

# Piezoelectric Generators for Vibrational Energy Harvesting

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**Abstract - In past times, energy harvesting used piezoelectric materials was very popular for new research. The devices shapes sizes and dimensions was tested, but was difficult to match power measurement as a device fabrication and other method which vary from paper to paper. In an standard comparison by changing the parameters , the dependence of on device dimension has been investigated. The devices like MEMS scale have been measured little work has been using aluminium nitride. (PZT). The project shows and utilises aim due to its ease in processing and potential on chip integration . By using at this at MEMS scale the advantage is that array of piezo generators can be placed on the same die. With the use of aim a large goal of integrated power harvesting chips become achievable. The theoretical result of its scaling shows that power output and even power per unit volume will be decrease with scaling. This shows that the generator takes up the almost same area as other small generators would produce a noticeably a large power output.**

From the conclusion the result of this experiment were not most favourable. The problem in measuring the electrical response of the device lie beam shows the design involved with energy harvesting on a small scale. Piezoelectric generators rely on resonance to produce useful quantity , and power output is very sensitive to the frequency of the type of vibration applied. Although this type of generators are useful if targeted to specific application if the frequency of natural vibration is known. A more versatile approach could use other design to decrease the frequency sensitivity. Broad band design used either non resonant or self tuning structures could be able to harvest energy more easily in changing environment.

**keywords: PZT, Energy harvestors.**

## I. INTRODUCTION

Low-power wireless distributed sensor networks are becoming attractive for monitoring different variables - such as temperature, strain in a material, or air pressure - over a wide area. However, one drawback of these networks is the power each node draws, though recent work has shown this can be lowered considerably [1]. Batteries can be used to power nodes for extended periods of time, but they have a limited life cycle and eventually need to be replaced. As this can be a costly and time consuming procedure for networks with many nodes, a means of powering the devices indefinitely would be a more practical solution.

Solar power provides a considerable amount of energy per area and volume, but unfortunately is limited to applications that are reliably sunlit [2]. A promising alternative takes advantage of the energy in ambient vibrations and converts it to electrical power. This approach compares very favorably with batteries, providing equal or greater power per unit volume.

There are multiple techniques for converting vibrational energy to electrical energy. The most prevalent three are electrostatic, electromagnetic, and piezoelectric conversion [3]. A majority of current research has been done on piezoelectric conversion due to the low complexity of its analysis and fabrication. Most research, however, has targeted a specific device scale [4-7]. Little research comparing power output across different scales has been done for piezo harvesters, though scaling effects have been discussed briefly in some works [4,8].

This paper aims to develop a theoretical understanding behind the scaling of piezoelectric cantilever generators, and to recommend a direction for future research in this area based on the conclusions.

### 1.1 Overview

Cantilever structure with tip mass is the most widely used configuration for piezoelectric energy harvesting device. The source of vibration is shown with an arrow at the base of the contact point. The stiffness of the structure depends on the loading condition, material, and cross-sectional area perpendicular to the direction of vibration. The governing equation of motion for the system shown in Fig. 1 can be obtained from energy balance equation or D'Alembert's principle. This configuration applies to both the energy harvesting mechanisms shown in Fig. 1 (a) and (b).

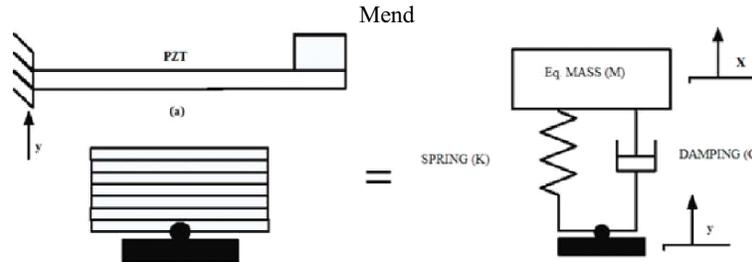


Figure 1: (a) Cantilever beam with tip mass, (b) multilayer PZT subjected to transverse vibration excited at the base, and (c) equivalent lumped spring mass system of a vibrating rigid body.

The governing equation of motion of a lumped spring mass system can be written as:

$$M\ddot{z} + C\dot{z} + Kz = -My \quad (0.1)$$

Where  $z = x - y$  is the net displacement of mass. Equation (1.1) can also be written in terms of damping constant and natural frequency. A damping factor,  $\zeta$ , is a dimensionless number defined as the ratio of system damping to critical damping as:

$$\zeta = \frac{c}{c_c} = \frac{c}{2\sqrt{mK}} \quad (0.2)$$

The natural frequency of a spring mass system is defined by Eq. (1.3) as

$$\omega = \sqrt{\frac{K}{M}} \quad (0.3)$$

where the stiffness  $K$  for each loading condition should be initially calculated. For example, in case of a cantilever beam, the stiffness  $K$  is given by  $K = 3EI/L^3$ , where  $E$  is the modulus of elasticity,  $I$  is the moment of inertia, and  $L$  is the length of beam. The moment of inertia for a rectangular cross-sectional can be obtained from expression,  $I = (1/12)bh^3$ , where  $b$  and  $h$  are the width and thickness of the beam in transverse direction, respectively. The power output of piezoelectric system will be higher if system is operating at natural frequency which dictates the selection of material and dimensions. The terms "natural frequency" and "resonant frequency" are used alternatively in literature, where natural frequency of piezoelectric system should not be confused with natural frequency of mechanical system. The ratio of output  $z(t)$  and input  $y(t)$  can be obtained by applying Laplace transform with zero initial condition on Eq. (1.1) as:

$$\left| \frac{Z(s)}{Y(s)} \right| = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (0.4)$$

The time domain of the response can be obtained by applying inverse Laplace transform on Eq. (1.4) and assuming that the external base excitation is sinusoidal given as:  $y = Y \sin(\omega t)$ ,

$$z(t) = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}} Y \sin(\omega t - \phi) \quad (0.5) \dots$$

The phase angle between output and input can be expressed as  $\phi = \arctan(C\omega/(K - \omega^2M))$ . The approximate mechanical power of a piezoelectric transducer vibrating under the above mentioned condition can be obtained from the product of velocity and force on the mass as:

$$P(t) = \frac{m\zeta Y^2 \left(\frac{\omega}{\omega_n}\right)^2 \omega^2}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}} \quad (0.6)$$

The maximum power can be obtained by setting the operating frequency as natural frequency in Eq. (1.6).

### 1.2 Modes of Vibration and Resonance

A cantilever beam can have many different modes of vibration, each with a different resonant frequency. The first mode of vibration has the lowest resonant frequency, and typically provides the most deflection and therefore electrical energy. A lower resonant frequency is desirable, since it is closer in frequency to physical vibration sources and generally more power is produced at lower frequencies [5]. Therefore, energy harvesters are generally designed to operate in the first resonant mode.

Each mode of vibration has a characteristic mode shape. This describes the deflection of the beam along its length. Figure 2 shows some examples of mode shapes for the first three vibrational modes of a beam. When a beam is vibrating in a particular mode, the deflection will vary sinusoidally with time, with the amplitude of the sine wave along the length of the beam given by the mode shape.

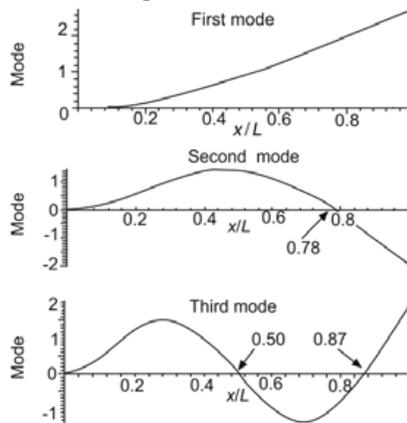


Figure 2: Different mode shapes of a vibrating beam.

The points where the mode shape is zero are stationary and are referred to as nodes. In general, the  $n$ th vibrational mode will have  $n$  nodes.

## II. ESTIMATION OF RESONANT FREQUENCY USING THE BEAM EQUATION

The resonant frequencies of a beam can be estimated using Euler-Bernoulli beam theory [9]. By solving the Euler-Bernoulli beam equation with the appropriate boundary conditions, the eigenvalues of the system can be determined, which then allow for the calculation of the resonant frequencies. The differential equation describing the motion of an Euler-Bernoulli beam is:

$$\frac{\partial^4 y}{\partial x^4} + \frac{\rho A}{EI} \frac{\partial^2 y}{\partial t^2} = 0, \quad (0.7)$$

where  $y$  is the beam deflection as a function of position along the beam and time,  $\rho$  is the density,  $A$  is the area of the cross section of the beam,  $E$  is the Young's modulus, and  $I$  is the area moment of inertia. For a beam of rectangular cross section, the relevant moment is  $I = (1/12) wt^3$ .

The general solution for sinusoidal vibration is as follows, with the constants and eigenvalues determined by the boundary conditions.

$$y(x, t) = (c_1 \sin \beta x + c_2 \cos \beta x + c_3 \sinh \beta x + c_4 \cosh \beta x), \quad (0.8)$$

where  $\beta^4 = \frac{\rho A \omega^2}{EI}$ .

For a fixed-free beam with no proof mass, the relevant boundary conditions for a beam of length  $L$  are:

$y(0, t) = y_x(0, t) = 0$  and  $y_{xx}(L, t) = y_{xxx}(L, t) = 0$ . These first two boundary conditions indicate that the fixed end of the beam is stationary, and that the beam is flat at the point of attachment. The free end conditions mean that there are no forces applied at that point and no bending moment. The first nontrivial eigenvalue of this system is  $\beta L \approx 1.875$ , so the equation for the resonant frequency of the first mode is:

$$f = \frac{\omega}{2\pi} = \frac{1.875^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A}} \quad (0.9)$$

By rewriting  $I$  and  $A$  in terms of the beam dimensions, the widths cancel and the expression reduces to:

$$f = \left( \frac{1.875^2}{2\pi} \sqrt{\frac{E}{12\rho}} \right) \frac{t}{L^2} \quad (0.10)$$

For a beam with a proof mass added on the tip, the mass can be modeled as a point load on the tip. The fourth boundary condition then becomes

$$y_{xxx}(L, t) = -\frac{m\omega^2}{EI} y(L, t), \quad (0.11)$$

where  $m$  is the mass of the beam. The calculation of eigenvalues in this case depends on the ratio of the added mass to the mass of the beam.

### III. RESONANT FREQUENCY VERIFICATION

To verify the estimates for the resonant frequency, the impedance of the device was measured across a range of frequencies using an Agilent impedance analyzer. The impedance measurements were done in a vacuum, as damping due to air would severely reduce the beam's movement at atmospheric pressure and make the resonance peak difficult to discern.

The following measurement was made for the beam measuring 400  $\mu\text{m}$  long by 100  $\mu\text{m}$  wide:

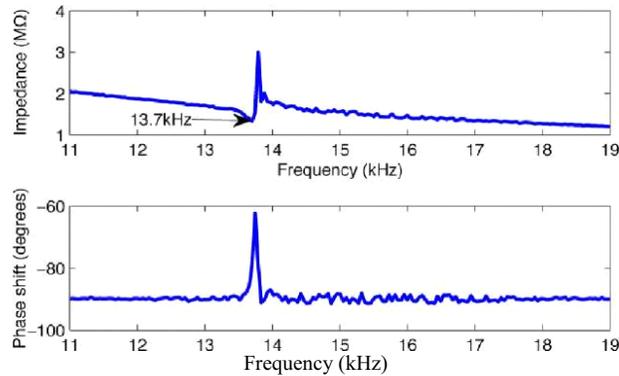


Figure 3: Impedance vs frequency.

Using the approximate relationships derived in section 2, the calculated resonant frequencies are 15.6 kHz (beam equation approach).

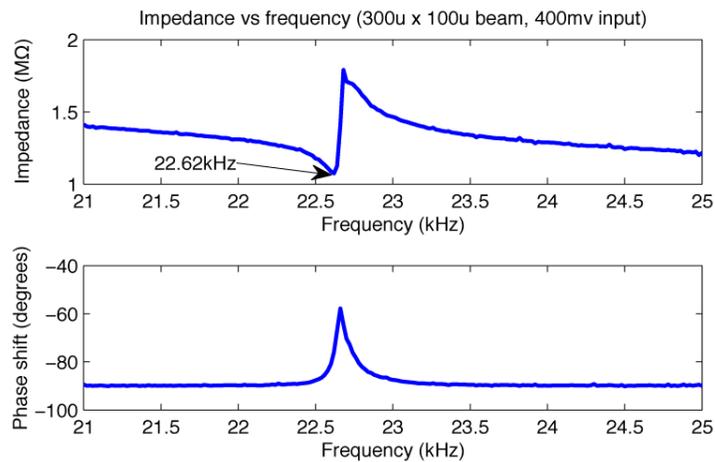


Figure 4: Impedance vs frequency.

In this case, the calculated resonant frequencies are 27.8 kHz (beam equation) and 28.2 kHz (stiffness), and the relative errors are 22% and 25%. Since the relative error increases significantly when a shorter beam is used, this suggests that the derived models are not as accurate for shorter beams.

In fact, one of the assumptions of the Euler-Bernoulli beam equation is that the length is significantly larger than the width and thickness. Keeping all other factors constant, better agreement would be expected with longer beams.

#### IV. CONCLUSIONS AND RECOMMENDATIONS

It has been shown that the current device structure does not have scaling advantages in power per unit area or volume. It is difficult to produce sufficient displacement at small scales to generate a considerable voltage. At the microscale, resonance frequencies are too low to effectively convert ambient frequencies as found in nature.

Due to the difficulty in reaching low frequencies with MEMS scale devices, these types of energy harvesters would be limited to applications with very high frequency vibrations. However, for compact systems with very low power requirements, MEMS microgenerators are a very attractive means of powering devices indefinitely.

Recommendations are to build devices of this form, with a proof mass added, while targeting lower resonant frequencies. Alternate geometries may help in lowering the resonant frequency, and gaining more power output. More effective solutions include designing a structure that is either not dependent on resonance, or has a means of tuning its resonant frequency. Examples of such devices have already been demonstrated by other researchers[10, 11]. To take advantage of the large deflections and strains that go with a beam oscillating at resonance, the tuning approach is recommended as the most useful for power output. The challenge will be in adapting existing tuning approaches to the MEMS scale, or in devising a new means to tune the beams' resonant frequency.

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