

# $g^*$ - Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

Rekha Chaturvedi

*Department of Engineering Mathematics  
St. Aloysius Institute of Technology  
Jabalpur, (M.P.), India*

**Abstract - The purpose of this paper to introduce the concept of Intuitionistic fuzzy  $g^*$  - continuous mappings in Intuitionistic Fuzzy Topological Spaces and investigate some of its basic properties.**

**Keywords: Intuitionistic fuzzy  $g^*$ -closed sets, intuitionistic fuzzy,  $g^*$ - continuous mappings, intuitionistic fuzzy  $g^*$ -closure. 2000, Mathematics Subject Classification, 54A99(03E99), 54C08**

## I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [15] in 1965 and fuzzy topology by Chang [4] in 1967. Several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 20 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [8], fuzzy connectedness [14], fuzzy multifunctions [9] fuzzy  $g$ -closed set and fuzzy  $g$ -continuity have been generalized for intuitionistic fuzzy topological spaces. In the present paper we introduce and study the concept of intuitionistic fuzzy  $g^*$  -continuous mappings in intuitionistic fuzzy topological space.

## II. PRELIMINARIES

Definition 2.1: [1] Let  $X$  be a nonempty fixed set. An intuitionistic fuzzy set  $A$  in  $X$  is an object having the form

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$$

where the functions  $\mu_A: X \rightarrow I$  and  $\gamma_A: X \rightarrow I$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of nonmembership (namely  $\gamma_A(x)$ ) of each element  $x \in X$  to the set  $A$ , respectively, and  $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$  for each  $x \in X$ .

Definition 2.2: [1] Let  $X$  be a nonempty set and the intuitionistic fuzzy sets  $A$  and  $B$  be in the form  $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$ ,  $B = \{ \langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X \}$  and let  $\{A_i : i \in J\}$  be an arbitrary family of intuitionistic fuzzy sets in  $X$ . Then

$$A \subseteq B \text{ if } \forall x \in X [\mu_A(x) \leq \mu_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x)];$$

$$A = B \text{ if } A \subseteq B \text{ and } B \subseteq A;$$

$$A^c = \{ \langle x, \gamma_A(x), \mu_A(x) \rangle : x \in X \};$$

$$\bigcap A_i = \{ \langle x, \wedge \mu_{A_i}(x), \vee \gamma_{A_i}(x) \rangle : x \in X \};$$

$$\bigcup A_i = \{ \langle x, \vee \mu_{A_i}(x), \wedge \gamma_{A_i}(x) \rangle : x \in X \};$$

$$\tilde{0} = \{ \langle x, 0, 1 \rangle : x \in X \}, \text{ and } \tilde{1} = \{ \langle x, 1, 0 \rangle : x \in X \}$$

Definition 2.3[5]: Let  $(X, \mathfrak{F})$  be an intuitionistic fuzzy topological space and

$A = \langle x, \mu_A(x), \gamma_A(x) \rangle$  be an intuitionistic fuzzy set in  $X$ . Then the fuzzy interior and fuzzy closure of  $A$  are defined by

$$cl(A) = \bigcap \{ K : K \text{ is an intuitionistic fuzzy closed set in } X \text{ and } A \subseteq K \},$$

$$int(A) = \bigcup \{ G : G \text{ is an intuitionistic fuzzy open set in } X \text{ and } G \subseteq A \}.$$

Definition 2.4 [5]: An intuitionistic fuzzy topology on a nonempty set  $X$  is a family  $\mathfrak{F}$  of intuitionistic fuzzy sets in  $X$  satisfy the following axioms:

(T1)  $\emptyset, \tilde{I} \in \mathfrak{S}$

(T2)  $G1 \cap G2 \in \mathfrak{S}$  for any  $G1, G2 \in \mathfrak{S}$ ,

(T3)  $\cup G_i \in \mathfrak{S}$  for any arbitrary family  $\{ G_i : i \in J \} \subseteq \mathfrak{S}$ .

In this case the pair  $(X, \mathfrak{S})$  is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in  $\mathfrak{S}$  is known as an intuitionistic fuzzy open set in  $X$ . The complement  $A^c$  of an intuitionistic fuzzy open set  $A$  is called an intuitionistic fuzzy closed set in  $X$ .

Definition 2.5 [5]: Two intuitionistic fuzzy sets  $A$  and  $B$  of  $X$  is said to be  $q$ -coincident ( $AqB$  for short) if and only if there exists an element  $x \in X$  such that

$$\mu_A(x) > \gamma_B(x) \text{ or } \gamma_A(x) < \mu_B(x)$$

Lemma 2.1 [5]: For any two intuitionistic fuzzy sets  $A$  and  $B$  of  $X$ .

$$\bigcap (AqB) \Leftrightarrow A \subseteq B^c.$$

Definition 2.6[5]: Let  $X$  and  $Y$  be two nonempty sets and  $f : X \rightarrow Y$  be a function. Then

(a) If  $B = \{ \langle y, \mu_B(y), \gamma_B(y) \rangle : y \in Y \}$  is an intuitionistic fuzzy set in  $Y$ , then the preimage of  $B$  under  $f$  denoted by  $f^{-1}(B)$ , is the IFS in  $X$  defined by

$$f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \rangle : x \in X \}$$

(b) If  $A = \{ \langle x, \lambda_A(x), \nu_A(x) \rangle : x \in X \}$  is an intuitionistic fuzzy set in  $X$ , then the image of  $A$  under  $f$  denoted by  $f(A)$  is the intuitionistic fuzzy set in  $Y$  defined by

$$f(A) = \{ \langle y, f(\lambda_A)(y), f(\nu_A)(y) \rangle : y \in Y \}$$

$$\text{where } f(\nu_A) = 1 - f(1 - \nu_A).$$

Definition 2.7[8] : Let  $(X, \mathfrak{S})$  and  $(Y, \Phi)$  be two intuitionistic fuzzy topological spaces and let  $f : X \rightarrow Y$  be a function. Then

$f$  is said to be intuitionistic fuzzy continuous if the preimage of each intuitionistic fuzzy open set in  $Y$  is an intuitionistic fuzzy open set in  $X$ .

$f$  is said to be intuitionistic fuzzy  $g$ -continuous if the inverse image of every intuitionistic fuzzy closed set of  $Y$  is intuitionistic fuzzy  $g$ -closed set in  $X$ .

Definition 2.8[12]: An intuitionistic fuzzy set  $A$  of an intuitionistic fuzzy topological space  $(X, \mathfrak{S})$  is called an intuitionistic fuzzy generalized closed (intuitionistic fuzzy  $g$ -closed) if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy open.

Definition 2.9[12]: Complement of an intuitionistic fuzzy  $g$ -closed set is called intuitionistic fuzzy  $g$ -open set.

Remark 2.1 [12]: Every intuitionistic fuzzy closed (intuitionistic fuzzy open) set is intuitionistic fuzzy  $g$ -closed (intuitionistic fuzzy  $g$ -open) set but its converse may not be true.

Definition 2.10[13]: An intuitionistic fuzzy topological space  $X$  is called intuitionistic fuzzy  $GO$ -connected if there is no proper intuitionistic fuzzy set of  $X$  which is both intuitionistic fuzzy  $g$ -open and intuitionistic fuzzy  $g$ -closed.

Remark 2.2: Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy  $g$ -continuous, but the converse may not be true.

Definition 2.11[13]: Let  $(X, \mathfrak{S})$  be an intuitionistic fuzzy topological space. The generalized closure of a intuitionistic fuzzy set  $A$  of  $X$  denoted by  $gcl(A)$  is the intersection of all intuitionistic fuzzy  $g$ -closed sets of  $X$  which contains  $A$ .

Definition 2.12[5]: Let  $(X, \mathfrak{S})$  be an intuitionistic fuzzy topological space and  $Y$  be a crisp subset of  $X$ . Then the induced intuitionistic fuzzy topology (relative intuitionistic fuzzy topology)  $\mathfrak{S}_Y$  for  $Y$  is defined by

$$\mathfrak{Y} = \{Y \cap O : O \in \mathfrak{X}\}.$$

The pair  $(Y, \mathfrak{Y})$  is called a subspace of  $(X, \mathfrak{X})$ .

Lemma 2.2[5]: Let  $(Y, \mathfrak{Y})$  be a subspace of an intuitionistic fuzzy topological space  $(X, \mathfrak{X})$  and  $A$  be an intuitionistic fuzzy set of  $Y$ . Then

$A$  is  $\mathfrak{Y}$ -intuitionistic fuzzy closed if and only if there exists an intuitionistic fuzzy open set  $F$  in  $X$  such that  $A = F \cap Y$

$$Cl_{\mathfrak{Y}}(A) = Y \cap Cl_{\mathfrak{X}}(A).$$

### III. INTUITIONISTIC FUZZY $g^*$ -CONTINUOUS MAPPINGS

Definition 3.1: A mapping  $f : (X, \mathfrak{X}) \rightarrow (Y, \sigma)$  is said to be intuitionistic fuzzy  $g^*$ -continuous if the inverse image of every intuitionistic fuzzy closed set of  $Y$  is intuitionistic fuzzy  $g^*$ -closed in  $X$ .

Remark(3.1): Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy  $g^*$ -continuous and intuitionistic fuzzy  $g^*$ -continuous mapping but the converses may not be true.

Example (3.1): Let  $X = \{a, b\}$  and  $Y = \{x, y\}$  and the intuitionistic fuzzy sets  $U, V, W$  are defined as follows:

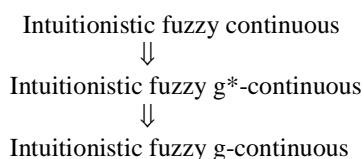
$$U = \{\langle a, 0.5, 0.4 \rangle, \langle b, 0.6, 0.4 \rangle\}$$

$$V = \{\langle x, 0.6, 0.3 \rangle, \langle y, 0.6, 0.4 \rangle\}$$

$$W = \{\langle x, 0.3, 0.6 \rangle, \langle y, 0.4, 0.6 \rangle\}$$

Let  $\mathfrak{X} = \{\tilde{0}, U, \tilde{1}\}$  be intuitionistic fuzzy topology on  $X$  and  $\sigma = \{\tilde{0}, V, \tilde{1}\}$  and  $\sigma^* = \{\tilde{0}, W, \tilde{1}\}$  be intuitionistic fuzzy topologies on  $Y$ . Then the mapping  $f : (X, \mathfrak{X}) \rightarrow (Y, \sigma)$  defined by  $f(a) = x$  and  $f(b) = y$  is intuitionistic fuzzy  $g^*$ -continuous but not intuitionistic fuzzy continuous and the mapping  $g : (X, \mathfrak{X}) \rightarrow (Y, \sigma^*)$  defined by  $g(a) = x$  and  $g(b) = y$  is intuitionistic fuzzy  $g$ -continuous but not intuitionistic fuzzy  $g^*$ -continuous.

Following diagram shows the implication between the various classes of generalized continuity in intuitionistic fuzzy topological space.



Theorem 3.1: A mapping  $f : (X, \mathfrak{X}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy  $g^*$ -continuous if and only if the inverse image of every intuitionistic fuzzy open set of  $Y$  is intuitionistic fuzzy  $g^*$ -open in  $X$ .

Proof: It is obvious because  $f^{-1}(U^c) = (f^{-1}(U))^c$  for every intuitionistic fuzzy set  $U$  of  $Y$ .

Theorem 3.2: If  $f : (X, \mathfrak{X}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy  $g^*$ -continuous then for each intuitionistic fuzzy point  $c(\alpha, \beta)$  of  $X$  and each intuitionistic fuzzy open set  $V$ ,  $f(c(\alpha, \beta)) \subseteq V$  there exists a intuitionistic fuzzy  $g^*$ -open set  $U$  such that  $c(\alpha, \beta) \subseteq U$  and  $f(U) \subseteq V$ .

Proof: Let  $c(\alpha, \beta)$  be an intuitionistic fuzzy point of  $X$  and  $V$  be an intuitionistic fuzzy open set such that  $f(c(\alpha, \beta)) \subseteq V$ , put  $U = f^{-1}(V)$  then by hypothesis  $U$  is intuitionistic fuzzy  $g^*$ -open set of  $X$  such that  $c(\alpha, \beta) \subseteq U$  and  $f(U) = f(f^{-1}(V)) \subseteq V$ .

Theorem 3.3: If  $f : (X, \mathfrak{X}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy  $g^*$ -continuous then for each intuitionistic fuzzy point  $c(\alpha, \beta)$  in  $X$  and each intuitionistic fuzzy open set  $V$  of  $Y$  such that  $c(\alpha, \beta) \subseteq V$  and  $f(U) \subseteq V$ .

Proof: Let  $c(\alpha, \beta)$  be an intuitionistic fuzzy point of  $X$  and  $V$  be an intuitionistic fuzzy open set of  $Y$  such that  $f(c(\alpha, \beta)) \subseteq V$ , put  $U = f^{-1}(V)$  then by hypothesis  $U$  is intuitionistic fuzzy  $g^*$ -open set of  $X$  such that  $c(\alpha, \beta) \subseteq U$  and  $f(U) = f(f^{-1}(V)) \subseteq V$ .

Definition 3.2: Let  $(X, \mathfrak{X})$  be an intuitionistic fuzzy topological space. The  $g^*$ -closure of an intuitionistic fuzzy set  $A$  of  $X$  denoted by  $g^*cl(A)$  is the intersection of all intuitionistic fuzzy  $g^*$ -closed sets of  $X$  which contains  $A$ .

Remark(3.2): It is clear that,  $A \subseteq g^*cl(A) \subseteq gcl(A) \subseteq cl(A)$  for any intuitionistic fuzzy set  $A$  of  $X$ .

Theorem 3.4: If  $f : (X, \mathfrak{X}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy  $g^*$ -continuous, then  $f(g^*cl(A)) \subseteq cl(f(A))$  for every intuitionistic fuzzy set  $A$  of  $X$ .

Proof: Let  $A$  be an intuitionistic fuzzy set of  $X$ . Then  $cl(f(A))$  is an intuitionistic fuzzy closed set of  $Y$ . Since  $f$  is intuitionistic fuzzy  $g^*$ -continuous  $f^{-1}(cl(f(A)))$  is intuitionistic fuzzy  $g^*$ -closed in  $X$ . Clearly  $A \subseteq f^{-1}(cl(f(A)))$ . Therefore  $g^*cl(A) \subseteq g^*cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$ . Hence  $f(g^*cl(A)) \subseteq cl(f(A))$ .

Definition 3.2: An intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is said to be  $T^{*1/2}$ -space if every intuitionistic fuzzy  $g^*$ -closed set in  $X$  is intuitionistic fuzzy closed in  $X$ .

Theorem 3.5: A mapping  $f$  from an intuitionistic fuzzy  $T^{*1/2}$ -space  $(X, \mathfrak{T})$  to an intuitionistic fuzzy topological space  $(Y, \sigma)$  is intuitionistic fuzzy continuous if and only if it is intuitionistic fuzzy  $g^*$ -continuous.

Proof: Necessity: Follows from Remark(3.2).

Sufficiency: If  $A$  is intuitionistic fuzzy closed in  $Y$ , then  $f^{-1}(A)$  is intuitionistic fuzzy  $g^*$ -closed in  $X$  because  $f$  is intuitionistic fuzzy  $g^*$ -continuous. Since  $X$  is  $T^{*1/2}$ -space. Therefore  $f^{-1}(A)$  is intuitionistic fuzzy closed in  $X$ . Hence  $f$  is intuitionistic fuzzy continuous.

Theorem 3.6: If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy  $g^*$ -continuous and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  is intuitionistic fuzzy continuous. Then  $g \circ f: (X, \mathfrak{T}) \rightarrow (Z, \eta)$  is intuitionistic fuzzy  $g^*$ -continuous

Proof: Let  $A$  be an intuitionistic fuzzy closed set in  $Z$ , then  $f^{-1}(A)$  is intuitionistic fuzzy closed in  $Y$  because  $g$  is intuitionistic fuzzy continuous. Therefore  $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$  is intuitionistic fuzzy  $g^*$ -closed in  $X$ . Hence  $g \circ f$  is intuitionistic fuzzy  $g^*$ -continuous.

Theorem 3.7: If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  is intuitionistic fuzzy  $g^*$ -continuous mappings and  $(Y, \sigma)$  is intuitionistic fuzzy  $T^{*1/2}$ -space then  $g \circ f: (X, \mathfrak{T}) \rightarrow (Z, \eta)$  is intuitionistic fuzzy  $g^*$ -continuous.

Proof: Obvious.

Theorem 3.8: Intuitionistic fuzzy  $g^*$ -continuous image of an intuitionistic fuzzy  $g^*$ -compact space is intuitionistic fuzzy compact.

Proof: Let  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  be an intuitionistic fuzzy  $g^*$ -continuous map from an Intuitionistic fuzzy  $g^*$ -compact space  $(X, \mathfrak{T})$  on to an intuitionistic fuzzy topological space  $(Y, \sigma)$ . Let  $\{A_i: i \in \Lambda\}$  be an intuitionistic fuzzy  $g^*$ -open cover of  $Y$  then  $\{f^{-1}(A_i): i \in \Lambda\}$  is an intuitionistic fuzzy  $g^*$ -open cover of  $X$ . Since  $X$  is intuitionistic fuzzy  $g^*$ -compact it has finite intuitionistic fuzzy subcover say  $\{f^{-1}(A_1), \dots, f^{-1}(A_n)\}$ . Since  $f$  is onto  $\{A_1, A_2, \dots, A_n\}$  is an intuitionistic fuzzy open cover of  $Y$  and so  $(Y, \sigma)$  is intuitionistic fuzzy compact.

Definition 3.3: An intuitionistic fuzzy topological space  $X$  is called intuitionistic fuzzy  $g^*$ -connected if there is no proper intuitionistic fuzzy set of  $X$  which is both intuitionistic fuzzy  $g^*$ -open and set in  $X$  is intuitionistic fuzzy  $g^*$ -closed in  $X$ .

Theorem: 3.9: An intuitionistic fuzzy  $T^{*1/2}$ -space is intuitionistic fuzzy  $C_5$ -connected if and only if it is intuitionistic fuzzy  $g^*$ -connected.

Proof: Obvious.

Theorem 3.10: If  $f: (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is an intuitionistic fuzzy  $g^*$ -continuous surjection and  $X$  is intuitionistic fuzzy  $g^*$ -connected then  $Y$  is intuitionistic fuzzy  $C_5$ -connected.

Proof: Suppose  $Y$  is intuitionistic fuzzy  $C_5$ -connected. Then there exists a proper intuitionistic fuzzy set  $G$  of  $Y$  which is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Therefore  $f^{-1}(G)$  is proper intuitionistic fuzzy  $g^*$ -closed and intuitionistic fuzzy  $g^*$ -open set of  $X$ , because  $f$  is intuitionistic fuzzy  $g^*$ -continuous surjection. Hence  $X$  is not intuitionistic fuzzy  $g^*$ -connected.

## REFERENCES

- [1] K. Atanassova, Intuitionistic Fuzzy Sets, In VII ITKR's Session, (V.Sgurev, Ed.) Sofia, Bulgaria, (1983)
- [2] K. Atanassov and S. Stoeva., Intuitionistic Fuzzy Sets, In Polish Symposium on Interval and Fuzzy Mathematics, Poznan, (1983), 23-26
- [3] K. Atanassov, Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 20(1986), 87-96.
- [4] C.L. Chang, Fuzzy Topological Spaces, J.Math.Anal.Appl. 24(1968) 182-190.
- [5] D. Coker, An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets and Systems .88(1997), 81-89.
- [6] D.Coker and A. Es. Hyder., On Fuzzy Compactness in Intuitionistic Fuzzy Topological Spaces, The Journal of Fuzzy Mathematics, 3-4(1995), 899-909.
- [7] H. Gurcay, D. Coker and Es., A.Haydar, On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces. The Journal of Fuzzy Mathematics Vol.5, no.2, 1997, 365-378.
- [8] N. Levine., Generalized Closed Sets In Topology, Rend. Cerc. Mat. Palermo.19(2), 1970, 571-599.
- [9] O. Ozbakir and D. Coker., Fuzzy Multifunctions in Intuitionistic Fuzzy Topological Spaces Notes on IFS 5(1999) No. 3.
- [10] S.S. Thakur and Malviya R., Generalized Closed Sets In Fuzzy Topology, Math. Notae 38(1995), 137-140.
- [11] S.S. Thakur and Khare R.K., Fuzzy regular Generalized Closed Sets. Varahmihir Journal of Mathematical sciences. Vol.3(1)(2003), 65-70.
- [12] S.S Thakur and Rekha Chaturvedi, Generalized Closed Sets in Intuitionistic Fuzzy Topology. The Journal of Fuzzy Mathematics Vol.16(2)(2008).
- [13] S.S Thakur and Rekha Chaturvedi, Generalized Continuity in Intuitionistic Fuzzy Topological Spaces. Notes on Intuitionistic Fuzzy Sets. Vol.12(1)(2006), 38-44.

- [14] N. Turnali and D. Coker, Fuzzy Connectedness in Intuitionistic Fuzzy Topological Spaces. Fuzzy Sets And Systems 116(2000) (3), 369-375.
- [15] L.A.Zadeh, Fuzzy Sets, Information and Control, 18(1965), 338-353.