a frequency of 250 Hz.

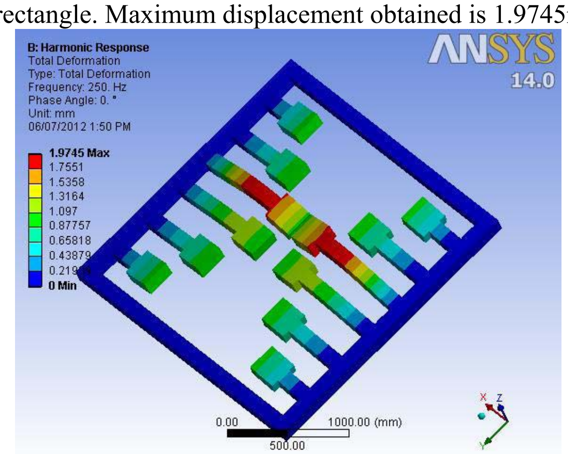


Figure 8 Harmonic analysis of array of cantilever

V. CONCLUSION

Static and dynamic analysis of piezoelectric cantilever is carried out in Ansys. Power generation is mainly dependent on system natural frequency and operating frequency. When these two frequencies are equal resonance occurs and at resonance large amount of power is generated. Static analysis of piezoelectric bimorph beam resulted in deflection of -33µm. Dynamic analysis of piezoelectric beam is carried. Modal analysis showed that the comparison of natural frequencies obtained from analytical and Ansys are having a close match. The maximum displacement obtained is 15.096 mm for maximum frequency of 1502.17 Hz with error of 1.329 %.
Thus the array of piezoelectric cantilever when attached to vibrating structure, vibrations occurs which deflects piezoelectric cantilever and results in generation of power.

REFERENCES

[1] Singiresu. S.Rao, Mechanical Vibrations, 4th edition, 2000, pp 639-642.
[2] Shashank Priya. Daniel J. Inman, Energy Harvesting Technologies, 2nd 2009, pp 35.
[3] N.G. Stephen, On energy harvesting from ambient vibration, Journal of Sound and Vibration, vol.293, 2006, pp.409–425.
[4] S. Kim, W. W. Clark, and Q.-M. Wang, "Piezoelectric energy harvesting with a clamped circular plate: Analysis, " Journal of Intelligent Material Systems and Structures, vol. 16, 2005, pp. 847-854.
[5] S. Roundy, "On the effectiveness of vibration-based energy harvesting," Journal of Intelligent Material Systems and Structures, vol. 16, 2005, pp. 809-823.