g*- Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract - The purpose of this paper to introduce the concept of Intuitionistic fuzzy g* - continuous mappings in Intuitionistic Fuzzy Topological Spaces and investigate some of its basic properties.

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I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [15] in 1965 and fuzzy topology by chang [4] in 1967. Several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets . In the last 20 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [8], fuzzy connectedness [14], fuzzy multifunctions [9] fuzzy g-closed set and fuzzy g-continuity have been generalized for intuitionistic fuzzy topological spaces. topological space.In the present paper we introduce and study the concept of intuitionistic fuzzy g* -continuous mappings in intuitionistic fuzzy topological space.

II. PRELIMINARIES

Definition 2.1: [1] Let X be a nonempty fixed set. An intuitionistic fuzzy set A in X is an object having the form

$$A=\{<x,\mu A(x), \gamma A(x)> : x\in X \}$$

where the functions $\mu A:X\to I$ and $\gamma A: X\to I$ denote the degree of membership (namely $\mu A(x)$) and the degree of nonmembership (namely $\gamma A(x)$) of each element $x\in X$ to the set A, respectively, and $0 \le \mu A(x)+ \gamma A(x) \le 1$ for each $x\in X$.

Definition 2.2: [1] Let X be a nonempty set and the intuitionistic fuzzy sets A and B be in the form $A = \{<x, \mu A(x), \gamma A(x)> : x \in X\}$, $B = \{<x, \mu B(x), \gamma B(x)> : x \in X\}$ and let $\{Ai : i \in J\}$ be an arbitrary family of intuitionistic fuzzy sets in X. Then

$A \subseteq B$ if $\forall x \in X\ [\mu A(x) \le \mu B(x)$ and $\gamma A(x) \ge \gamma B(x)]$;
$A = B$ if $A \subseteq B$ and $B \subseteq A$;
$Ac= \{<x, \gamma A(x), \mu A(x)> : x \in X\}$;
$\cap Ai = \{<x, \wedge\mu Ai(x), \vee\gamma Ai(x)> : x \in X\}$;
$\cup Ai = \{<x, \vee\mu Ai(x), \wedge\gamma Ai(x)> : x \in X\}$;
$\tilde{0} = \{<x, 0, 1> : x \in X\}$, and $\tilde{1} =\{<x, 1, 0> : x \in X\}$

Definition 2.3[5]: Let $(X, \Im)$ be an intuitionistic fuzzy topological space and
$A = <x, \mu A(x),\gamma A(x)>$ be an intuitionistic fuzzy set in X. Then the fuzzy interior and fuzzy closure of A are defined by

$cl(A) = \cap\{K : K$ is an intuitionistic fuzzy closed set in X and $A \subseteq K\}$,
$int(A) = \cup\{G : G$ is an intuitionistic fuzzy open set in X and $G \subseteq K\}$.

Definition 2.4 [5]: An intuitionistic fuzzy topology on a nonempty set X is a family $\Im$ of intuitionistic fuzzy sets in X satisfy the following axioms:

(T1) . $\tilde{0}, \tilde{1} \in \Im$

(T2) $G1 \cap G2 \in \Im$ for any G1, G2 $\in \Im$,
(T3) $\cup Gi \in \Im$ for any arbitrary family { Gi : i $\in$ J}$\subseteq \Im$.

In this case the pair (X, $\Im$) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is known as an intuitionistic fuzzy open set in X. The complement Ac of an intuitionistic fuzzy open set A is called an intuitionistic fuzzy closed set in X.

Definition 2.5 [5]: Two intuitionistic fuzzy sets A and B of X is said to be q-coincident(AqB for short) if and only if there exists an element $x \in X$ such that
$\mu A(x) > \gamma B(x)$ or $\gamma A(x) < \mu B(x)$

Lemma 2.1 [5]: For any two intuitionistic fuzzy sets A and B of X.
$\rceil$(AqF) $\Leftrightarrow A \subseteq Bc$.

Definition 2.6[5]: Let X and Y be two nonempty sets and f : X $\rightarrow$ Y be a function.Then
(a) If B = {<y, μB(y), γB(y)> : y $\in$ Y}is an intuitionistic fuzzy set in Y, then the preimage of B under f denoted by f -1(B), is the IFS in X defined by
f -1(B) = <x, f -1(μB)(x), f -1(γB)(x)> : x $\in$ X}
(b) If A = {<x, λA(x), νA(x)> : x $\in$ X}is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by
f(A) = {<y, f (λA)(y), f-(νA)(y)> : y $\in$ Y}
where f-(νA) = 1 – f(1- νA).

Definition 2.7[8] : Let (X,$\Im$) and (Y,Φ) be two intuitionistic fuzzy topological spaces and let f: X$\rightarrow$Y be a function. Then
f is said to be intuitionistic fuzzy continuous if the preimage of each intuitionistic fuzzy open set in Y is an intuitionistic fuzzy open set in X.
f is said to be intuitionistic fuzzy g-continuous if the inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy g-closed set in X.

Definition 2.8[12]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\Im$) is called an intuitionistic fuzzy generalized closed (intuitionistic fuzzy g-closed) if cl(A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.

Definition 2.9[12]:Complement of an intutionistic fuzzy g-closed set is called intutionistic fuzzy g-open set.

Remark 2.1 [12]: Every intuitionistic fuzzy closed(intutionistic fuzzy open) set is intuitionistic fuzzy g-closed(intutionistic fuzzy g-open) set but its converse may not be true.

Definition 2.10[13]: An intuitionistic fuzzy topological space X is called intuitionistic fuzzy GO-connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy g-open and intuitionistic fuzzy g-closed.

Remark 2.2: Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true.

Definition 2..11[13]: Let (X, $\Im$) be an intuitionistic fuzzy topological space. The generalized closure of a intuitionistic fuzzy set A of X denoted by gcl(A) is the intersection of all intuitionistic fuzzy g-closed sets of X which contains A.

Definition 2..12[5]: Let (X, $\Im$) be an intuitionistic fuzzy topological space and Y be a crisp subset of X. Then the induced intuitionistic fuzzy topology(relative intuitionistic fuzzy topology) $\Im$Y for Y is defined by

$\Im Y = \{Y \cap O : O \in \Im\}$.

The pair $(Y, \Im Y)$ is called a subspace of $(X, \Im)$.

Lemma 2.2[5]: Let $(Y, \Im Y)$ be a subspace of an intuitionistic fuzzy topological space $(X, \Im)$ and A be an intuitionistic fuzzy set of Y. Then
A is $\Im Y$-intuitionistic fuzzy closed if and only if there exists an intuitionistic fuzzy open set F in X such that $A = F \cap Y$
$ClY(A) = Y \cap ClX(A)$.

III. INTUITIONISTIC FUZZY G*-CONTINUOUS MAPPINGS

Definition 3.1: A mapping $f : (X,\Im) \to (Y,\sigma)$ is said to be intuitionistic fuzzy g*-continuous if the inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy g* -closed in X.
Remark(3.1): Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g*-continuous and intuitionistic fuzzy g-continuous mapping but the converses may not be true.
Example (3.1): Let $X = \{a,b\}$ and $Y = \{x,y\}\}$ and the intuitionistic fuzzy sets U,V,W are defined as follows:
$U = \{<a,0.5,0.4>,<b,0.6,0.4>\}$
$V = \{<x,0.6,0.3>,<y,0.6,0.4>\}$
$W = \{<x,0.3,0.6>,<y,0.4,0.6>\}$
Let $\Im = \{\tilde{0}, ,U,\tilde{1}\}$ be intuitionistic fuzzy topology on X and $\sigma = \{\tilde{0},V,\tilde{1}\}$ and $\sigma^* = \{\tilde{0}, W,\tilde{1}\}$ be intuitionistic fuzzy topologies on Y. Then the mapping $f : (X,\Im) \to (Y,\sigma)$ defined by $f(a) = x$ and $f(b) = y$ is intuitionistic fuzzy g*-continuous but not intuitionistic fuzzy continuous and the mapping $g : (X,\Im) \to (Y,\sigma^*)$defined by $g(a) = x$ and $g(b) = y$ is intuitionistic fuzzy g-continuous but not intuitionistic fuzzy g*-continuous.

Following diagram shows the implication between the various classes of generalized continuity in intuitionistic fuzzy topological space.

Intuitionistic fuzzy continuous
⇓
Intuitionistic fuzzy g*-continuous
⇓
Intuitionistic fuzzy g-continuous

Theorem 3.1: A mapping $f:(X,\Im) \to (Y,\sigma)$ is intuitionistic fuzzy g*-continuous if and only if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy g*- open in X.
Proof: It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$ for every intuitionistic fuzzy set U of Y.
Theorem 3.2: If $f:(X,\Im) \to (Y,\sigma)$ is intuitionistic fuzzy g*-continuous then for each intuitionistic fuzzy point $c(\alpha,\beta)$ of X and each intuitionistic fuzzy open set V, $f(c(\alpha,\beta)) \subseteq V$ there exists a intuitionistic fuzzy g*- open set U such that $c(\alpha,\beta) \subseteq U$ and $f(U) \subseteq V$.
Proof: Let $c(\alpha,\beta)$ be an intuitionistic fuzzy point of X and V be an intuitionistic fuzzy open set such that $c(\alpha,\beta) \subseteq V$, put $U = f^{-1}(V)$ then by hypothesis U is intuitionistic fuzzy g*- open set of X such that $c(\alpha,\beta) \subseteq U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.
Theorem 3.3: If $f:(X,\Im) \to (Y,\sigma)$ is intuitionistic fuzzy g*-continuous then for each intuitionistic fuzzy point $c(\alpha,\beta)$ in X and each intuitionistic fuzzy open set V of Y such that $c(\alpha,\beta)qU$ and $f(U) \subseteq V$.
Proof: Let $c(\alpha,\beta)$ be an intuitionistic fuzzy point of X and V be an intuitionistic fuzzy open set of Y such that $f(c(\alpha,\beta))qV$, put $U = f^{-1}(V)$ then by hypothesis U is intuitionistic fuzzy g*- open set of X such that $c(\alpha,\beta)qU$ and $f(U) = f(f^{-1}(V)) \subseteq V$.
Definition 3.2:Let $(X,\Im)$ be an intuitionistic fuzzy topological space. The g*-closure of an intuitionistic fuzzy set A of X denoted by $g^*cl(A)$ is the intersection of all intuitionistic fuzzy g*-closed sets of X which contains A.
Remark(3.2): It is clear that, $A \subseteq g^*cl(A) \subseteq gcl(A) \subseteq cl(A)$ for any intuitionistic fuzzy set A of X.
Theorem 3.4: If $f:(X,\Im) \to (Y,\sigma)$ is intuitionistic fuzzy g*-continuous, then $f(g^*cl(A)) \subseteq cl(f(A))$ for every intuitionistic fuzzy set A of X.

Proof: Let A be an intuitionistic fuzzy set of X. Then cl(f(A)) is an intuitionistic fuzzy closed set of Y. Since f is intuitionistic fuzzy g*-continuous f-1(cl(f(A))) is intuitionistic fuzzy g*-closed in X. Clearly A⊆f-1(cl(f(A))).Therefore g*cl(A))⊆g*cl(f-1(cl(f(A)))=f-1(cl(f(A))).Hence f(g*cl(A))⊆cl(f(A)).

Definition 3.2: An intuitionistic fuzzy topological space (X,ℑ) is said to be T*1/2 –space if every intuitionistic fuzzy g*-closed set in X is intuitionistic fuzzy closed in X.

Theorem 3.5: A mapping f from an intuitionistic fuzzy T*1/2-space (X,ℑ) to an intuitionistic fuzzy topological space (Y,σ) is intuitionistic fuzzy continuous if and only if it is intuitionistic fuzzy g*- continuous.

Proof: Necessity: Follows from Remark(3.2).

Sufficiency: If A is intuitionistic fuzzy closed inY, then f-1(A) is intuitionistic fuzzy g*-closed in X because f is intuitionistic fuzzy g*- continuous. Since X is T*1/2-space. Therefore f-1(A) is intuitionistic fuzzy closed in X. Hence f is intuitionistic fuzzy continuous.

Theorem 3.6: If f:(X,ℑ)→(Y,σ) is intuitionistic fuzzy g*-continuous and g: (Y,σ) →(Z,η) is intuitionistic fuzzy continuous. Then gof: (X,ℑ) →(Z,η) is intuitionistic fuzzy g*-continuous

Proof: Let A be an intuitionistic fuzzy closed set in Z, then f-1(A) is intuitionistic fuzzy closed in Y because g is intuitionistic fuzzy continuous. Therefore (gof)-1(A) = f-1(g-1(A)) is intuitionistic fuzzy g*-closed in X. Hence gof is intuitionistic fuzzy g*-continuous.

Theorem 3.7: If f:(X,ℑ)→(Y,σ) and g: (Y,σ) →(Z,η) is intuitionistic fuzzy g*-continuous mappings and (Y,σ) is intuitionistic fuzzy T*1/2-space then gof: (X,ℑ) →(Z,η) is intuitionistic fuzzy g*-continuous.

Proof: Obvious.

Theorem 3.8: Intuitionistic fuzzy g*-continuous image of an intuitionistic fuzzy g*-compact space is intuitionistic fuzzy compact.

Proof: Let f:(X,ℑ)→(Y,σ) be an intuitionistic fuzzy g*-continuous map from an Intuitionistic fuzzy g*-compact space (X,ℑ)on to an intuitionistic fuzzy topological space (Y,σ). Let {Ai:i∈Λ} be an intuitionistic fuzzy g*-open cover of Y then{ f-1(Ai):i ∈Λ} is an intuitionistic fuzzy g*-open cover of X. Since X is intuitionistic fuzzy g*-compact it has finite intuitionistic fuzzy subcover say{f-1(A1),…………….,f-1(An)}.Since f is onto{A1,A2,………An}is an intuitionistic fuzzy open cover of Y and so (Y,σ) is intuitionistic fuzzy compact.

Definition 3.3: An intuitionistic fuzzy topological space X is called intuitionistic fuzzy g*-connected if there is no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy g*-open and set in X is intuitionistic fuzzy g*-closed in X.

Theorem: 3.9: An intuitionistic fuzzy T*1/2 –space is intuitionistic fuzzy C5-connected if and only if it intuitionistic fuzzy g*-connected.

Proof: Obvious.

Theorem 3.10: If f:(X,ℑ)→(Y,σ) is an intuitionistic fuzzy g*-continuous surjection and X is intuitionistic fuzzy g*-connected then Y is intuitionistic fuzzy C5-connected.

Proof: Suppose Y is intuitionistic fuzzy C5-connected. Then there exists a proper intuitionistic fuzzy set G of Y which is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Therefore f-1(G) is proper intuitionistic fuzzy g*-closed and intuitionistic fuzzy g*-open set of X, because f is intuitionistic fuzzy g*-continuos surjection. Hence X is not intuitionistic fuzzy g*-connected.

REFERENCES

[1] K. Atanassova, Intuitionistic Fuzzy Sets, In VII ITKR's Session, (V.Sgurev,Ed.) Sofia, Bulgaria, (1983)

[2] K. Atanassov and S. Stoeva., Intuitionistic Fuzzy Sets, In Polish Symposium on Interval and Fuzzy Mathematics , Poznan, (1983), 23-26

[3] K. Atnassov, Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 20(1986), 87-96.

[4] C.L. Chang, Fuzzy Topological Spaces, J.Math.Anal.Appl. 24(1968) 182-190.

[5] D. Coker, An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets and Systems .88(1997), 81-89.

[6] D.Coker and A. Es. Hyder., On Fuzzy Compactness in Intuitionistic Fuzzy Topological Spaces, The Journal of Fuzzy Mathematics, 3-4(1995),899-909.

[7] H. Gurcay, D. Coker and Es., A.Haydar, On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces. The Journal of Fuzzy Mathematics Vol.5, no.2, 1997, 365-378.

[8] N. Levine., Generalized Closed Sets In Topology, Rend. Cerc. Mat. Palermo.19(2), 1970, 571-599.

[9] O. Ozbakir and D. Coker., Fuzzy Multifunctions in Intuitionistic Fuzzy Topological Spaces Notes on IFS 5(1999) No. 3.

[10] S.S. Thakur and Malviya R., Generalized Closed Sets In Fuzzy Topology, Math. Notae 38(1995), 137-140.

[11] S.S. Thakur and Khare R.K., Fuzzy regular Generalized Closed Sets. Varahmihir Journal of Mathematical sciences.Vol.3(1)(2003),65-70.

[12] S.S Thakur and Rekha Chaturvedi, Generelized Closed Sets in Intuitionistic Fuzzy Topology. The Journal of Fuzzy Mathematics Vol.16(2)(2008).

[13] S.S Thakur and Rekha Chaturvedi, Generelized Continuity in Intuitionistic Fuzzy Topological Spaces. Notes on Intuitionistic Fuzzy Sets. Vol.12(1)(2006), 38-44.

[14] N. Turnali and D. Coker, Fuzzy Connectedness in Intuitionistic Fuzzy Topological Spaces. Fuzzy Sets And Systems 116(2000) (3), 369-375.
[15] L.A.Zadeh, Fuzzy Sets, Information and Control, 18(1965), 338-353.

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