Image Restoration using Accelerated Proximal Gradient method

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Abstract - Digital images are becoming a preponderant tool in communication and especially in data transmission today. But one major problem in this method is that the image obtained after transmission is often corrupted by a variety of noise sources. Noise refers to stochastic variations are received image needs processing before it can be used for further applications. Image restoration involves elimination of noise. Filtering techniques were adapted so far to restore images since five decades. Here we consider the problem of image restoration degraded by a blur function and corrupted by random noise. A method for reducing additive noise in images by explicit analysis of local image statistics is introduced and compared to other reduction methods. The proposed method which makes use of a priori noise model has been evaluated on various types of images.

The total variation regularizer is well suited to piecewise smooth images. If we add the fact that these regularizers are convex, the reason for the resurgence of interest on TV-based approaches to inverse problems.TV-based algorithm for image deconvolution, under the assumption linear observations and additive white Gaussian noise. To compute the TV estimate. Bayesian based algorithms and techniques of image processing have been described and substantiated with experimentation using MATLAB.

KEY WORDS: additive white Gaussian Image restoration, linear observations, thresholding.

I. INTRODUCTION

MANY image processing problems can be formalized as estimating the original image **x** from a corrupted observation b produced by first applying a linear operator A to **x** and then adding noise. Subsampling may also follow, resulting in missing values. A typical linear operator A is usually ill-conditioned or even singular. Thus image restoration is a classical linear inverse problem [1]. To solve the linear inverse problem, one needs to involve a regularization term in the objective function to utilize the prior knowledge to recover the original image. Image restoration is usually formulated as the following convex minimization problem:

$$\min_{x \in Bl;u} (fx) = \frac{1}{2} \left|\left|A(x) - b\right|\right| 2f + \lambda \Phi\,(\mathrm{x}), \qquad (1)$$

where $\Phi(\mathbf{x})$ is a convex regularizer, λ is a regularization parameter [2], and Bl,u are the boundedness constraints on the restored image **x**:

$$Bl,u = \{\mathbf{x} \in R^{m\times n} | l \leq x_{i,j} \leq u,\ \forall i, j\}, \qquad (2)$$

in which l and u are the lower and upper bounds, respectively, and $m \times n$ is the size of the image. Several regularizers, e.g., total variation (TV) [3], waveletbased sparsity [4] [5], and non-local graph regularization [6] [7], have been proposed for image restoration. In this paper, I focus on TV-based image restoration. The TV model was first introduced in [3] as an effective regularizer. To date, this model has been widely adopted in many image processing problems, such as image denoising, blind deconvolution, compressed MR imaging , and microarray processing. Because of the non-smooth and non-differentiable property of the TV regularizer, it is difficult to solve TVbased image restoration by conventional optimization methods.

To date numerous methods, e.g., gradient-based methods ,dual methods, graph cut, and second-order cone programming, have been developed in different contexts. Now adays images acquired by digital cameras usually contain tens of megapixels, which makes TV-based image restoration inherently a large scale optimization problem. Recently continuous efforts have been spent on designing effective algorithms with less requirements on computational load and memory. One gradient-based method, the iterative shrinkage/ thresholding algorithm (IST), is developed for waveletbased deconvolution . Other researchers have also independently proposed IST in different contexts. Rigorous proofs of the convergence of IST have been provided in and. Subsequently, a generalized expectation maximization algorithm, also named as the iterative reweighted shrinkage method was proposed for image deconvolution.

Although simple and effective, IST has been known as a slow method, especially under some assumptions on the operator *A*. Recently, a number of accelerated IST algorithms have been proposed, and several of them, e.g., the two-step iterative shrinkage/thresholding (TwIST) algorithm and the fast iterative shrinkage/thresholding algorithm (FISTA), have been successfully applied to image restoration. FISTA is also known as the accelerated proximal gradient (APG) based method, which has an attractive convergence rate of $O(k^{-2})$, where *k* is the number of iterations. APG has also been applied to matrix completion and robust principal component analysis (RPCA).
In this paper, i extend the original APG method to a more general and efficient class, called the generalized accelerated proximal gradient (GAPG) method, which maintains the convergence rate of $O(k^{-2})$.

When applied to TV-based image restoration, the TwIST and FISTA algorithms involve both outer and inner iterations, where the inner iteration is to approximately solve an image denoising subproblem. Recently, several image denoising methods, e.g., the dual approach by Chambolle and the maximum flow algorithm by Goldfarb and Yin, have been proposed for solving the TV-based denoising problem. We expect that the efficiency of image restoration would be further improved if the image denoising subproblem could be avoided.

In this paper, by generalizing the FISTA method with a constant step size , I propose a GAPG method with a proven $O(k^{-2})$ convergence rate. Motivated by the variable splitting method, we further introduce two auxiliary variables to approximate the partial derivatives and reformulate the TV-based image restoration problem as an unconstrained convex optimization problem. Moreover, the problem reformulation is more suitable for improving the efficiency of GAPG, and we can also avoid solving the image denoising subproblem. Finally the GAPG framework is combined with the continuation technique to solve the resulting optimization problem. Our method works for both anisotropic and isotropic discrete TV-based image restoration. Numerical results demonstrate the efficiency of the proposed method: our algorithms are much faster than the monotone version of TwIST (MTwIST) and the monotone version of FISTA (MFISTA) for TV-based image restoration. Further, our GAPG converges faster than the original APG and MTwIST when solving the same optimization problem.

The remainder of this paper is organized as follows. Section II introduces some background knowledge that is necessary for our paper, including the TV-based image restoration models.
Section III presents our GAPG method and introduce the problem formalizations and algorithms for anisotropic and isotropic discrete TV-based image restoration, respectively. Then Section IV presents the experimental results.Finally Section V concludes the paper.

II-THE GENERALIZED ACCELERATED PROXIMAL GRADIENT METHOD

In this section, we propose a generalized accelerated proximal gradient (GAPG) method to solve the minimization problem with an objective function.

$$f(x) \le f(y) + (x-y, \nabla f(y)) + \frac{L_f}{2}\|x-y\|_F^2, \forall x,y \qquad (3)$$

is the key to proving the $O(k^{-2})$ convergence rate of the original APG method, and the Lipschitz gradient condition is just to ensure that (3) holds. Given a positive definite matrix **L**, we may introduce the **L**-inner product$(x,y)_L = X^T Ly$ and the **L**-norm $\|X\|_L = \sqrt{(X,X)}L$. Then the inequality (3) can be generalized as

$$f(x) \leq f(y) + (x-y, \nabla f(y)) + \frac{1}{2}||x-y||^2_{L_f}, \forall x,y, \quad (4)$$

where **L**f is a positive definite matrix. Such **L**f exists for a broad class of f. For example, if f satisfies the Lipschitz gradient condition (15), then **L**f can be chosen as Lf **I**, where **I** is the identity matrix. We will give other choices of **L**f in our image restoration problems. The motivation to replace the inequality (3) with (4) is that a smaller Lipschitz constant may lead to faster convergence (c.f. (2)).

Analogously, GAPG updates **x** by minimizing a quadratic approximation Q**L**f (**x**, **y**) of F(**x**) at specially chosen points **y**, where

$$Q_{L_f}(X,Y)=f(Y)+(\nabla f(y),X-Y)+\frac{1}{2}||X-Y||^2_{L_f}+G(X) \quad (5)$$

and $\mathbf{y}_k$ is still chosen. For the subproblem $p_{\mathbf{L}_f}(\mathbf{y}_k) = \mathrm{argmin}_{\mathbf{x}}\, Q_{\mathbf{L}_f}(\mathbf{x}, \mathbf{y}_k)$ to be easy to solve, we usually choose a diagonal $\mathbf{L}_f$. Then we straightforwardly generalize the original APG method with a constant step size by the GAPG algorithm .

The Generalized Accelerated Proximal Gradient Algorithm

1: while not converged do
2: $\mathbf{x}k \leftarrow p\mathbf{L}f(\mathbf{y}k)$,
3: $tk+1 \leftarrow \frac{1+\sqrt{1+4t_k^2}}{2}$
4: $\mathbf{y}k+1 \leftarrow \mathbf{x}k + \frac{t_k-1}{t_k+1}(\mathbf{x}k - \mathbf{x}k\text{-}1)$
5: $k \leftarrow k + 1$.
6: end while

By replacing $Lf\,\langle\cdot,\cdot\rangle$ and $Lf\,||.||^2_F$ in the proofs in [2] with $\langle\cdot,\cdot\rangle\mathbf{L}f$ and $||.||^2_{L_f}$ respectively, we can easily prove the $O(k^{-2})$convergence rate of GAPG as stated in the following theorem.

Theorem 2: Let {**x**k} and {**y**k} be generated by GAPG.Then for any $k \geq 1$, where **x_** is any optimal solution.

$$F(X_k)- F(X^*)\leq \frac{2||x_0-x^*||^2_{L_f}}{(k+1)^2}, \quad (6)$$

One can easily see that if **L**$f = Lf$ **I**, then GAPG reduces to the original APG. However, as one will see **L**f can have other choices such that (5) holds and $||X||\mathbf{L}f \leq Lf\,||X||_F$ for any **x**, resulting in faster convergence.

now summarize our GAPG-based anisotropic TV-based image restoration algorithm . Note that in real computation the matrix-vector products in Algorithm 2 should be replaced by operators acting on images.
For example, $\mathbf{A}^T(\mathbf{A}\mathbf{y}_x^k - \mathbf{b})$ is actually computed as $A^*(A(\mathbf{y}_x^k)\mathbf{-b})$.

Compared with MTwIST and MFISTA, the proposed algorithm is significantly faster with better or at least comparable restoration quality. The proposed algorithm is more suitable for fast image restoration thanks to three techniques. The first is the problem formalization (6) that involves auxiliary variables to decouple the problem, with which we need not solve the TV-based denoising problem (4) in each iteration. The second is the GAPG method, which is expected to be more efficient than the original APG by choosing an appropriate matrix **L**f such that different variables can have different "Lipschitz constants". The third is the continuation technique to provide good initial solutions for our GAPG method.

It is also worth noting that both MFISTA and MTwIST need to evaluate the objective function values during the iterations in order to ensure that the objective function values decrease with iterations. In contrast, our GAPG algorithm does not require such extra computation.

Algorithm 2 Anisotropic TV-based Image Restoration via GAPG

Input: Observed image **b**, λ, $\mu 0$, **A**, **D**v, and **D**h
Output: $\mathbf{x} \leftarrow \mathbf{x}k$.
1: $\mathbf{x}0 \leftarrow PBl;u\,(\mathbf{b})$, $d_v^0 \leftarrow \mathbf{D}v\,, X_0$, $d_h^0 \leftarrow \boldsymbol{D}_h X_0, t_0 \leftarrow 1$
$\mu \leftarrow \delta\mu 0$, $y_{d_u}^1 \leftarrow \mathbf{x}0$, $y_{d_u}^1 \leftarrow d_h^0, k \leftarrow 1$.
2: while not converged do
3: // Line 4 solves subproblem (46).
4: $\mathbf{x}k \leftarrow PB_{l,u}\{ Y_x^k - \lambda_{max}^{-1}[A^T(\mathbf{A}Y_X^V - \mathbf{b}) + \boldsymbol{D}_U^T(\mathbf{D}_U\ Y_x^k - Y_{D_U}^k) + D_h^T(D_h\,Y_x^k - Y_{D_U}^k)]\}$
5: // Lines 6-7 solve subproblems (47) and (48).
6: $D_U^K \leftarrow TT_{\lambda\mu k}\ /n(Y_{D_V}^k - \frac{1}{n}\,(Y_{D_V}^k - \mathbf{D}_U\ Y_x^k))$,
7. $D_h^K \leftarrow TT_{\lambda\mu k}\ /n(Y_{D_h}^k - \frac{1}{n}\,(Y_h^k - \mathbf{D}_h\ Y_x^k))$,
8: $tk{+}1 \leftarrow \frac{1+\sqrt{1+4(t_K)^2}}{2}$
9: // Lines 10-12 update $Y_X^{K+1}, Y_{d_u}^{K+1}, Y_{d_h}^{K+1}$
10. $Y_X^{K+1} \leftarrow X_k + \frac{t_k - 1}{t_k + 1}(X_k - X_{k-1})$,
11. $Y_{d_v}^{K+1} \leftarrow d_v^k + \frac{t_k - 1}{t_k + 1}(d_v^k - d_v^{k-1})$,
12. $Y_{d_h}^{K+1} \leftarrow d_h^k + \frac{t_k - 1}{t_k + 1}(d_h^k - d_h^{k-1})$,
13: Update μk to $\mu k{+}1$,
14: $k \leftarrow k + 1$
15: end while

Problem Formalization
The isotropic TV-based image restoration problem is formalized As

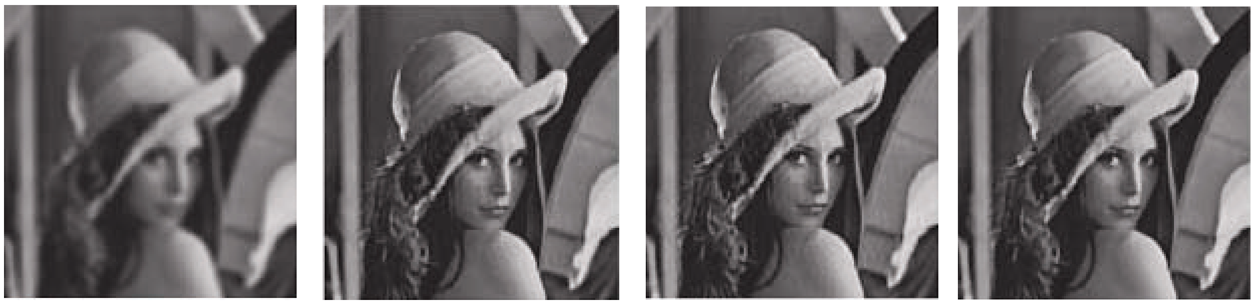
$$\min_{X \epsilon B_{l,u}} \frac{1}{2}||AX - b||_F^2 + \lambda TV_{iso}\,(X), \qquad (7)$$

where TViso(**x**) is defined as (4). Like the anisotropic case,by introducing two new variables **d**v and **d**h, imposing the boundedness constraints on **d**v and **d**h and using isotropic TV induced norm , the problem can be rewritten as:

$$\min_{X, d_v, d_h} \frac{\mu}{2}\,||\mathrm{AX} - b||_F^2 + \frac{1}{2}\,||\ d_v - d_v\mathrm{X}\ ||_F^2 + \frac{1}{2}\,||\ d_h - d_h\mathrm{X}\ ||_F^2 + \lambda\mu\ (\ ||d_v, d_h)||_{It} + \mathrm{x}\ B_{t,u}(\mathrm{x})).$$

III. EXPERIMENTAL RESULTS

In this section, we present the results of the proposed method for image restoration from blurred and noisy images and from incomplete samples (inpainting). In our experiments, we only test the algorithms for solving isotropic TV-based image restoration problems. Then we compare the computational time and the peak signal-to-noise ratio (PSNR) of the proposed method with two recent methods, MTwIST and MFISTA . In order to verify the efficiency of GAPG, we also compare the convergence speed of GAPG, the original APG, and MTwIST using the same image restoration problem formulation.
All the methods are implemented in MATLAB and are tested on a computer with a Core 2 Quad Q6600 processor running at 2.40GHz.



Blurred and noisy MTwIST MFISTA GAPG

Fig. 1. Isotropic TV-based image restoration from a blurred and noisy Lena image. top left: blurred and noisy Lena; top right: restored image using MTwIST; bottom left: restored image using MFISTA; bottom right: restored image using GAPG.

Blurred and noisy MTwIST MFISTA GAPG

Fig. 2. Isotropic TV-based image restoration from a blurred and noisy Cameraman image. top left: blurred and noisy Cameraman; top right: restored image using MTwIST; bottom left: restored image using MFISTA; bottom right: restored image using GAPG.

TABLE I
IMAGE RESTORATION: CPU TIMES AND PSNR VALUES OBTAINED USING MTWIST, MFISTA, AND GAPG FOR THE LENA IMAGE

	PSNR (dB)	CPU times (*s*)
MTwIST (*k*=100) [14] 2	29.00	20.21
MFISTA (*k*=100) [21]	29.13	30.79
GAPG (*k*=150)	29.23	4.86

TABLE II
IMAGE RESTORATION: CPU TIMES AND PSNR VALUES OBTAINED USING MTWIST, MFISTA, AND GAPG FOR THE CAMERAMAN IMAGE

	PSNR (dB)	CPU times (*s*)
MTwIST (*k*=100) [14] 2	27.33	20.83
MFISTA (*k*=100) [21]	27.56	31.86
GAPG (*k*=150)	27.66	4.91

IV.CONCLUSION

Here presented a new Bayesian framework for image restoration that uses a product-based Student's-t type of priors. The main theoretical contribution of this work is that by constraining the approximation of the posterior in the variational framework, we bypass the need for knowing the normalization constant of this prior. Thus, we avoid having to use improper priors, i.e. priors whose normalization constant is empirically selected .Furthermore, the proposed methodology does not require empirical parameter selection as in the MAP methodology that uses a similar-in-spirit prior. it also presented a Lanczos-based computational scheme tailored to the computations required by our algorithm. More specifically, it appears that the herein proposed approach is more competitive in the higher *BSNR* cases. Here to solve anisotropic and isotropic TV-based image restoration. First, we extend the original APG method with a constant step size to propose a generalized accelerated proximal gradient method, such that different variables can have different "Lipschitz constants". The appealing convergence rate $O(k^{-2})$ is maintained by the GAPG method and our numerical results show that GAPG converges faster than the original APG. Second, by introducing two auxiliary variables, we are able to decompose the problem into much smaller problems that can be solved with high parallelism. Finally, we also adopt the common continuation technique to gradually reduce the relaxation parameter in order to provide good initial solutions.

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