Fuzzy Markov Model for Arrival Process in Web Server Queues

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Abstract - In this paper the arrival process has been studied using fuzzy Markov model. The arrival process per attributed to node is found to be the sum of independent fuzzy Poisson cluster process. Also the univariate probability generating function has been derived.

Key Words: Fuzzy Markov Renewal Process, Semi Fuzzy Markov Kernel, non-homogenous fuzzy Poisson process

I. INTRODUCTION

The World Wide Web has become one of the most popular and important applications on the Internet. The need for communication networks capable of providing an ever increasing spectrum of services calls for efficient techniques for the analysis, monitoring, evaluation and design of the networks. Analysis are perpetually faced with incomplete & ever increasing user demands and uncertainty about the evolution of the network systems. To meet the requirements of users and to provide guarantees on reliability & affordability, system models must be developed that can capture the characteristics of the actual network. Traffic Analysis is a vital component to understand the requirements and capabilities of a network. The past years have seen innumerous traffic models proposed for understanding and analyzing the traffic characteristics of networks. In this paper we have studied some special characters of the arrival process of the customers along with the attributes.
A web server receives numerous requests for its services, whose arrival process varies per attribute. One of the most widely used and oldest traffic model is the Poisson Model. The memoryless Poisson distribution is the predominant model used for analyzing traffic. Poisson processes are common in traffic applications scenarios that comprise of a large number of independent traffic streams. Non-Homogeneous Poisson process are used to study a queueing network model for integration of the mobility and teletraffic aspects that are characteristic of wireless network [4] .
Markov models attempt to model the activities of a traffic source on a network, by a finite number of states. The accuracy of the model increases linearly with the number of states used in the model. In a simple Markov Traffic model, each of the state transition represents a new arrival process on the network. ***Markov-renewal*** models are more general: they allow the interarrival times to be arbitrarily distributed, and constrain the distribution to depend upon both the current state of the system and the interarrival interval..A *Semi-Markov* model is one that is obtained by allowing the time between state transitions to follow an arbitrary probability distribution. A Markov model has been used for studying the characteristic of the arrival process to the wirelss network [4][5].
For many systems due to uncertainties and imprecision of data, the estimation of precise values of probabilities is very difficult. For this reason, the concept of fuzzy reliability have been introduced and formulated in the context of the possibility measures[2][3][6] .
The paper is constructed as follows: Section 2 model description section 3 recalls the preliminaries needed for this paper and section 4 Studies the characteristics of the arrival process to Input node using a non homogeneous fuzzy probabilistic semi-Markov model with transition fuzzy probabilities. Section 5 presents the characteristics of the arrival process to intermediate node section 6 presents that of outside node.

II. PRELIMINARIES

A fuzzy set is the generalization of the crisp sets. A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to a degree to which that individual is similar or compatible with the concept represented by the fuzzy set.

Let $\tilde{A}$ be a fuzzy set, the membership function [6] $\mu_{\tilde{A}}(x) \in [0,1]$ is evaluated for $\tilde{A}$ at $x \in R$ where [0,1] denotes the interval of real number from 0 to 1 including 0 and 1.Then the fuzzy sets are the subsets of real number system.

Let Γ be the universe of discourse and ψ be the power set of Γ. Then by [6] the possibility measure σ is a mapping defined as follows

$\sigma : \psi \to [0,1]$

such that the following properties holds

i. $\sigma(\emptyset) = 0$ and $\sigma(\Gamma) = 1$

ii. $\sigma(\cup A_i) = \sup(\sigma(\cup A_i))$

for every arbitrary collection A_i of ψ. The triplet (Γ, ψ, σ) is called as possibility space.

III. Model Description

The model consists of a fuzzy web system of infinite work stations. A client entering for 'job n' to kth station. Let D= I ∪M∪O denotes the set of nodes of the network where,

I - denotes the input node which will be visited by the client only when they enters the network

O - denotes the output node which will be visited by the client when he leaves the network, once he leaves the system he is supposed to reenter the network only through I

M – Denoted the intermediate nodes where the client may stay and has a freedom of moving different area's in search of information he requires.

3.1 Mobility of the customer:

Let X_l^n denotes location node of the customer at T_l^n time.

$\{(X_l^n, T_l^n), n \geq 0\} \sim \{(X^n, T^n), n \geq 0\}$

where X_n represents the steps at the n transition

T_n represents the time at the n transition

The process $\{(X_n, T_n)\ n \in N\}$ is called non- homogeneous Fuzzy Markov Renewal process if

$\sigma(X_{n+1} = j, T_{n+1} \leq t | X_0, X_1, X_2, \ldots, X_n = i, T_0, T_1, \ldots, T_n)$

$\quad = \sigma(X_{n+1} = j, T_{n+1} \leq t | X_n = i)$; for $i \neq j$

The Transition distribution measure follows a non- homogenous fuzzy semi markov kernel [2].

$$\tilde{Q}_{ij} = \sigma(X_{n+1} = j, T_{n+1} \leq t | X_n = i)$$

$$\tilde{r}_{ij}(t) = \lim_{n\to\infty} \tilde{Q}_{ij}(s,t), i, j \in E, j \neq i$$

$\tilde{r}_{ij}(t)$ represent the possibility of the client making his next transition to state j given that he entered state i at time t. However before the entrance to j the process holds for a time s in state i.

For i,j∈E. By [2] define $\{\tilde{G}_{ij}(t), t \geq 0\}$ as the fuzzy distribution function of the time spent by the customer at the visiting node i, given that its next visiting node is j.

$$\therefore \tilde{G}_{ij}(t) = \begin{cases} \frac{\tilde{Q}_{ij}}{\tilde{r}_{ij}(t)}, \text{if } \tilde{Q}_{ij} \neq 0 \\ 1, \quad \text{if } \tilde{Q}_{ij} = 0 \end{cases}$$

where

$\tilde{G}_{ij}(t) = \frac{\tilde{Q}_{ij}}{\tilde{r}_{ij}(t)} = \sigma(T_{n+1} - T_n \leq t | X_{n+1} = j, X_n = i)$ if $\bar{p}_{ij} > 0$.

Let $N^* = \inf\{n \geq 1 : X_n \in O\}$ denotes the number of node transition made by an arbitrary customer before leaving the network.

T_N^* denotes the time of departure so that X_N^* is the outside node reached by the customer at departure.

$Y^l(t)$ denotes the location node of the l^{th} customer at t unit of time after arriving the network.

$\{Y^l(t), t > 0\} \sim \{Y(t), t > 0\}$

Where $\{Y(t), t > 0\}$, being the minimal semi markov process associated to $\{(X_n, T_n), n \geq 0\}$ defined by

$Y^l(t) = \begin{cases} X_n, T_n \leq t \leq T_n \\ X_N \quad , t \geq T_N \end{cases}$ for all t>0 and i□I, j □O.

We have

$\tilde{p}_{ij}(t) = \sigma[(Y(t) = j | Y(0) = i)] = \min[\tilde{p}_{ij}(t), \tilde{r}_{ij}(t)]$

3.2 Customers Attributes:

Suppose that each customer in the network has associated an attributes that may change with time. Let $A^l(t)$ denotes the attributes of l^{th} customers at t unit of time after arriving the network[7].

Suppose if

$\{A^l(t), t > 0\} \sim \{A(t), t > 0\}$ with $\{A^l(t), t > 0\}$ being a continuous time markov chain with finite state space S.

For t ≥ 0 and i, j $\in$ S we have

$\tilde{q}_i(t) = P[A^l(t) = j]$

$\tilde{q}_{ij}(t) = P[\{A^l(t) = j | A^l(0) = i\}]$

Let us also suppose that the attribute of the customer and their locations in the network are independent (i.e)

$\{A^l(t); t \geq 0, l \geq 1\} \perp \{Y^l(t), t > 0, l \geq 1\}$

IV. ARRIVAL PROCESS TO INPUT NODE

The study of arrival process to an individual node for each type of attributes is done by considering the input node first followed by the intermediate and outside nodes.

Let $\{S_i^{\alpha(l)}, l \geq 1\}$ denotes the sequence of arrival time to the network through the input node 'i' with attribute α and $\{N_i^{\alpha}(t), t \geq 0\}$ be the associate counting process .

It follows that the arrival process to the input node per attribute $\{N_i^{\alpha}(t), t \geq 0\}$ i∈I,α∈A are independent non homogeneous Fuzzy Poisson Process with intensity measure by [1]

$$\Lambda_i^{\alpha}(dt) = \tilde{q}_i(0)\tilde{\vartheta}_{\alpha}(t)\lambda(t)\,dt, i \in I, \alpha \in A, t \geq 0 \quad (1)$$

V. ARRIVAL PROCESS TO INTERMEDIATE NODE:

To study the arrival process to intermediate and output node it is convenient to first enhance the fuzzy markov renewal process previously defined by incorporating the attribute of the customer at transition epoch. Let us suppose $\hat{X}_n = (X_n, A(T_n))$ denotes the node and attribute of a general customer after its n^{th} transition. The process $\{(T_n, \hat{X}_n; n \geq 0)\}$ is a fuzzy Markov Renewal process on state space D x A, whose transition distribution measure is

$$\tilde{Q}_{ij}^{\alpha\beta}(t) = \tilde{P}(\hat{X}_{n+1} = (j, \beta), T_{n+1} - T_n \leq t | \hat{X}_n = (i, \alpha))$$

$$= \tilde{r}_{ij} \int_0^t \tilde{q}_{\alpha\beta}(s) \,.\, \tilde{G}_{ij}(ds) \text{ ; for i,j } \in I, \alpha, \beta \in A, t \geq 0. \quad (2)$$

The Possibility of a general customer reaches the node β from α given that he entered the network at time t , is an embedded discrete time Fuzzy markov chain (DTFMC) at transition epochs is $\hat{X} = \{\hat{X}_n, n \geq 0\}$ which has state space D×A and is given by

$$\tilde{r}_{ij}^{\alpha\beta} = \tilde{P}(\hat{X}_{n+1} = (j, \beta) | \hat{X} = (i, \alpha))$$

$= \tilde{r}_{ij} \int_0^{\infty} \tilde{q}_{\alpha\beta}(s) . \tilde{G}_{ij}(ds)$; for i,j $\in D, \alpha, \beta \in A, t \geq 0$. (3)

Let $\hat{X}_n = (j, \beta)$ denotes a visit to node j with attribute β occurs at time T_n.

For i ∈ I, j ∈M ∪O and α,β∈ A then $s_{ij}^{\alpha\beta(1)}, s_{ij}^{\alpha\beta(2)}, s_{ij}^{\alpha\beta(3)}, \ldots$ denotes the successive time T_n for which $\hat{X}_n = (j, \beta)$ given $\hat{X}_0 = (i, \alpha)$. For the sequence $\{s_{ij}^{\alpha\beta(n)}; n \geq 1\}$,let $\tilde{F}_{ij}^{\alpha\beta}(t)$ be the fuzzy distribution function of the first visit time from node i with attribute α to node j with attribute β.

$$\tilde{F}_{ij}^{\alpha\beta}(t) = \tilde{P}(s_{ij}^{\alpha\beta(1)} \leq t) \qquad (4)$$

and $\tilde{F}_{jj}^{\beta\beta}(t)$ be the fuzzy distribution function of the time between successive visit to node j with attribute β.

$$\tilde{F}_{jj}^{\beta\beta}(t) = \tilde{P}\left(s_{ij}^{\alpha\beta(n+1)} - s_{ij}^{\alpha\beta(n)} \leq t\right), n \geq 1. \qquad (5)$$

Let $w_{ij}^{\alpha\beta}([t, t+h])$ denotes the number of visits to node j with attribute β in $[t, t+h]$ made by customer with input node i and attribute α arriving in $[t, t+h]$. The total number of visits to the node 'j' with attribute β in $[t, t+h]$ is given by

$w_{ij}^{A\beta}([0, t+h], [t, t+h]) = \sum_{i \in I} \sum_{\alpha \in A} w_{ij}^{\alpha\beta}([0, t+h], [t, t+h])$.
(6)

The total number of visits to node β has two type of contribution .

i. The arrival to the node β is made in [t, t+h] which **correspondence** to $w_{ij}^{\alpha\beta}([t, t+h]$.
ii. The arrival to the node β occurs in the interval [0,t] whose **corresponding** number is almost equal to $w_{ij}^{\alpha\beta}([0, t], [t, t+h]$.

5.1 Visit made by customer arriving before time 't':

Let $H_{ij}^{\alpha\beta}(t)$ denotes the number of visits to node j with attribute β made by an arbitrary customer in a period of t units of time starting strictly after entering node I with attribute α also

$H_{ij}^{\beta\beta}(0) = 0$,--------------(7)

The waiting time between two successive occurance of a renewal is atleast one unit of time. Consequently in a finite interval of time of length say $N_i^{\alpha}(t)$ we have atmost $N_i^{\alpha}(t)$ renewals.

Let $Z_{ij}^{\alpha\beta}(t)$ denotes the number of customers in the network that arrives until time 't' having input node 'I' and attribute α and do a visit to node j with attribute β at time t or later

$$Z_{ij}^{\alpha\beta}(t) = \begin{cases} 1, & N_i^{\alpha}(t) = s_i^{\alpha(1)} + J_{ij}^{\alpha\beta(1)}; i, j \geq 0 \\ 0, & otherwise \end{cases} \qquad (7)$$

$$=\sum_{l=1}^{N_i^{\alpha(t)}} 1_{\{s_i^{\alpha(l)} \le t \le s_i^{\alpha(l)} + J_{ij}^{\alpha\beta(1)}\}} \qquad (8) \qquad\qquad (8)$$

Where $s_i^{\alpha(l)} + J_{ij}^{\alpha\beta(1)}$ is the time of occurrence of last visit to node j with attribute β by the lth customer who enters the network through the input node i with attributeα.

The time is equal to $s_i^{\alpha(l)}$ if $J_{ij}^{\alpha\beta(l)} = 0$ (ie) if the lth customer with input node I and attribute α does not make any visit to the node j with attribute β.

Here { $J_{ij}^{\alpha\beta(l)}$ } are iid to $J_{ij}^{\alpha\beta}$ where

$$J_{ij}^{\alpha\beta} = \begin{cases} \sup\{ S_{ij}^{\alpha\beta(n)} : S_{ij}^{\alpha\beta(n)} < \infty \}, & S_{ij}^{\alpha\beta(1)} < \infty \\ 0, & S_{ij}^{\alpha\beta(1)} = \infty \} \end{cases} \qquad (9) \qquad (9)$$

and $J_{ij}^{\alpha\beta}$ may be identified as the total lifetime of the defective delayed renewal process$\{ S_{ij}^{\alpha\beta(n)} \}$ or its associated counting process $\{H_{ij}^{\alpha\beta}(t) t \ge 0\}$

Let $U_{ij}^{\alpha\beta}\left(U_{jj}^{\beta\beta}\right)$denotes the probability of a customer ever reaching node j with attribute β starting from node i (j) and attributeα(β) , which may be computed using the DTMC $\widetilde{X_n}$ with Transition Probability given by

Theorem 5.1.1:

For i∈I,j∈M α,β∈A and t ≥0

$Z_{ij}^{\alpha\beta}(t)$ is a Poisson Random Variable with mean

$$\mu_{ij}^{\alpha\beta}(t) = \int_0^t P\left(J_{ij}^{\alpha\beta} \ge t - t_0\right) \Lambda_i^{\alpha}(du) \qquad (10)$$

where $P\left(J_{ij}^{\alpha\beta} \ge 0\right) = 1$

and $P\left(J_{ij}^{\alpha\beta} \ge s\right) = U_{ij}^{\alpha\beta} - \left(1 - U_{jj}^{\beta\beta}\right) E\left[H_{ij}^{\alpha\beta}(t,h)\right]$; S> 0.

<u>*Proof:*</u>

Consider i∈I,j∈M α,β∈A and t ≥0 fixed. we first note that

(10) is trivially verified for t =0.

Now for t>0 and m ∈ N_0 we have

$$P\left(Z_{ij}^{\alpha\beta}(t) = m\right) = \sum_{n=m}^{\infty} P\left(\sum_{l=1}^{n}\left(1_{\{s_i^{\alpha(l)} \le t \le s_i^{\alpha(l)} + J_{ij}^{\alpha\beta(l)}\}} = m \middle| N_i^{\alpha}(t) = n\right) . P(N_i^{\alpha}(t) = n)\right.$$

(11)

Under the condition that n customers with input node i and attribute α have arrived in [0,t] the times $s_i^{\alpha(l)}$ l =1,2,3,…n considered as unordered random variables which are distributed as n iid random variables with common distribution function $(\Lambda_i^{\alpha}[0,u] | \Lambda_i^{\alpha}[0,t])$; 0≤u≤t.

The probability that a customer who arrives through input node i and attribute α and whose arrival instant is distributed in [0,t] with distribution function $(\Lambda_i^{\alpha}[0,u] \mid \Lambda_i^{\alpha}[0,t])$ will make one (or the last vist) to node j with attribute β at time t or later is

$$\frac{1}{\Lambda_i^{\alpha}[0,t]} \int_0^t \tilde{P}\,(J_{ij}^{\alpha\beta} \geq t-u)\, \Lambda_i^{\alpha}[du]$$

As for $\gamma > 0, j \epsilon M \ and \ \beta \epsilon A$.

$$\tilde{P}(J_{jj}^{\beta\beta} \leq v) = \left(1 - U_{jj}^{\beta\beta}\right)\left(1 + E\left[\,H_{jj}^{\beta\beta}(v)\right]\right)$$

We conclude for $v > 0$

$$\tilde{P}\left(J_{ij}^{\alpha\beta}(t) \geq v\right) = \tilde{F}_{ij}^{\alpha\beta} + \int_0^v \tilde{F}_{ij}^{\alpha\beta}(ds)\tilde{P}\left(J_{jj}^{\beta\beta}(t) \geq v-s\right)$$

$$= \tilde{P}\left(S_{ij}^{\alpha\beta(l)} \leq v\right) + \int_0^v \tilde{F}_{ij}^{\alpha\beta}(ds)\tilde{P}\left(J_{jj}^{\beta\beta}(t) \geq v-s\right)$$

$$= \tilde{U}_{ij}^{\alpha\beta} - \left(1 - \tilde{U}_{jj}^{\beta\beta}\right) \int_0^v \tilde{F}_{ij}^{\alpha\beta}(ds)\ \left(1 + E\left[\,H_{jj}^{\beta\beta}(v-s)^{-}\right]\right.$$

$$= \tilde{U}_{ij}^{\alpha\beta} - \left(1 - \tilde{U}_{jj}^{\beta\beta}\right)E\left[\,H_{ij}^{\alpha\beta}(v^{-})\right]$$

From [1] we conclude that $Z_{ij}^{\alpha\beta}(t)$ is Poisson with mean $\mu_{ij}^{\alpha\beta}(t)$.

Theorem 5.1.2:

For i ∈ I , β∈ M and α,β∈ A { $s_{ij}^{\alpha\beta(n)}$; n≥1} is a delayed renewal process and {$\hat{H}_{ij}^{\alpha\beta}(t,h)$; h≥0} denotes the number of visits to node j with attribute β made over the interval [t , t+h] by a customer chosen randomly among those that arrive to the network in [0,t] through input node i with attributeα and which do at least one visit to node j with attribute β at time t or later and it is the associated (delayed renewal) counting process. $\hat{H}_{ij}^{\alpha\beta}(t,h) with\ distribution\ function\ \ \boldsymbol{F}_{ij}^{\alpha\beta}. \hat{H}_{ij}^{\alpha\beta}(t,h)$ has a probability generating function is given by

$$1 - \tilde{\boldsymbol{F}}_{ij}^{\alpha\beta}(t,h) + \sum_{k=0}^{\infty} s^k \left[\tilde{\boldsymbol{F}}_{ij}^{\alpha\beta^k}(t,h) * \tilde{F}_{jj}^{\beta\beta}(t) - \tilde{\boldsymbol{F}}_{ij}^{\alpha\beta^{k-1}}(t,h) * \tilde{F}_{jj}^{\beta\beta}(t)\right] \qquad (12)$$

Proof:

$$\tilde{g}_{\hat{H}_{ij}^{\alpha\beta}(t,h)} = \sum_{k=0}^{\infty} s^k \tilde{P}_k(s)$$

where $\tilde{P}_k(s) = P[N(t,h) = k]$

$$= \tilde{G}_{k-1}(t,h) - \tilde{G}_k(t,h)$$

Again, $\tilde{P}_k(s) = 1 - \tilde{G}(t,h)$

Hence,

$$\tilde{g}_{\tilde{H}_{ij}^{\alpha\beta}} = 1 - \tilde{G}(t,h) + \sum_{k=0}^{\infty} s^k \tilde{P}_k(s)$$

$$\tilde{g}_{\tilde{H}_{ij}^{\alpha\beta}} = 1 - \tilde{F}_{ij}^{\alpha\beta}(t,h)$$

$$+ \sum_{k=0}^{\infty} s^k \, [\tilde{F}_{ij}^{\alpha\beta^{k-1}}(t,h) * \tilde{F}_{jj}^{\beta\beta}(t) - \tilde{F}_{ij}^{\alpha\beta^{k}}(t,h) * \tilde{F}_{jj}^{\beta\beta}(t)]$$

(13)

By renewal function,

$E[\tilde{H}_{ij}^{\alpha\beta}(t,h)] = E[N(t,h)=k]$

$$= \sum_{k=0}^{\infty} k.[N(t,h) = k]$$

$$= \sum_{k=0}^{\infty} k.\left[\tilde{G}_{k-1}(t,h) - \tilde{G}_k(t,h)\right]$$

$$= \sum_{k=1}^{\infty} \tilde{G}_k(t,h)$$

$$= \sum_{k=0}^{\infty} \tilde{F}_{ij}^{\alpha\beta} * \tilde{F}_{jj}^{\beta\beta}(h) \qquad (14)$$

Theorem 5.1.3:

For t>0, j∈ M and $\beta \in$ A $\{w_{ij}^{\alpha\beta}([0,t+h],[t,t+h])$, t ≥0} , i∈I and α∈A are independent poisson cluster process with probability generating function

$$\tilde{g}_{W_{ij}^{\alpha\beta}([0,t+h],[t,t+h])} = \exp\left[-\mu_{ij}^{\alpha\beta}(t)\left[1 - \tilde{g}_{\tilde{H}_{ij}^{\alpha\beta}(t,h)}(s)\right]\right]$$

(15)

and expected value $E\left[W_{ij}^{\alpha\beta}([0,t],[t,t+h])\right] = \mu_{ij}^{\alpha\beta}(t)\, E\left[\tilde{H}_{ij}^{\alpha\beta}(t,h)\right]$ (16)

Proof:

Consider t>0 , i∈I j∈M and α,β∈A(fixed). then for some m ∈N

$\tilde{P}\left[W_{ij}^{\alpha\beta}([0,t+h],[t,t+h])\right] =$

$$\tilde{g}_{W_{ij}^{\alpha\beta}([0,t+h],[t,t+h])} = \sum_{m=0}^{\infty} s^m \tilde{P}\left(W_{ij}^{\alpha\beta}([0,t+h],[t,t+h]) = m\right)$$

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} s^m \tilde{P}\left(W_{ij}^{\alpha\beta}([0,t+h],[t,t+h]) = m \middle| Z_{ij}^{\alpha\beta}(t) = n\right) . \tilde{P}[Z_{ij}^{\alpha\beta}(t) = n]$$

$$= \sum_{n=0}^{\infty}\left[\tilde{g}_{\tilde{H}_{ij}^{\alpha\beta}(t,h)}(s)\right]^n . \tilde{P}[Z_{ij}^{\alpha\beta}(t) = n]$$

$$= \exp\left[-\mu_{ij}^{\alpha\beta}(t)\left[1 - \tilde{g}_{\tilde{H}_{ij}^{\alpha\beta}(t,h)}(s)\right]\right]$$

$$\mathrm{E}\left[W_{ij}^{\alpha\beta}([0,t],[t,t+h])\right] = E\left[\tilde{H}_{ij}^{\alpha\beta}(t,h)\right]\mu_{ij}^{\alpha\beta}(t)$$

Capitalizing on the results of the previous two subsection concerning respectively the number of visits to intermediate nodes in the time interval [t, t+h] made by customers arriving to the network before time t and after time t, we are now able to characterize the total number of vistis to intermediate nodes in the time interval [t, t+h].

5.2 Visit made by the customer arriving after time ‘t’:

Theorem 5.2.1:

For i ∈ I , β∈ M and α,β∈ A { $s_{ij}^{\alpha\beta(n)}$; n≥1} is a delayed renewal process and {$H_{ij}^{\alpha\beta}(t)$; t≥0} is the associated (delayed renewal) counting process. Moreover for t ≥0

$H_{ij}^{\alpha\beta}(t)$ has a probability generating function equal to

$$1 - \tilde{F}_{ij}^{\alpha\beta}(t) + \sum_{k=0}^{\infty} s^k [\tilde{F}_{ij}^{\alpha\beta^k}(t) * \tilde{F}_{jj}^{\beta\beta}(t) - \tilde{F}_{ij}^{\alpha\beta^{k-1}}(t) * \tilde{F}_{jj}^{\beta\beta}(t)] \quad (17)$$

Proof:

$$\tilde{g}_{H_{ij}^{\alpha\beta}(t)} = \sum_{k=0}^{\infty} s^k \tilde{P}_k(s)$$

where $\tilde{P}_k(s) = P[N(t) = k]$

$$= \tilde{G}_{k-1}(t) - \tilde{G}_k(t)$$

Again, $\tilde{P}_k(0) = 1 - \tilde{G}(t)$

Hence,

$$\tilde{g}_{H_{ij}^{\alpha\beta}(t)} = 1 - \tilde{G}(t) + \sum_{k=0}^{\infty} s^k \tilde{P}_k(s)$$

$$\tilde{g}_{H_{ij}^{\alpha\beta}(t)} = 1 - \tilde{F}_{ij}^{\alpha\beta}(t) + \sum_{k=1}^{\infty} s^k [\tilde{F}_{ij}^{\alpha\beta^{k-1}}(t) * \tilde{F}_{jj}^{\beta\beta}(t) - \tilde{F}_{ij}^{\alpha\beta^k}(t) * \tilde{F}_{jj}^{\beta\beta}(t)]$$

By renewal function,

$E[H_{ij}^{\alpha\beta}(t)] = E[N(t)=k]$

$$= \sum_{k=0}^{\infty} k.[N(t) = k]$$

$$= \sum_{k=0}^{\infty} k.\left[\tilde{G}_{k-1}(t) - \tilde{G}_k(t)\right]$$

$$= \sum_{k=1}^{\infty} \tilde{G}_k(t)$$

$$= \sum_{k=0}^{\infty} \tilde{F}_{ij}^{\alpha\beta} * \tilde{F}_{jj}^{\beta\beta}(t)$$

Theorem 5.2.2:

For t>0, j∈ M and $\beta \in$ A $\{w_{ij}^{\alpha\beta}[t, t+h])$, h ≥0} , i∈I and α∈A are independent poisson cluster process with probability generating function

$$\tilde{g}_{W_{ij}^{\alpha\beta}(,[t,t+h])}(s) = \exp\left[-\Lambda_i^{\alpha}([t, t+h]) + \left[\int_t^{t+h} \tilde{g}_{H_{ij}^{\alpha\beta}(t,h)}(s)\Lambda_i^{\alpha}(du)\right]\right] \tag{16}$$

and expected value $E\left[W_{\alpha\beta}^{ij}([t, t+h])\right] = \mu_{\alpha\beta}^{ij}(t)\, E\left[H_{ij}^{\alpha\beta}(t, h)\right]$ (17)

Proof:

Consider t>0 , i∈I j∈M and α,β∈A(fixed). then for some m ∈N

$$\tilde{P}\left[W_{ij}^{\alpha\beta}([0, t+h], [t, t+h])\right]$$

$$= \sum_{n=0}^{\infty} \tilde{P}\left(W_{ij}^{\alpha\beta}([t, t+h]) = m \middle| N_i^{\alpha}([t, t+h]) = n\right) . \tilde{P}[N_i^{\alpha}([t, t+h]) = n]$$

$$\tilde{g}_{W_{ij}^{\alpha\beta}([0,t+h],[t,t+h])} = \sum_{m=0}^{\infty} s^m \tilde{P}\left(W_{ij}^{\alpha\beta}(,[t, t+h]) = m\right)$$

$$= \sum_{m=0}^{\infty} s^m \sum_{n=0}^{\infty} \tilde{P}\left(W_{ij}^{\alpha\beta}([t, t+h]) = m \middle| N_i^{\alpha}([t, t+h]) = n\right) . \tilde{P}[N_i^{\alpha}([t, t+h]) = n]$$

$$= \sum_{n=0}^{\infty} \left[\tilde{g}_{H_{ij}^{\alpha\beta}(t,h)}(s)\right]^n . \tilde{P}[Z_{ij}^{\alpha\beta}(t) = n]$$

$$= \sum_{n=0}^{\infty} \left[\frac{1}{\Lambda_i^{\alpha}([t,t+h])} \int_t^{t+h} \tilde{g}_{H_{ij}^{\alpha\beta}(t,h)}(s)\Lambda_i^{\alpha} du\right]^n . \tilde{P}[N_i^{\alpha}([t, t+h]) = n]$$

$$= \exp\left[\frac{1}{\Lambda_i^{\alpha}([t,t+h])} \int_t^{t+h} \tilde{g}_{H_{ij}^{\alpha\beta}(t,h)}(s)\Lambda_i^{\alpha} du\right]$$

where we have used the fact that under the condition that n customers arrive to the system in [t , t+h] through node with attribute I, a random permutation of the corresponding arrival times is distributed as a sequene of n iid random variable with common distribution function $\Lambda_i^{\alpha}([t,t+u]) / \Lambda_i^{\alpha}([t,t+h])$,0<u<h.

$$\mathrm{E}\left[W_{ij}^{\alpha\beta}([0,t],[t,t+h])\right] = \frac{d}{ds}\tilde{g}_{W_{ij}^{\alpha\beta}(,[t,t+h])}(s) = \int_t^{t+h} E\left[H_{ij}^{\alpha\beta}(t+h-u)\right]\Lambda_i^{\alpha}(du)$$

5.3 Total number of visits to intermediate nodes:

Capitalizing on the results of the previous two subsections concerning, respectively, the number of visits to intermediate nodes in the time interval [t, t+h] made by customers arriving to the network before time t and after time t, we are now able to characterize the total number of visits to intermediate nodes in the time interval [t, t+h].

Theorem5.3.1:

For t>0 β∈M and j ∈A . $\{W_{ij}^{*\alpha\beta}([0,t],[t,t+h]), t>0\}$, α∈M and i∈A are independent poisson cluster process with probability generating functions

$$\tilde{g}_{W_{ij}^{\alpha\beta}([0,t+h],[t,t+h])}(s) =$$

exp

$$\left[-\mu_{ij}^{\alpha\beta}(t)\left[1-\tilde{g}_{\hat{H}_{ij}^{\alpha\beta}(t,h)}(s)\right]-\Lambda_i^{\alpha}([t,t+h]) + \left[\int_t^{t+h}\tilde{g}_{H_{ij}^{\alpha\beta}(t,h)}(s)\Lambda_i^{\alpha}(du)\right]\right] \qquad (18)$$

and expected value

$$\mathrm{E}\left[W_{\alpha\beta}^{ij}([t,t+h])\right] = \int_t^{t+h} E\left[H_{ij}^{\alpha\beta}(t+h-u)\right]\Lambda_i^{\alpha}(du) + E\left[\hat{H}_{ij}^{\alpha\beta}(t,h)\right]\mu_{ij}^{\alpha\beta}(t)$$

(19)

(19)

VI. ARRIVAL PROCESS TO OUTSIDE NODE

As a customer can make at most one visit to an outside node, the arrival processes to outsides are simpler than the corresponding processes to intermediate nodes. In fact, the arrival processes to outside nodes have properties similar to those of the arrival processes to input nodes as shown in the next theorem.

Theorem 6.1:

$\{\left[W_{\alpha\beta}^{ij}([0,t])\right], t\geq 0\}, i\in I, j\in O$ and α,β∈A are independent NFHPP with mean measure

$\mathrm{E}\left[W_{\alpha\beta}^{ij}([0,t])\right] = \int_t^{t+h} F_{ij}^{\alpha\beta}(t-s)\,\Lambda_i^{\alpha}(ds), t\geq 0$.Moreover, for fixed t , h≥0, $W_{ij}^{\alpha\beta}([0,t+h],[t,t+h]),\ i\in I, j\in O \text{ and } \alpha,\beta\in A$, are independent Fuzzy Poisson random variable with means.

$\mathrm{E}\left[W_{ij}^{\alpha\beta}([0,t+h],[t,t+h])\right]$

$$=\int_t^{t+h}[\tilde{F}_{ij}^{\alpha\beta}(t+h-s)-\tilde{F}_{ij}^{\alpha\beta}(t-s)^{-}]\Lambda_i^{\alpha}(ds)$$

$$+\int_t^{t+h}\tilde{F}_{ij}^{\alpha\beta}(t+h-s)\,\Lambda_i^{\alpha}(ds) \qquad (20)$$

Proof:

Consider t≥0, $i\in I, j\in O$ and $\alpha,\beta\in A$ fixed. We first note that the probability generating function given by is valid for $j\in O$, for which $H_{ij}^{\alpha\beta}(u)$, u ≥0, can assume only the values 0 and 1. therefore

$$\tilde{g}_{W_{ij}^{\alpha\beta}(,[0,t])}(s) = \exp\left[-\Lambda_i^{\alpha}([0,t]) + \left[\int_0^t \tilde{g}_{H_{ij}^{\alpha\beta}(t-u)}(s)\Lambda_i^{\alpha}(du)\right]\right]$$

$$= \exp\left[-\Lambda_i^{\alpha}([0,t]) + \left[\int_0^t[1-\tilde{F}_{ij}^{\alpha\beta}(t-u)+s\tilde{F}_{ij}^{\alpha\beta}(t-u)]\Lambda_i^{\alpha}(du)\right]\right]$$

$$= \exp\left[-(1-s)\left[\int_0^t[s\tilde{F}_{ij}^{\alpha\beta}(t-u)]\Lambda_i^{\alpha}(du)\right]\right]$$

which is the probability generating function of a Poisson random variable with mean $\int_0^t[s\tilde{F}_{ij}^{\alpha\beta}(t-u)]\Lambda_i^{\alpha}(du)$. Furher more by thinning of Poisson processes we conclude that $\{\left[W_{\alpha\beta}^{ij}([0,t])\right], t\geq 0\}, i\in I, j\in O$ and α,β∈ A are independent NFHPP

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