

# Low Complexity Decoder for Space Time Block Code with 4 Transmit Antennas

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**Abstract-** Space-time block coding represents a practical and Pragmatic method to mitigate the fading effects and to increase the channel capacity, by using spatial diversity. The space-time Coding techniques are still preferred in communication systems because of the very simple transmit coding rule and of the relative reduced complexity of the Maximum likelihood (ML) receivers with respect to other coding methods. One of the drawbacks when increasing the number of transmit antennas is increase in complexity. This paper deals with this issue and introduces a decoding algorithm which decomposes the original complex-valued system into a parallel system, thus allowing for a simple and independent detection of the real and imaginary parts of each complex transmitted symbol.

**Keywords -** Space-time block coding , Maximum-Likelihood, MIMO.

## I-INTRODUCTION

Space Time Coding (STC) is a transmit diversity scheme in which coding is performed both in both spatial and temporal domain introducing redundancy between signals transmitted from various antennas at various time periods. It can achieve diversity gain and coding gain over spatially uncoded systems without sacrificing bandwidth.

Alamouti [1] proposed a simple but most effective transmit diversity technique which is called space-time block coding because the source data are coded and transmitted through different antennas in different time slots. Initial and simple examples of implementation of space time coding were given in [1], where two transmit antennas and two receive antennas were used. Tarokh, et al generalized the transmission scheme to an arbitrary number of transmit antennas, which can achieve the full diversity promised by the transmit and receive antennas [2], [3]. From their papers, we can find the key points of space-time coding. One of those is that more than one antenna could be used in both sides of the transmitter and receiver. It means that the radio links in space-time Coding systems are multi-input multi-output (MIMO) radio channels.

Decoding of space time block codes requires knowledge of channels at the receiver. In some of initial papers about space-time coding, the channel parameters are assumed known. In practice, due to the changing environment, an estimation of the channel has to be made to extract the received signal. Orthogonal pilot sequences are often used for channel estimation in the literature. The use of pilot sequences at the beginning of the Transmission is exploited in [3], where the first transmitted symbols are the pilot then followed by the data in the next sequence.

Space time block code (STBC) is preferred because of very simple transmit coding rule and relative reduced complexity of Maximum Likelihood receiver with respect to other coding methods (e.g. Space time trellis code or spatial multiplexing). One of the drawbacks when using STBC is increase in complexity at the ML receiver when increasing number of transmit antenna and the number of constellation points. Hence, by simple signal processing technique, the symbol metric to be minimized by the ML receiver is separated along real and imaginary parts. The reduction of the search space leads to significant reduction of receiver complexity.

## II SPACE TIME BLOCK CODING

In addressing the issue of decoding complexity, Alamouti discovered a remarkable scheme for transmission using two transmits antennas [1]. Space-time block coding, introduced in [2] and [4], generalizes the transmission scheme discovered by Alamouti to an arbitrary number of transmit antennas and is able to achieve

the full diversity promised by the transmit and receive antennas. These codes retain the property of having a very simple maximum likelihood decoding algorithm based only on linear processing at the receiver.

*A Transmission model*

A space time block code is defined by a  $p \times n$  transmission matrix  $G$ . the entries of the matrix  $G$  are linear combinations of the combinations of the variables  $s_1, s_2, \dots, s_k$  and their conjugates.  $G_2$  represents a code which utilizes two transmit antennas and is defined by

$$G_2 = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \tag{1}$$

Since  $p$  time slots are used to transmit  $k$  symbols, the rate  $R$  of the code to be  $R=k/p$ . For example, the rate of  $G_2$  is one.

We consider a wireless communication system with 2 antennas at the base station and 1 antenna at the remote. At each time slot  $t$ , signals  $s_t^i, i=1,2$  are transmitted simultaneously from the two transmit antennas. The coefficient  $h_{ij}$  is the path gain from transmit antenna  $i$  to receive antenna  $j$ .

At time  $t$ , the signal  $r_t^j$  received at antenna  $j$  is given by

$$r_t^j = \sum_{i=1}^2 h_{ij} s_t^i + \eta_t^j \tag{2}$$

where the noise samples  $\eta_t^j$  are independent samples of a zero-mean complex Gaussian random variable with variance  $1/(2SNR)$  per complex dimension. Assuming perfect channel state information is available; the receiver computes the decision metric

$$\sum_{t=1}^l \sum_{j=1}^m \left| r_t^j - \sum_{i=1}^n h_{i,j} s_t^i \right|^2 \tag{3}$$

Over all code words  $s_1^1 s_1^2 \dots s_1^n s_2^1 s_2^2 \dots s_2^n \dots s_l^1 s_l^2 \dots s_l^n$  and decides in favor of the code word that minimizes the sum.

*B Detection scheme*

ML decoding of any space time block code can be achieved using only linear processing at the receiver. Assuming that the channel  $h$  is known at the receiver, the ML estimate is obtained at the decoder by performing  $\min \|R - HS\|_F^2$  where  $\| \cdot \|_F$  is the Frobenius norm and  $R = [r_t^j]$  is the received signal of size  $T \times M$  and whose entry  $r_t^j$  is the signal received at antenna  $j$  at time  $t, t = 1, 2, \dots, T, j = 1, 2, \dots, M$  and  $S = [s_t^i]$  is transmitted signal matrix of size  $T \times N$  whose entry  $s_t^i$  is the signal transmitted at antenna  $i$  at time  $t, t = 1, 2, \dots, T, i = 1, 2, \dots, N$ . and the matrix  $H = [h_{ij}]$  is the channel coefficient matrix of size  $N \times M$  whose entry  $h_{ij}$  is the channel coefficient from transmit antenna  $i$  to receive antenna  $j$ .

We consider the Alamouti OSTBC defined by [1] with  $N = K = T = 2$  and  $M = 1$ . The received signal is

$$\begin{aligned} r_1 &= h_{11}s_1 + h_{21}s_2 + n_0 \\ r_2 &= -h_{11}s_2^* + h_{21}s_1^* + n_1 \end{aligned} \tag{4}$$

where  $n_0$  and  $n_1$  represent the complex noise and interference. The combiner then combines the signals

$$\begin{aligned} z_1 &= h_{11}^* r_1 + h_{21} r_2^* \\ z_2 &= h_{21}^* r_1 + h_{11} r_2^* \end{aligned} \tag{5}$$

Substituting  $r_1$  and  $r_2$

$$z_1 = (|h_{11}|^2 + |h_{21}|^2) s_1 + h_{11}^* n_0 + h_{21} n_1^*$$

$$z_2 = \left( |h_{11}|^2 + |h_{21}|^2 \right) s_2 - h_{11} n_1^* + h_{21}^* n_0 \quad (6)$$

The two signals coming from the combiner is simplified as

$$\begin{aligned} \tilde{s}_1 &= (z_1 - s_k)^2 + (h_{ij} s_k)^2 \\ \tilde{s}_2 &= (z_2 - s_k)^2 + (h_{ij} s_k)^2 \end{aligned} \quad (7)$$

The two signals are sent to maximum likelihood detector

$$\begin{aligned} d^2(\tilde{s}_1, s_i) &\leq d^2(\tilde{s}_1, s_k) \quad \forall i \neq k \\ d^2(\tilde{s}_2, s_i) &\leq d^2(\tilde{s}_2, s_k) \quad \forall i \neq k \end{aligned} \quad (8)$$

Where  $d^2(x, y) = (x - y)(x^* - y^*)$

### III. LOW COMPLEXITY MAXIMUM LIKELIHOOD DETECTION OF STBC

One of the drawbacks when using space-time block codes is the increase in complexity at the ML receiver when increasing the number of transmit antennas and the number of constellation points. Hence, the Maximum Likelihood decoding is simplified by using real valued lattice representation to get optimal ML decoding. The number of computation required will be less compared to conventional ML.

Since one complex symbol is equivalent to two real symbols, we could decode real and imaginary value of the signal separately.

Consider the received signal as given in (4)

$$\begin{aligned} r_1 &= h_{11} s_1 + h_{21} s_2 + n_0 \\ r_2 &= -h_{11} s_2^* + h_{21} s_1^* + n_1 \end{aligned} \quad (9)$$

At the combiner the following computation is done

$$\begin{aligned} z_{11} &= \Re(h_{11}^* r_1) + \Re(h_{21} r_2^*) \\ z_{12} &= \Re(h_{21}^* r_1) + \Re(h_{11} r_2^*) \end{aligned} \quad (10)$$

and

$$\begin{aligned} z_{21} &= \Im(h_{11}^* r_1) + \Im(h_{21} r_2^*) \\ z_{22} &= \Im(h_{21}^* r_1) + \Im(h_{11} r_2^*) \end{aligned} \quad (11)$$

Comparing (10) and (11) with (5), we can observe the combination of real and imaginary part of signal is done separately. Hence the combination of the two signals does not involve the multiplication of real value with the imaginary value. Hence the number of multiplication required for computation at the combiner is reduced to half compared to the earlier approach. Further simplifying, results to the following computations at the decoder

$$\begin{aligned} z_{11} &= \Re\left(|h_{11}|^2 + |h_{21}|^2\right) \Re(s_1) + \Re(h_{11}^* n_0 + h_{21} n_1^*) \\ z_{12} &= \Re\left(|h_{11}|^2 + |h_{21}|^2\right) \Re(s_2) - \Re(h_{11} n_1^* + h_{21}^* n_0) \end{aligned} \quad (12)$$

$$\begin{aligned} z_{21} &= \Im\left(|h_{11}|^2 + |h_{21}|^2\right) \Im(s_1) + \Im(h_{11}^* n_0 + h_{21} n_1^*) \\ z_{22} &= \Im\left(|h_{11}|^2 + |h_{21}|^2\right) \Im(s_2) - \Im(h_{11} n_1^* + h_{21}^* n_0) \end{aligned} \quad (13)$$

The four signals are sent to maximum likelihood detector

$$\begin{aligned}
 d^2(\Re(\tilde{s}_1), \Re(s_i)) &\leq d^2(\Re(\tilde{s}_1), \Re(s_k)) \quad \forall i \neq k \\
 d^2(\Re(\tilde{s}_2), \Re(s_i)) &\leq d^2(\Re(\tilde{s}_2), \Re(s_k)) \quad \forall i \neq k
 \end{aligned}
 \tag{14}$$

and

$$\begin{aligned}
 d^2(\Im(\tilde{s}_1), \Im(s_i)) &\leq d^2(\Im(\tilde{s}_1), \Im(s_k)) \quad \forall i \neq k \\
 d^2(\Im(\tilde{s}_2), \Im(s_i)) &\leq d^2(\Im(\tilde{s}_2), \Im(s_k)) \quad \forall i \neq k
 \end{aligned}
 \tag{15}$$

Minimization of the four real metrics leads to estimates for  $\tilde{s}_{1R}, \tilde{s}_{2R}$  giving the real part of symbol  $s_1, s_2$  and  $\tilde{s}_{1I}, \tilde{s}_{2I}$  giving the imaginary part of symbol  $s_1$  and  $s_2$  and further final two estimates  $\tilde{s}_1 = \tilde{s}_{1R} + j\tilde{s}_{1I}$  and  $\tilde{s}_2 = \tilde{s}_{2R} + j\tilde{s}_{2I}$ .

#### IV. SIMULATIONS

We apply the reduced complexity ML algorithm and conventional ML for two space time block codes. The results are shown in Figure 1 and Figure 2, for four transmit antennas with one and two receive antennas respectively for 16-QAM modulation. There is degradation in performance of reduced complexity ML decoder as compared to ML decoder but the number of computations required is reduced to half. We observe that the performance for the two scheme remains same for SNR=6dB and SNR=8dB and the degradation of performance is more for SNR<13dB.

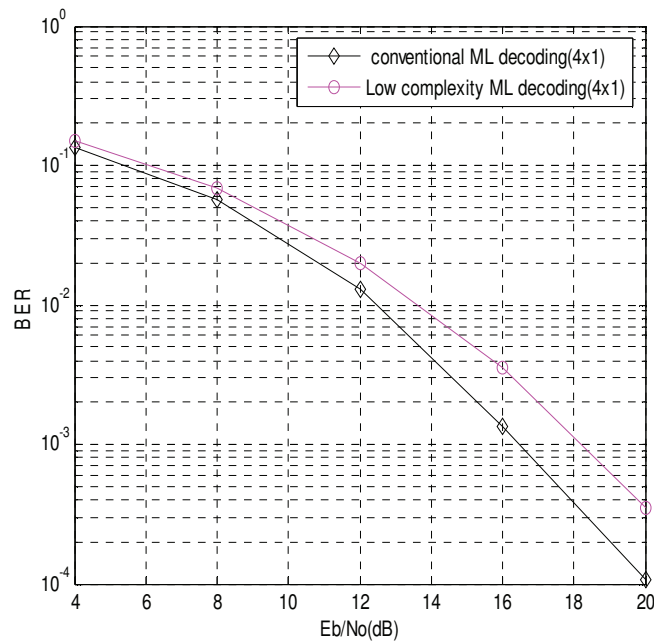


Figure.1 Bit Error Rate Vs Signal to Noise ratio for ML and reduced complexity ML decoder with 4 Tx and 1Rx

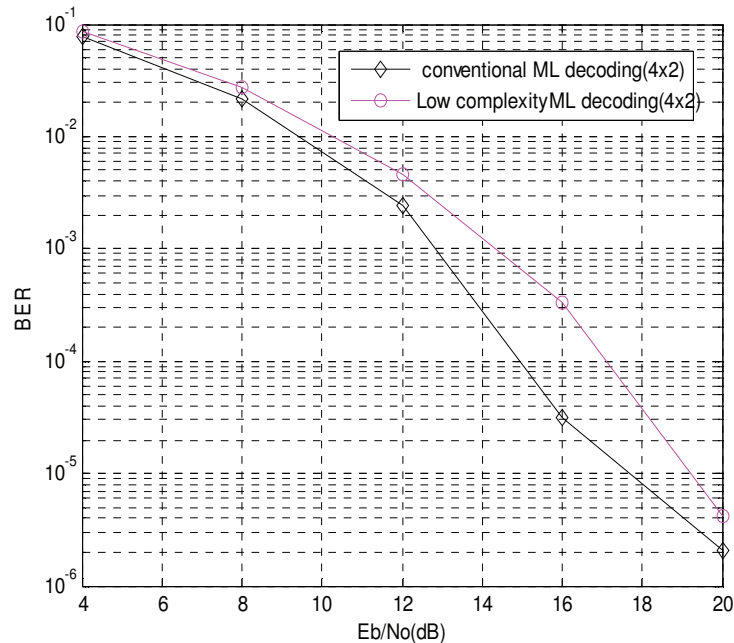


Figure 2. Bit Error Rate Vs Signal to Noise ratio for ML and reduced complexity ML decoder with 4 Tx and 2Rx

#### IV. CONCLUSION

We have introduced a simple decoding algorithm for orthogonal space-time codes. The decoder relies on ML detection, but unlike the direct applying of the algorithm, it performs some additional signal preprocessing in order to obtain separable metrics not only for each symbol, but also for the real and imaginary parts. This leads to a significant reduction in the search space of the ML detector and furthermore to the global complexity and computation time of the receiver.

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