Rotating MHD Convective Flow of Oldroyd-BFluid Through a Porous Medium in a Vertical Porous Channel with Thermal Radiation

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Abstract -An analysis of an oscillatory magnetohydrodynamic (MHD) convective flow of anOldroyd-B, incompressible and electrically conducting fluid through a porous medium bounded within two infinite vertical parallel porous plates is carried out. The fluid is injected with constant velocity through the stationary porous plate and simultaneously sucked with same constant velocity through the other oscillating plate. The temperature of the stationary plate is also oscillating. The temperature difference of the two plates is assumed high enough to induce heat transfer due to radiation. The entire system rotatesabout the axis normal to the planes of the plates with uniform angular velocity Ω. A magnetic field of uniform strength is applied in the direction perpendicular to the plates. The induced magnetic field is neglected due to the assumption of small magnetic Reynolds number. Adopting complex variable notations, a closed form analytical solution of the problem is obtained. The analytical results are evaluated numerically and then presented graphically to discuss in detail.For small andlarge rotations the dependence of the steady and unsteady resultant velocities and their phasedifferences on various parameters are discussed with the help of figures and tables.

Key words -Oldroyd-B Fluid,Porous Medium,Injection/suction,Magnetohydrodynamic (MHD),Convective, Oscillating, Rotating, Radiation.

I. INTRODUCTION

The theory of non-Newtonian fluids has become a field of very active research for the last few decades as this class of fluids represents, mathematically, many industrially important fluids such as plastic films and artificial fibers in industry. Flows of non-Newtonian fluids have gained considerable attention of engineers and scientist in recent past due to their important applications in various branches of science and technology particularly in chemical and nuclear industries, material processing, geophysics, and bio-engineering. In view of these applications an extensive range of mathematical models such as the Rivlin-Ericksen second order modelOldroyd [1], Oldroyd modelRao[2], Johnson-Seagalman modelHayat et al.[3]have been developed to analyze the diverse hydrodynamic behavior of these non-Newtonian fluids. Different visco-elastic fluid models have been presented by many investigators for variety of geometries using various types of analytical and computational schemes. These fluid flows are encountered in numerous areas of petrochemical, biomedical and environmental engineering including polypropylene coalescence sinteringTichy[4], Vlastos [5] and Wouteret. al. [6].The flow of viscoelastic fluids through porous media has also attracted the attention of a large number of scholars owing to their application in the fields of extraction of energy from geothermal regions and in the flow of oil through porous rocks. Many common liquids such as certain paints, polymer solutions, some organic liquids and many new materials of industrial importance exhibit both viscous and elastic properties. The fluids with such characteristics are called viscoelastic fluids. The scientific treatment of the problem of irrigation, soil erosion etc. are present developments of porous media. Gupta and Sridhar [7] studied the viscoelstic effects in non-Newtonian flow through porous medium.

Petrov[8] investigated the development of the flow of viscous and viscoelastic media between two parallel plates. Rajgopal et al [9] analyzed oscillatory motion of an electrically conducting viscoelastic fluid over a stretching sheet in saturated porous medium with suction/blowing.

Rajgopal[10] studied the unsteady unidirectional flows of a non-Newtonian fluid. Rajgopal and Gupta[11] obtained an exact solution for the flow of a non-Newtonian fluid past an infinite porous plate. Ariel [12] analyzed the flow of a viscoelastic fluid past a porous plate. Ariel [13] also obtained an exact solution of flow problems of a second grade fluid through two porous walls.Labropulu[14] obtained another exact solution of non-Newtonian fluid flows with prescribed vorticity. Pillai et al [15] studied the heat transfer in a viscoelastic boundary layer flow through a porous medium. Choudhury and Das[16] investigated magnetohydrodunamic boundary layer flow of non-Newtonian fluid past a flat plate. Metzner and White[17] investigated the flow behavior of viscoelastic fluid in the inlet region of a channel. Samria et al. [18] analyzed free convection flow of an elasto-viscous fluid past an infinite vertical plate.

The study of flow in rotating porous media is motivated by its practical applications in geophysics and engineering. Among the applications of rotating flow in porous media to engineering disciplines, one can find the food processing industry, chemical process industry, centrifugation filtration processes and rotating machinery. The presence of magnetic field significantly affects the flow of electrically conducting fluid. The magnetic field effect on the non-Newtonian fluid flow has wide applications in chemical engineering, metallurgical engineering, and various industries. Researchers have considerable interest in the study of flow phenomenon between two parallel plates. Because of its occurrence in rheometric experiments to determine the constitutive properties of the fluid, in lubrication engineering, and in transportation and processing encountered in chemical engineering, the flow on non-Newtonian visco-elastic fluid is worthwhile to investigate. The rotating flow of electrically conducting, incompressible, viscous and viscoelastic fluids has gained considerable attention because of its numerous applications in physics and engineering. A number of scholars have shown their interest towards the application of visco-elastic fluid flows through various types of channel in the presence of magnetic field.In geophysics, it is applied to measure and study the positions and velocities with respect to a fixed frame of reference on the surface of earth, which rotate with respect to an inertial frame in the presence of its magnetic field. The subject of geophysical dynamics nowadays has become an important branch of fluid dynamics due to the increasing interest to study environment. In astrophysics, it is applied to study the stellar and solar structure, inter planetary and inter stellar matter, solar storms etc. In engineering, it finds its application in MHD generators, ion propulsion, MHD bearings, MHD pumps, MHD boundary layer control of re-entry vehicles etc. Several scholars have studied such flows because of their varied importance. Devnath[19] studied magnetohydrodynamic boundary layers in a rotating flow.Devika et al [20] analyzed theMHD oscillatory flow of a viscoelastic fluid in a porous channel with Chemical Reaction.Hossanien and Mansour [21]investigated unsteady magnetic flow through a porous medium between two infinite parallel plates.Puri[22] analyzed rotating flow on an elastic-viscous fluid on an oscillating plate. Puri and Kulshreshtha[23] investigated rotating flow of non-Newtonian fluids. Hayat et al [24] studied unsteady hydromagnetic rotating flow of a conducting second grade fluid. Rahman and Sarkar[25] investigated the unsteady MHD flow of a viscoelastic Oldroyd fluid under time varying body forces through a rectangular channel. Attia and Abdeen[26] investigated unsteady Hartmann flow with heat transfer of a viscoelastic fluid under exponential decaying pressure gradient.Singh and Mathew [27] studied the oscillatory hydromagnetic flow in a rotating horizontal porous channel.Singh [28] obtained an exact solution of MHD mixed convection periodic flow in a rotating vertical channel with heat radiation.Singh [29] studied viscoelastic mixed convection MHD oscillatory flow through a porous medium filled in a vertical channel.Hayat et al [30] have also studied hydromagneticCouette flow of an Oldroyd-B fluid in a rotating system.

The objective of the present analysis is to study MHD oscillatory convection flow of an incompressible, electrically conducting and viscoelastic (Oldroyd-B) fluid in a vertical porouschannel. Constantinjection and suction is applied at the stationary and the oscillating infinite porous plates respectively. The entire system rotates about an axis perpendicular to the planes of the plates and a uniform magnetic field is also applied along this axis of rotation.

A general solution of the partial differential equations governing the flow problem is obtained and the effects of various flow parameters on the resultant velocity field and the resultant skin friction along with their phase angles are discussed in the last section of the paper with the help of figures and tables.

II. BASIC EQUATIONS

Consider the flow of a viscoelastic(Oldroyd-B), incompressible and electrically conducting fluid filled in a vertical channel.In order to derive the basic equations for the problem under consideration following assumptions are made:

(i) The two infinite vertical parallel plates of the channel are permeable and electrically non-conducting.
(ii) The vertical channel is filled with a porous medium.
(iii) The flow considered is fully developed, laminar and oscillatory.
(iv) The fluid is viscoelastic (Oldroyd-B), incompressible and finitely conducting.
(v) All fluid properties are assumed to be constant except that of the influence of density variation with temperature is considered only in the body force term.
(vi) A magnetic field of uniform strength B_0 is applied perpendicular to the plates of the channel.
(vii) The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
(viii) Hall effect, electrical and polarization effects are also neglected.
(ix) The temperature of a plate is non-uniform and oscillates periodically with time.
(x) The temperature difference of the two plates is also assumed to be high enough to induce heat transfer due to radiation.
(xi) The fluid is assumed to be optically thin with relatively low density.
(xii) The entire system (consisting of channel plates and the fluid) rotates about an axis perpendicular to the plates.

Under these assumptions we write hydromagnetic governing equations of motion and continuity in a rotating frame of reference as:

$$\nabla . \boldsymbol{V} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}.\nabla \boldsymbol{V}) + 2\boldsymbol{\Omega} \times \boldsymbol{V} = -\frac{1}{\rho}\nabla p^{*} + \vartheta_1 \nabla^2 \boldsymbol{V} + \frac{\vartheta_1}{K^{*}}\boldsymbol{V} + \nabla . \exists + \frac{1}{\rho}(\boldsymbol{J} \times \boldsymbol{B}) + \boldsymbol{F}, \tag{2}$$

$$\rho c_p \left[\frac{\partial T}{\partial t} + (\boldsymbol{V}.\nabla)T\right] = k\nabla^2 T - \nabla q . \tag{3}$$

In equation (2) the last term on the left hand side is the Coriolis force. On the right hand side of (2) the last term $\boldsymbol{F}(= g\beta T^{*})$ accounts for the force due to buoyancy. The second last term is the Lorentz Force due to magnetic field **B**and is given by

$$\boldsymbol{J} \times \boldsymbol{B} = \sigma(\boldsymbol{V} \times \boldsymbol{B}) \times \boldsymbol{B}, \tag{4}$$

and the modified pressure $p^{*} = p' - \frac{\rho}{2}|\boldsymbol{\Omega} \times \boldsymbol{R}|^2$, where **R** denotes the positionvector from the axis of rotation, p' denotes the fluid pressure, $\boldsymbol{J}$ is the current density, and all other quantities have their usual meaning and have been defined from time to time in the text.

In the term third from last of equation (2), $\exists$ is the Cauchy stress tensor and the constitutive equation derived by Coleman and Noll [31] for an incompressible homogeneous fluid of second order is

$$\exists = -p_1 I + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2. \tag{5}$$

Here $-p_1 I$ is the interdeterminate part of the stress due to constraint of incompressibility, μ_1, μ_2 and μ_3 are the material constants describing viscosity, elasticity and cross-viscosity respectively. The kinematic A_1 and A_2 are the Rivelen Ericson constants defined as

$$A_1 = (\nabla V) + (\nabla V)^T, \; A_2 = \frac{dA_1}{dt} + (\nabla V)^T A_1 + A_1(\nabla V), \quad \text{where}$$

∇ denotes the gradient operator and d/dt the material time derivative. According to Markovitz and Coleman [32] the material constants μ_1, μ_3 are taken as positive and μ_2 as negative. The modified pressure $p^* = p' - \frac{\rho}{2}|\Omega \times R|^2$, where **R** denotes the position vector from the axis of rotation, p' denotes the fluid pressure.

III. FORMULATION OF THE PROBLEM

We consider an unsteady flow of a viscoelastic incompressible and electrically conducting fluid bounded by two infinite electrically non-conducting vertical porous plates distance 'd' apart.A coordinate system is chosen such that the X^*-axis is oriented upward along the stationary plate and Z^*-axis taken perpendicular to the planes of the plates.A schematic diagram of the flow problem is shown in figure 1.The fluid is injected through the porous plate at $z^* = 0$ with constant velocity $w_0$and simultaneous removedwith the same velocity w_0 through the other oscillating porous plate at $z^* = d$. The non-uniform temperature of the plate at $z^* = d$ is assumed to be varying periodically with time. The temperature difference between the plates is high enough to induce the heat due to radiation. The Z^*- axis is considered to be the axis of rotation about which the fluid and the plates are assumed to be rotating as a solid body with a constant angular velocity Ω^*. A transverse magnetic field of uniform strength **B** (0, 0, B_0) is also applied along the axis of rotation. All physical quantities depend on z^* and t^* only for this fully developed laminar flowproblem. The equation of continuity $\nabla . V = 0$ gives on integration $w^* = w_0$.Thenthe velocity may reasonably be assumed with its components along x^*, y^*, z^* directions as **V** (u^*, v^*, w_0).

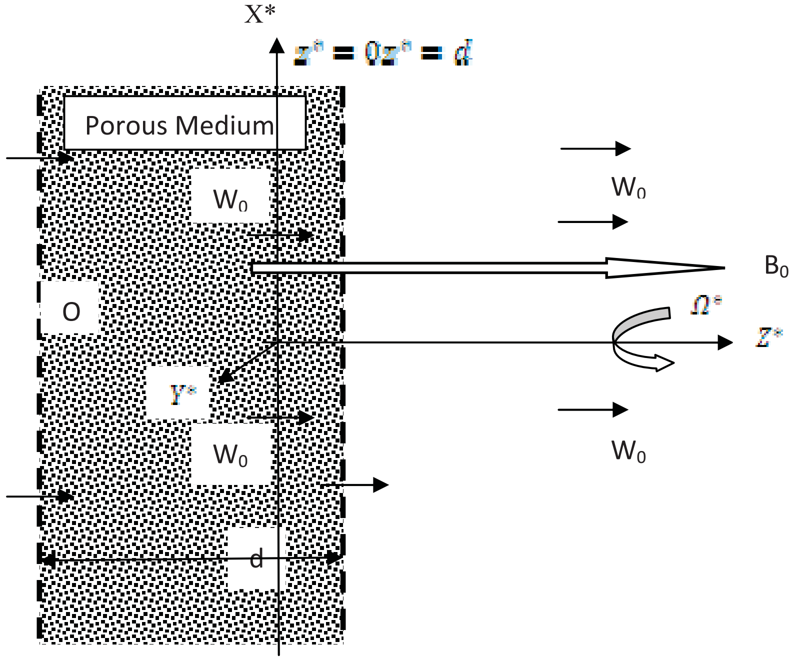

Figure 1.Physical configuration of the flow.

Under the usual Boussinesq approximation MHD flow in the rotating channel is governed by the following equations:

$$\frac{\partial u^*}{\partial t^*} + w_0 \frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta_1 \frac{\partial^2 u^*}{\partial z^{*2}} + \vartheta_2 \left(\frac{\partial^3 u^*}{\partial z^{*2}\partial t^*} + w_0 \frac{\partial^3 u^*}{\partial z^{*3}}\right) + 2\Omega^* v^* - \frac{\sigma B_0^2}{\rho} u^* - \frac{\vartheta_1 u^*}{K^*} + g\beta(T^* - T_d)$$

, (5)

$$\frac{\partial v^*}{\partial t^*} + w_0 \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \vartheta_1 \frac{\partial^2 v^*}{\partial z^{*2}} + \vartheta_2 \left(\frac{\partial^3 v^*}{\partial z^{*2}\partial t^*} + w_0 \frac{\partial^3 v^*}{\partial z^{*3}}\right) - 2\Omega^* u^* - \frac{\sigma B_0^2}{\rho} v^* - \frac{\vartheta_1 u^*}{K^*},$$

(6) $0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}$,

(7) $$\frac{\partial T^*}{\partial t^*} + w_0 \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}} - \frac{1}{\rho c_p}\frac{\partial Q}{\partial z^*}.$$

(8)

where ρ is the density, ϑ_1 is the kinematic viscosity, ϑ_2 is viscoelasticity, p^* is the modified pressure, t^* is the time, σ is the electric conductivity, B_0 is the component of the applied magnetic field along the z^*-axis, g is the acceleration due to gravity, k is the thermal conductivity, c_p is the specific heat at constant pressure and the last term in equation (8) is the radiative heat flux.

Following Cogley et al [33] it is assumed that the fluid is optically thin with a relatively low density and the heat flux due to radiation in equation (8) is given by

$$\frac{\partial Q}{\partial z^*} = 4\alpha^2(T^* - T_d). \tag{9}$$

where α is the mean radiation absorption coefficient. After the substitution of equation (9) equation (8) becomes

$$\frac{\partial T^*}{\partial t^*}+w_0\frac{\partial T^*}{\partial z^*}=\frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}}-\frac{4\alpha^2}{\rho c_p}\left(T^*-T_d\right). \qquad (10)$$

Equation (7) shows the constancy of the hydrodynamic pressure along the axis of rotation.

The boundary conditions for the problem are

$$z^*=0: \; u^*=v^*=0, w=w_0, T^*=T_0+\varepsilon(T_0-T_d)\cos\omega^* t^*, \qquad (12)$$

$$z^*=d: \; u^*=U^*(t^*)=U_0(1+\varepsilon\cos\omega^* t^*), v^*=0, w=w_0, T^*=T_d,$$

(13) where T_0 is the temperature of the plate at $z^*=0$, T_d is the mean temperature of the plate at $z^*=d$ and ω^* is the frequency of oscillations.

Eliminating the pressure gradient appearing in equations (5) and (6), we get

$$\frac{\partial u^*}{\partial t^*}+w_0\frac{\partial u^*}{\partial z^*}=\frac{dU^*}{dt^*}+\vartheta_1\frac{\partial^2 u^*}{\partial z^{*2}}+\vartheta_2\left(\frac{\partial^3 u^*}{\partial z^{*2}\partial t^*}+w_0\frac{\partial^3 u^*}{\partial z^{*3}}\right)+2\Omega^* v^*-\frac{\sigma B_0^2(u^*-U^*)}{\rho}-\frac{\vartheta_1(u^*-U^*)}{K^*}+g\beta(T^*-T_d)$$

, (14)

$$\frac{\partial v^*}{\partial t^*}+w_0\frac{\partial v^*}{\partial z^*}=\vartheta_1\frac{\partial^2 v^*}{\partial z^{*2}}+\vartheta_2\left(\frac{\partial^3 v^*}{\partial z^{*2}\partial t^*}+w_0\frac{\partial^3 v^*}{\partial z^{*3}}\right)-2\Omega^*(u^*-U^*)-\frac{\sigma B_0^2}{\rho}v^*-\frac{\vartheta_1 u^*}{K^*}.$$

(15)

Introducing the following non-dimensional quantities:

$$\eta=\frac{z^*}{d},\; t=\omega^* t^*,\; u=\frac{u^*}{U_0},\; v=\frac{v^*}{U_0},\; \theta=\frac{T^*-T_d}{T_0-T_d},\; \omega=\frac{\omega^* d^2}{\vartheta_1},$$

(14) into equations (14), (15) and (10), we get

$$\omega\frac{\partial u}{\partial t}+\lambda\frac{\partial u}{\partial \eta}=\omega\frac{\partial U}{\partial t}+\frac{\partial^2 u}{\partial \eta^2}+\omega\gamma\frac{\partial^3 u}{\partial \eta^2\partial t}+\lambda\gamma\frac{\partial^3 u}{\partial \eta^3}+2\Omega v-M^2(u-U)-K^{-1}(u-U)+\lambda^2 Gr\,\theta, \qquad (15)$$

$$\omega\frac{\partial v}{\partial t}+\lambda\frac{\partial v}{\partial \eta}=\frac{\partial^2 v}{\partial \eta^2}+\omega\gamma\frac{\partial^3 v}{\partial \eta^2\partial t}+\lambda\gamma\frac{\partial^3 v}{\partial \eta^3}-2\Omega(u-U)-M^2 v-K^{-1}v,$$

(16) $\omega Pr\frac{\partial \theta}{\partial t}+\lambda\frac{\partial \theta}{\partial \eta}=\frac{\partial^2 \theta}{\partial \eta^2}-N^2\theta,$

(17) where

$\lambda=\frac{w_0 d}{\vartheta_1}$ is the injection/suction parameter,

$\gamma=\frac{\vartheta_2\lambda}{d^2}$ is the visco-elastic parameter, $\Omega=\frac{\Omega^* d^2}{\vartheta_1}$ is the rotation parameter, $M=B_0 d\sqrt{\frac{\sigma}{\rho\vartheta_1}}$ is the Hartmann number,

$K=\frac{K^*}{d^2}$ is the permeability of the porous medium,

$Gr=\frac{g\beta\vartheta_1(T_0-T_d)}{U_0 w_0^2}$ is the Grashof number,

$Pr=\frac{\rho\vartheta_1 c_p}{k}$ is the Prandtl number,

$N = \frac{2ad}{\sqrt{k}}$ is the radiation parameter.

The boundary conditions in the dimensionless form become

$$\eta = 0: \quad u = v = 0, \;\; \theta = 1 + \varepsilon \cos t, \tag{18}$$

$$\eta = 1: \quad u = U(t) = 1 + \varepsilon \cos t, v = 0, \;\; \theta = 0. \tag{19}$$

IV. SOLUTION OF THE PROBLEM

Now combining equations (15) and (16) into single equation by introducing a complex function of the form q = u + iv, we get

$$\omega \frac{\partial q}{\partial t} + \lambda \frac{\partial q}{\partial \eta} = \omega \frac{\partial U}{\partial t} + \frac{\partial^2 q}{\partial \eta^2} + \omega\gamma \frac{\partial^3 q}{\partial \eta^2 \partial t} + \lambda\gamma \frac{\partial^3 q}{\partial \eta^3} - (2\Omega i + M^2 + K^{-1})(q - U) + \lambda^2 Gr\, \theta, \tag{21}$$

with corresponding boundary conditions in complex form as

$$\eta = 0: \quad q = 0, \;\; \theta = 1 + \cos t = 1 + \frac{\varepsilon}{2}(e^{it} + e^{-it}), \tag{22}$$

$$\eta = 1: \quad q = 1 + \varepsilon \cos t = 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right), \;\; T = 0. \tag{23}$$

In view of theboundary conditions (22) and (23) that is, periodic variations of the platevelocity ($\eta = 1$) and the plate temperature ($\eta = 0$), the solution of the problem isassumed to be of the form

$$q(\eta, t) = q_0(\eta) + \frac{\varepsilon}{2}\left[q_1(\eta)e^{it} + q_2(\eta)e^{-it}\right],$$

(24) $$\theta(\eta, t) = \theta_0(\eta) + \frac{\varepsilon}{2}\left[\theta_1(\eta)e^{it} + \theta_2(\eta)e^{-it}\right]. \tag{25}$$

Substituting equations (24) and (25) into equations (17) and (21) andcomparing the harmonic and non harmonic terms,we get

$$\lambda\gamma \frac{d^3 q_0}{d\eta^3} + \frac{d^2 q_0}{d\eta^2} - \lambda \frac{dq_0}{d\eta} - \Delta q_0 = -\Delta - \lambda^2 Gr\, \theta_0, \tag{26}$$

$$\lambda\gamma \frac{d^3 q_1}{d\eta^3} + (1 + i\omega\gamma)\frac{d^2 q_1}{d\eta^2} - \lambda \frac{dq_1}{d\eta} - (\Delta + i\omega) q_1 = -(\Delta + i\omega) - \lambda^2 Gr\, \theta_1, \tag{27}$$

$$\lambda\gamma \frac{d^3 q_2}{d\eta^3} + (1 + i\omega\gamma)\frac{d^2 q_2}{d\eta^2} - \lambda \frac{dq_2}{d\eta} - (\Delta - i\omega) q_2 = -(\Delta - i\omega) - \lambda^2 Gr\, \theta_2, \tag{28}$$

$$\frac{d^2\theta_0}{d\eta^2} \quad \lambda Pr \frac{d\theta_0}{d\eta} \quad N^2 \theta_0 = 0, \tag{29}$$

$$\frac{d^2\theta_1}{d\eta^2} - \lambda Pr \frac{d\theta_1}{d\eta} - (N^2 + i\omega Pr)\theta_1 = 0, \tag{30}$$

$$\frac{d^2\theta_2}{d\eta^2} - \lambda Pr \frac{d\theta_2}{d\eta} - (N^2 - i\omega Pr)\theta_2 = 0, \tag{31}$$

where $\Delta = (2\Omega i + M^2 + K^{-1})$.

The transformed boundary conditions can be written as

$$\eta = 0: \quad q_0 = q_1 = q_2 = 0, \; \theta_0 = \theta_1 = \theta_2 = 1, \tag{32}$$

$$\eta = 1: \quad q_0 = q_1 = q_2 = 1, \;\; \theta_0 = \theta_1 = \theta_2 = 0. \tag{33}$$

The solutions of the second order differential equations (29)to (31) under the boundary conditions (32) and (33) are

$$\theta_0(\eta) = \frac{e^{m_1+m_2\eta} - e^{m_2+m_1\eta}}{e^{m_1} - e^{m_2}}, \quad (34)$$

$$\theta_1(\eta) = \frac{e^{m_3+m_4\eta} - e^{m_4+m_3\eta}}{e^{m_3} - e^{m_4}}, \quad (35)$$

$$\theta_2(\eta) = \frac{e^{m_5+m_6\eta} - e^{m_6+m_5\eta}}{e^{m_5} - e^{m_6}}. \quad (36)$$

Before proceeding for the solutions of equations (26), (27) and (28), it is necessary to remark here that these are third order differential equations for non-zero γ. But for $\gamma = 0$ these equations reduce to classical case of viscous fluids. Mathematically and computationally, it is more challenging to analyze the flow of second grade fluids because of the peculiarity of the equations governing the fluid motion. The order of the differential equations characterizing the flow of second grade fluid is higher than the number boundary conditions available. This difficulty can be accentuated by the fact the viscoelastic parameter, γ, for a second grade fluid usually occurs in the coefficient of the highest derivative and can be resolved by seeking perturbation solution assuming the viscoelastic parameter to be small as is treated by Beard and Walters[34] considering the two-dimensional stagnation point flow of Walter's B fluid. One may also refer to Hayat and Hutter[35] and Hayat et al [24]. In the present analysis also the difficulty is removed by assuming perturbation solution of the following form:

$$q_0(\eta) = q_{00}(\eta) + \gamma q_{01}(\eta) + O(\gamma^2), \quad (37)$$

$$q_1(\eta) = q_{10}(\eta) + \gamma q_{11}(\eta) + O(\gamma^2), \quad (38)$$

$$q_2(\eta) = q_{20}(\eta) + \gamma q_{21}(\eta) + O(\gamma^2), \quad (39)$$

which is valid for small values of γ only. Substituting these relations into equations (26), (27) and (28) and equating the coefficients of like powers of γ. We obtain the following system of equations along with boundary conditions:

System of order zero

$$q_{00}'' - \lambda q_{00}' - \Delta q_{00} = -\Delta - \lambda^2 Gr\,\theta_0, \quad (40)$$

$$q_{10}'' - \lambda q_{10}' - (\Delta + i\omega) q_{10} = -(\Delta + i\omega) - \lambda^2 Gr\,\theta_1, \quad (41)$$

$$q_{20}'' - \lambda q_{20}' - (\Delta - i\omega) q_{20} = -(\Delta - i\omega) - \lambda^2 Gr\,\theta_2, \quad (42)$$

with boundary conditions

$$\eta = 0 : q_{00} = q_{10} = q_{20} = 0, \quad (43)$$

$$\eta = 1 : q_{00} = q_{10} = q_{20} = 1. \quad (44)$$

System of order one

$$q_{01}'' - \lambda q_{01}' - \Delta q_{01} = \lambda q_{00}''', \quad (45)$$

$$q_{11}'' - \lambda q_{11}' - (\Delta + i\omega) q_{11} = \lambda q_{10}''', \quad (46)$$

$$q_{21}'' - \lambda q_{21}' - (\Delta - i\omega) q_{21} = \lambda q_{20}''', \quad (47)$$

with boundary conditions

$$\eta = 0 : q_{01} = q_{11} = q_{21} = 0, \quad (48)$$

$$\eta = 1 : q_{01} = q_{11} = q_{21} = 0. \quad (49)$$

Zeroth-order solution

System of zero-order equations describes the basic problem of viscous fluids. The solution of the this problem is obtained as

$$q_{00}(\eta) = 1 + B_1 e^{n_1\eta} + B_2 e^{n_2\eta} + A_1 e^{m_1\eta} + A_2 e^{m_2\eta}, \quad (50)$$

$$q_{10}(\eta) = 1 + B_3 e^{n_3\eta} + B_4 e^{n_4\eta} + A_3 e^{m_3\eta} + A_4 e^{m_4\eta}, \quad (51)$$

$$q_{20}(\eta) = 1 + B_5 e^{n_5\eta} + B_6 e^{n_6\eta} + A_5 e^{m_5\eta} + A_6 e^{m_6\eta}. \tag{52}$$

First-order solution

$$q_{01}(\eta) = \frac{m_3^2 A_1}{\left(m_3^2 - \lambda m_3 - \Delta\right)}\left\{\frac{\left(e^{n_2} - e^{m_3}\right)e^{n_1\eta} - \left(e^{n_1} - e^{m_3}\right)e^{n_2\eta} + \left(e^{n_1} - e^{n_2}\right)e^{m_3\eta}}{\left(e^{n_1} - e^{n_2}\right)}\right\} + \frac{m_4^2 A_2}{\left(m_4^2 - \lambda m_4 - \Delta\right)}\left\{\frac{\left(e^{n_2} - e^{m_4}\right)e^{n_1\eta} - \left(e^{n_1} - e^{m_4}\right)e^{n_2\eta} + \left(e^{n_1} - e^{n_2}\right)e^{m_4\eta}}{\left(e^{n_1} - e^{n_2}\right)}\right\}, \tag{53}$$

$$q_{11}(\eta) = \frac{m_4^2 A_3}{\left(m_4^2 - \lambda m_4 - (\Delta + i\omega)\right)}\left\{\frac{\left(e^{n_4} - e^{m_4}\right)e^{n_3\eta} - \left(e^{n_3} - e^{m_4}\right)e^{n_4\eta} + \left(e^{n_3} - e^{n_4}\right)e^{m_4\eta}}{\left(e^{n_3} - e^{n_4}\right)}\right\} + \frac{m_5^2 A_4}{\left(m_5^2 - \lambda m_5 - (\Delta + i\omega)\right)}\left\{\frac{\left(e^{n_4} - e^{m_5}\right)e^{n_3\eta} - \left(e^{n_3} - e^{m_5}\right)e^{n_4\eta} + \left(e^{n_3} - e^{n_4}\right)e^{m_5\eta}}{\left(e^{n_3} - e^{n_4}\right)}\right\}, \tag{54}$$

$$q_{21}(\eta) = \frac{m_5^2 A_5}{\left(m_5^2 - \lambda m_5 - (\Delta - i\omega)\right)}\left\{\frac{\left(e^{n_6} - e^{m_5}\right)e^{n_5\eta} - \left(e^{n_5} - e^{m_5}\right)e^{n_6\eta} + \left(e^{n_5} - e^{n_6}\right)e^{m_5\eta}}{\left(e^{n_5} - e^{n_6}\right)}\right\} + \frac{m_6^2 A_6}{\left(m_6^2 - \lambda m_6 - (\Delta - i\omega)\right)}\left\{\frac{\left(e^{n_6} - e^{m_6}\right)e^{n_5\eta} - \left(e^{n_5} - e^{m_6}\right)e^{n_6\eta} + \left(e^{n_5} - e^{n_6}\right)e^{m_6\eta}}{\left(e^{n_5} - e^{n_6}\right)}\right\}. \tag{55}$$

V. RESULTS AND DISCUSSION

The Problem of MHD oscillatory, viscoelstic, convective and radiative flow in a vertical porous channel is analyzed. The fluid is injected through the stationary porous plate and simultaneously removed with same velocity through the oscillating porous plate of the channel. The entire system (consisting of porous channel and the fluid) rotates about an axis perpendicular to the Plates. The approximate solutions for the velocity and the temperature fields are obtained analytically. The steady and unsteady resultant velocities and amplitudes of the shear stress along with their phase differences are shown with the help of graphs. To have better insight of the physical problem different curves are compared with the basic dotted curve II in all the figures.

Now for the resultant velocities and the shear stresses of the steady and unsteady flow, we write

$$u_0(\eta) + iv_0(\eta) = q_0(\eta) \text{ and} \tag{56}$$

$$u_1(\eta) + iv_1(\eta) = q_1(\eta)e^{it} + q_2(\eta)e^{-it}. \tag{57}$$

The solution (37) corresponds to the steady part which gives u_0 as the primary and v_0 as the secondary velocity components. The amplitude and the phase difference due tothese primary and secondary velocities for the steady flow are given by

$$R_0 = \sqrt{u_0^2 + v_0^2} \text{ and } \alpha_0 = \tan^{-1}\left(\frac{v_0}{u_0}\right) \tag{58}$$

The resultant velocity $R_0$for the steady part of the flow in a vertical porous channel is presented in Fig.2.The different curves in this figure represent the various sets of parameter valueslisted in table 1. Comparison of curves I and II reveal that the resultant R_0increases with the increase of the injection/suction parameter λ. It is very clear from this figure that R_0increases with the increasing Grashof number Gr (curves II &III), the Hartmann number M (curves II & IV) and the Prandtl number Pr (curves II & V).The two valuesof the Prandtl number Pr as 0.7 and 7.0 are chosen to represent air and waterrespectively. However, the increase of the permeability parameter K and radiation parameter N lead to a decrease in the resultant R_0.The rotational effect of the flow is clearly visible with increasing rotation Ω of the channel.

The phase angle α_0 of the steady resultant R_0 is presented in Fig. 3. A significant decrease in the phase angle is noticed with the increase of the injection/suction parameter λ (curves I & II), convection current Gr (II & III), Hartmann number M (curves II & IV) and the Prandtl number Pr (curves II & VI). The phase angle decreases with the increase of the permeability parameter K (curves II & V) and the radiation parameter N (curves II & VII). A phase shift from the phase lead to phase lag is observed from this figure but this phase shift for the larger rotation of the channel is very sharp.

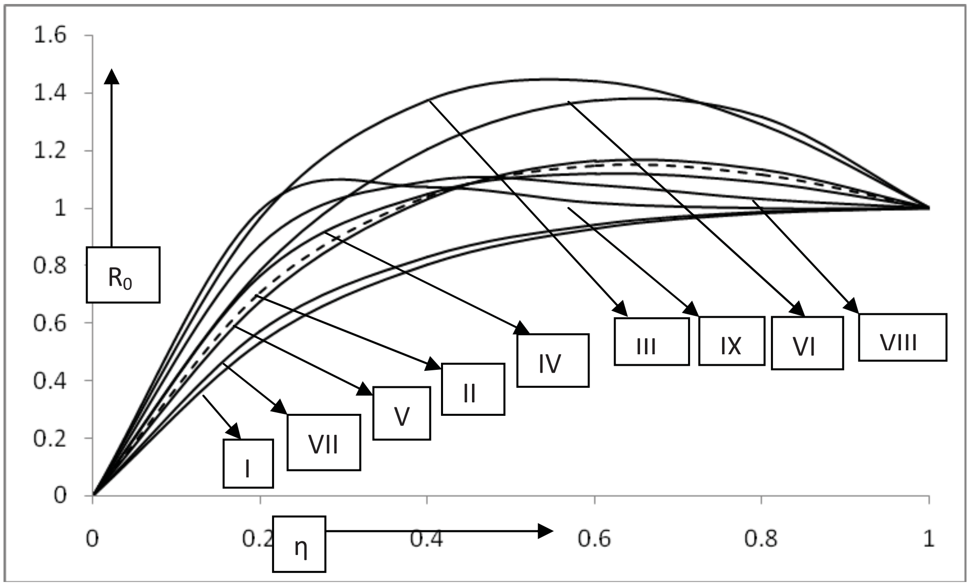

Figure 2.Resultant velocity R_0 due to steady components u_0 and v_0.

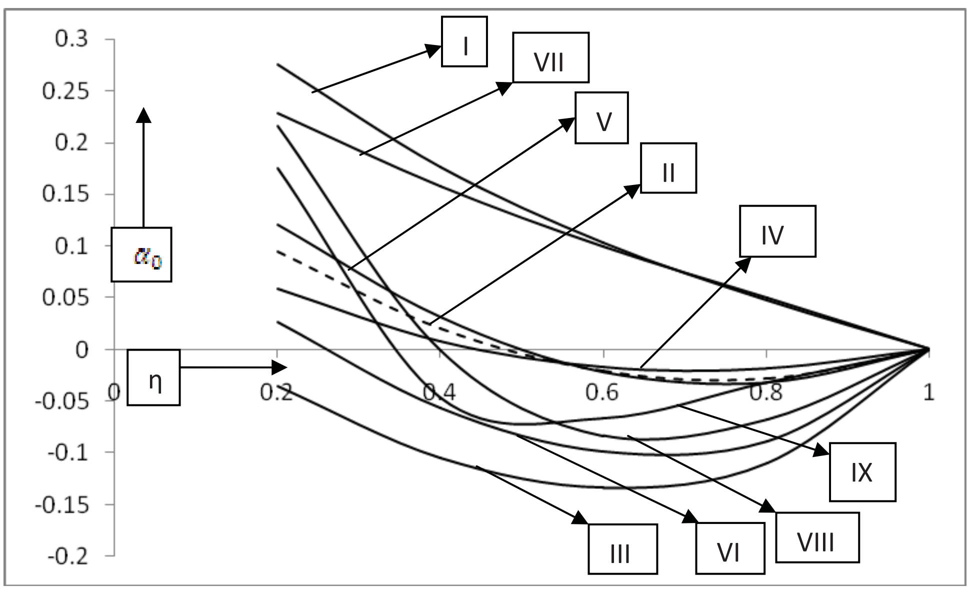

Figure 3. Phase angle α_0 due to u_0 and v_0.

Table 1.Values of parameters representing different curves in figures2 & 3.

Ω	λ	Gr	M	K	Pr	N	γ	Curves
5	0.5	5	2	0.2	0.7	1	-0.05	I
5	1.5	5	2	0.2	0.7	1	-0.05	II
5	1.5	10	2	0.2	0.7	1	-0.05	III
5	1.5	5	4	0.2	0.7	1	-0.05	IV
5	1.5	5	2	5.0	0.7	1	-0.05	V
5	1.5	5	2	0.2	7.0	1	-0.05	VI
5	1.5	5	2	0.2	0.7	5	-0.05	VII
25	1.5	5	2	0.2	0.7	1	-0.1	VIII
50	1.5	5	2	0.2	0.7	1	-0.1	IX

The amplitude and the phase difference of the steady shear stress at the stationary plate for the steady flow can be obtained as

$$\tau_{0r} = \sqrt{\tau_{0x}^2 + \tau_{0y}^2} \text{ and } \beta_0 = \tan^{-1}\left(\frac{\tau_{0y}}{\tau_{0x}}\right), \tag{59}$$

where

$$\begin{aligned}\left(\frac{\partial q_0}{\partial \eta}\right)_{\eta=0} &= \tau_{0x} + i\tau_{0y} = B_1 n_1 + B_2 n_2 + A_1 m_2 + A_2 m_1 \\ &+ \frac{m_3^2 A_3}{(m_3^2 - \lambda m_3 - \Delta)}\left\{\frac{n_1(e^{n_2} - e^{m_3}) - n_2(e^{n_1} - e^{m_3}) + m_3(e^{n_1} - e^{n_2})}{(e^{n_1} - e^{n_2})}\right\} \\ &+ \frac{m_4^2 A_4}{(m_4^2 - \lambda m_4 - \Delta)}\left\{\frac{n_1(e^{n_2} - e^{m_4}) - n_2(e^{n_1} - e^{m_4}) + m_4(e^{n_1} - e^{n_2})}{(e^{n_1} - e^{n_2})}\right\}.\end{aligned} \tag{60}$$

Here τ_{0x} and τ_{0y} are, respectively the shear stresses at the stationary plate due to the primary and secondary velocity components. The numerical values of the amplitude τ_{0r} and the phase difference $\beta_0$of the steady shear stress at the stationary plate (η=0) for the steady flow are presented in Table 2.

Table 2. Amplitude τ_{0r} and the phase difference β_0 of the steady shear stress at the stationary plate (η=0).

Ω	λ	Gr	M	K	Pr	N	γ	τ_{0r}	β_0
5	0.5	5	2	0.2	0.7	1	-0.05	3.6251	0.40336
5	1.5	5	2	0.2	0.7	1	-0.05	4.7716	0.20259
5	1.5	10	2	0.2	0.7	1	-0.05	6.7000	0.06916
5	1.5	5	4	0.2	0.7	1	-0.05	5.7413	0.13910
5	1.5	5	2	5.0	0.7	1	-0.05	4.3545	0.24595
5	1.5	5	2	0.2	7.0	1	-0.05	5.0986	0.14503
5	1.5	5	2	0.2	0.7	5	-0.05	4.4047	0.31005
25	1.5	5	2	0.2	0.7	1	-0.05	7.0240	0.57942
50	1.5	5	2	0.2	0.7	1	-0.05	9.6764	0.68881
75	1.5	5	2	0.2	0.7	1	-0.05	11.842	0.72727
100	1.5	5	2	0.2	0.7	1	-0.05	13.700	0.74623

It is noticed from the numerical values for τ_{0r} and β_0 in table 2 that the amplitude τ_{0r} increases with the increase of the injection/suction parameter λ, Grashof number Gr, Hartmann number M, Prandtl number Pr and increasing rotation Ω of the channel. However, amplitude decreases with the increase of permeability of the porous medium K and the radiation parameter N. Similarly, from the values for β_0 it is clear that the phase angledecreases with the increase of theinjection/suction parameter λ, Grashof number Gr, Hartmann number M and the Prandtl number Pr. An increase in the phase angle is also observed due to the increase of the permeability of the porous mediumK and radiation parameter N.

Equations (38) and (39) with the help of equations (51), (52), (54) and (55) together give the unsteady part of the flow. The unsteady primary and secondary velocity components $u_1(\eta)$ and $v_1(\eta)$, respectively for the fluctuating flow can be obtained as

$$u_1(\eta) = \{Real\ q_1(\eta) + Real\ q_2(\eta)\}\cos t - \{Im\ q_1(\eta) - Im\ q_2(\eta)\}\sin t, \tag{61}$$

$$v_1(\eta) = \{Real\ q_1(\eta) - Real\ q_2(\eta)\}\cos t + \{Im\ q_1(\eta) + Im\ q_2(\eta)\}\sin t. \tag{62}$$

The resultant velocity or the amplitude and the phase difference of the unsteady flow are given by

$$R_1 = \sqrt{u_1^2 + v_1^2} \text{ and } \alpha_1 = \tan^{-1}\left(\frac{v_1}{u_1}\right). \tag{63}$$

The resultant velocity R_1 for unsteady part is presented in Figure 4. The curves in the figure represent various sets of parameter values listed in table 3. Different curves are compared with the basic colored curve II. It is observed from this figure that the unsteady resultant R_1 decreases consistently with the increase of radiation parameter N (curves II & VII) and the frequency of oscillation ω (curves II & VIII). The figure also shows that the resultant increases over the entire width of the channel with the increase of the injection/suction parameter λ (curves II & I), Grashof number Gr (curves II & III) and Prandtl number Pr (curves II & VI). It is noticed that with the increasing rotation of the channel the unsteady resultant near the stationary plate increases first then decreases there after (curves II & IX, X). The figure also reveals that with the increase of the viscoelasticity of the fluid velocity decreases near the stationary plate and increases slightly thereafter (curves II & XI).

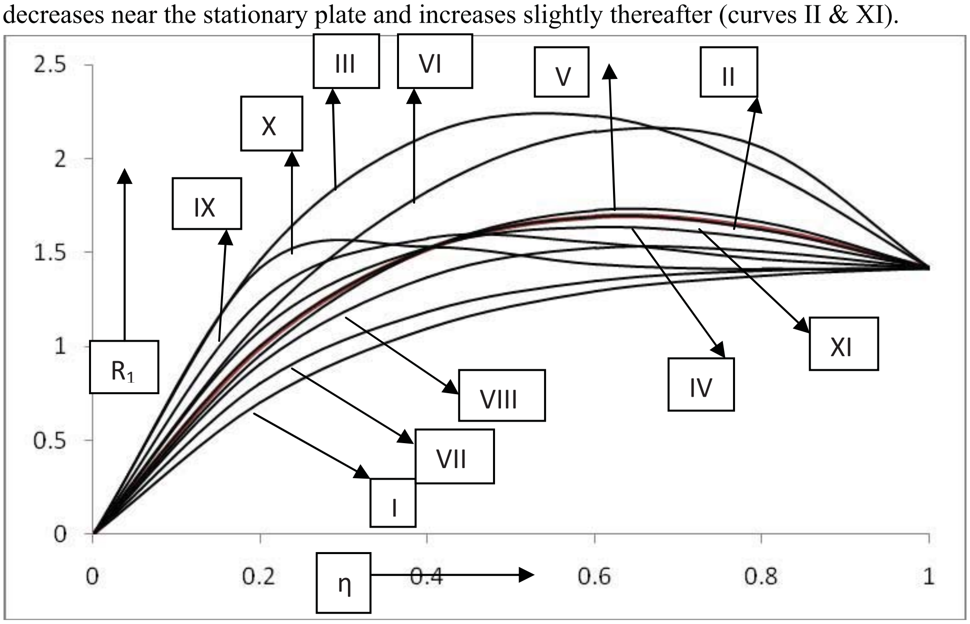

Figure 4.Resultant velocity $R_1 = \sqrt{u_1^2 + v_1^2}$ due to u_1 and v_1.

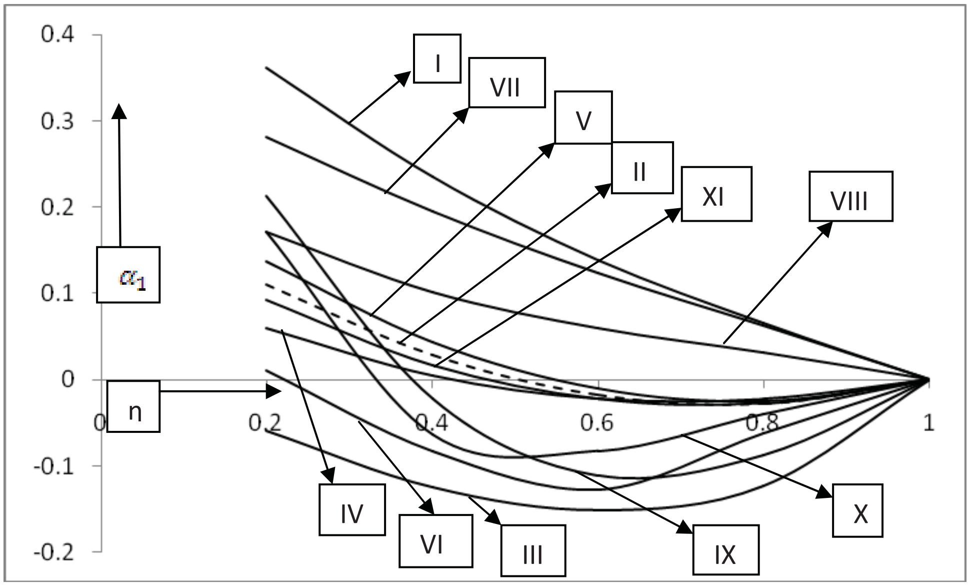

Figure 5. Phase angle $\alpha_1 = \tan^{-1}\left(\frac{v_1}{u_1}\right)$ due to u1 and v1.

Table 3.Values of parameters representing different curves in figure 4 & 5.

Ω	λ	Gr	M	K	Pr	N	ω	γ	Curves
5	0.5	5	2	0.2	0.7	1	5	-0.05	I
5	1.5	5	2	0.2	0.7	1	5	-0.05	II
5	1.5	10	2	0.2	0.7	1	5	-0.05	III
5	1.5	5	4	0.2	0.7	1	5	-0.05	IV
5	1.5	5	2	5.0	0.7	1	5	-0.05	V
5	1.5	5	2	0.2	7.0	1	5	-0.05	VI
5	1.5	5	2	0.2	0.7	5	5	-0.05	VII
5	1.5	5	2	0.2	0.7	1	10	-0.05	VIII
25	1.5	5	2	0.2	0.7	1	5	-0.05	IX
80	1.5	5	2	0.2	0.7	1	5	-0.05	X
5	1.5	5	2	0.2	0.7	1	5	-0.1	XI

The phase difference α_1 for the unsteady part of the flow is presented in figure 5. In this figure the comparison of the curves with the basic curve II reveal that the phase difference increases with the increase of permeability K of the porous medium (curves II & V), the radiation parameter N (curves II & VII) and the frequency of oscillations ω (curves II & VIII). However, for the increase of the rest of the parameters like the Grashof number Gr (curves II & III), Hartmann number M (curves II & IV), Prandtl number Pr (curves II & VI), rotation parameter Ω (curves II & IX, X) and the viscoelastic parameter γ (curves II & XI). The phase lead near the stationary plate shifts to phase lag near the oscillating plate.

For the unsteady part of the flow, the amplitude and the phase difference of sheer stresses at the stationary plate (η = 0) can be obtained as

$$\tau_{1r} = \sqrt{\tau_{1x}^2 + \tau_{1y}^2},\ \beta_1 = \tan^{-1}\left(\frac{\tau_{1y}}{\tau_{1x}}\right), \tag{64}$$

where $\tau_{1x} + i\tau_{1y} = \left(\frac{\partial q_0}{\partial \eta}\right)_{\eta=0} e^{it} + \left(\frac{\partial q_1}{\partial \eta}\right)_{\eta=0} e^{-it}$

$$= (B_3 n_3 + B_4 n_4 + A_3 m_4 + A_4 m_3)e^{it} + (B_5 n_5 + B_6 n_6 + A_5 m_6 + A_6 m_5)e^{-it}$$

$$+\gamma\left[\frac{m_4^2 A_3}{\left(m_4^2 - \lambda m_4 - (\Delta + i\omega)\right)}\left\{\frac{(e^{n_4} - e^{m_4})n_3 - (e^{n_3} - e^{m_4})n_4 + (e^{n_3} - e^{n_4})m_4}{(e^{n_3} - e^{n_4})}\right\} + \frac{m_3^2 A_4}{\left(m_3^2 - \lambda m_3 - (\Delta + i\omega)\right)}\left\{\frac{(e^{n_4} - e^{m_3})n_3 - (e^{n_3} - e^{m_3})n_4 + (e^{n_3} - e^{n_4})m_3}{(e^{n_3} - e^{n_4})}\right\}\right]e^{it} +$$

$$\gamma\left[\frac{m_6^2 A_5}{\left(m_6^2 - \lambda m_6 - (\Delta - i\omega)\right)}\left\{\frac{(e^{n_6} - e^{m_6})n_5 - (e^{n_5} - e^{m_6})n_6 + (e^{n_5} - e^{n_6})m_6}{(e^{n_5} - e^{n_6})}\right\} + \frac{m_5^2 A_6}{\left(m_5^2 - \lambda m_5 - (\Delta - i\omega)\right)}\left\{\frac{(e^{n_6} - e^{m_5})n_5 - (e^{n_5} - e^{m_5})n_6 + (e^{n_5} - e^{n_6})m_5}{(e^{n_5} - e^{n_6})}\right\}\right]e^{-it}$$

. (65)

The unsteady shear stress amplitude τ_{1r} is shown in figure 6. The figure clearly shows that the amplitude increases with the increase of Grashof number Gr, Hartmann number M, Prandtl number Pr, frequency of oscillations ω, injection/suction parameter λ, rotation Ω of the channel and the viscoelastic parameter γ. However, the amplitude decreases with the increase of permeability of the porous medium K and the radiation parameter N.

The phase difference β_1 of the unsteady shear stress is presented in figure 7. In order to know the variation in the phase difference due to the increase of different flow parameters all curves are compared with the dotted curve II. The phase increase is noticed with the increase of permeability of the porous medium K (curves II & V), radiation parameter N(curves II & VII) and rotation parameter Ω (curves II & IX, X). But the increase of injection/suction parameter λ (curves II & I), Grashof number Gr(curves II & III), Hartmann number M(curves II & IV), Prandtl number Pr(curves II & VI) and viscoelstic parameter γ(curves II & XI). It is evident that there remains phase lead by and large.

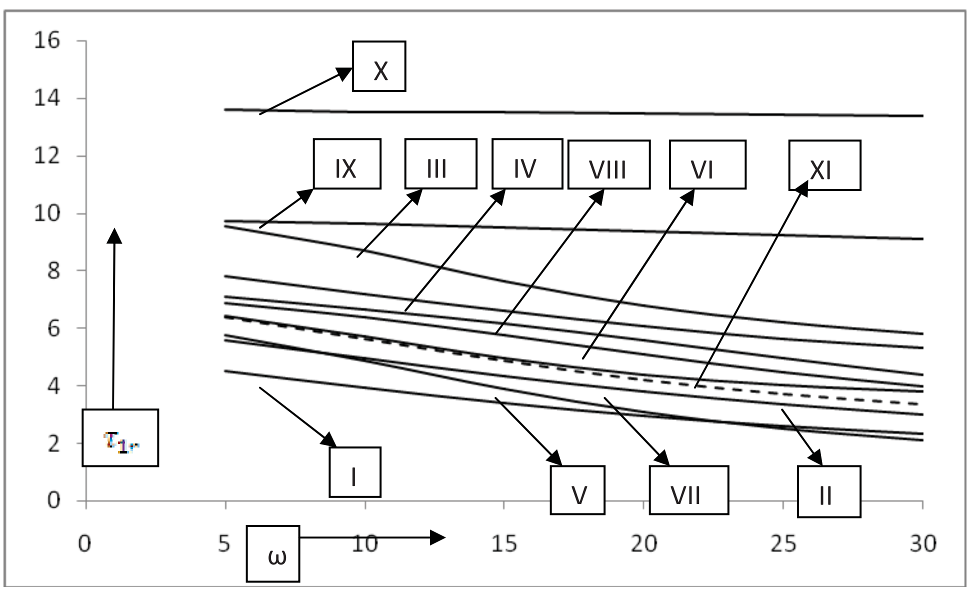

Figure 6.Amplitude τ_{1r} of unsteady shear stress at $t = \frac{\pi}{4}$.

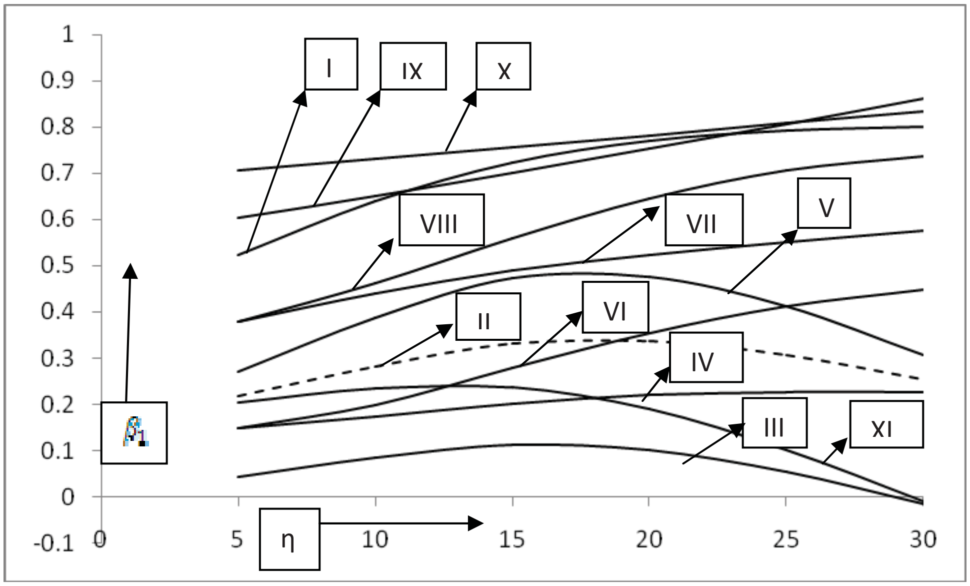

Figure 7.The phase difference β_1 of unsteady shear stress for $t = \frac{\pi}{4}$.

Table 4.Values of parameters representing different curves in figure 6 & 7.

Ω	λ	Gr	M	K	Pr	N	γ	Curves
5	0.5	5	2	0.2	0.7	1	-0.05	I
5	1.5	5	2	0.2	0.7	1	-0.05	II
5	1.5	10	2	0.2	0.7	1	-0.05	III
5	1.5	5	4	0.2	0.7	1	-0.05	IV
5	1.5	5	2	5.0	0.7	1	-0.05	V
5	1.5	5	2	0.2	7.0	1	-0.05	VI
5	1.5	5	2	0.2	0.7	5	-0.05	VII
5	1.5	5	2	0.2	0.7	1	-0.05	VIII
25	1.5	5	2	0.2	0.7	1	-0.05	IX
80	1.5	5	2	0.2	0.7	1	-0.05	X
5	1.5	5	2	0.2	0.7	1	-0.1	XI

APPENDIX

$$\Delta = 2\Omega i + M^2 + K^{-1}, m_1 = \frac{\lambda Pr + \sqrt{\lambda^2 Pr^2 + 4N^2}}{2},$$

$$m_2 = \frac{\lambda Pr - \sqrt{\lambda^2 Pr^2 + 4N^2}}{2}, m_3 = \frac{\lambda Pr + \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega)}}{2}, m_4 = \frac{\lambda Pr - \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega)}}{2},$$

$$m_5 = \frac{\lambda Pr + \sqrt{\lambda^2 Pr^2 + 4(N^2 - i\omega)}}{2}, \quad m_6 = \frac{\lambda Pr - \sqrt{\lambda^2 Pr^2 + 4(N^2 - i\omega)}}{2},$$

$$n_1 = \frac{\lambda + \sqrt{\lambda^2 + 4\Delta}}{2}, n_2 = \frac{\lambda - \sqrt{\lambda^2 + 4\Delta}}{2}, n_3 = \frac{\lambda + \sqrt{\lambda^2 + 4(\Delta + i\omega)}}{2},$$

$$n_4 = \frac{\lambda - \sqrt{\lambda^2 + 4(\Delta + i\omega)}}{2}, n_5 = \frac{\lambda + \sqrt{\lambda^2 + 4(\Delta - i\omega)}}{2}, n_6 = \frac{\lambda - \sqrt{\lambda^2 + 4(\Delta - i\omega)}}{2},$$

$$A_1 = \frac{-Gr\lambda^2 e^{m_1}}{(e^{m_1} - e^{m_2})(m_1^2 - \lambda m_1 - \Delta)},$$

$$A_2 = \frac{-Gr\lambda^2 e^{m_2}}{(e^{m_1} - e^{m_2})(m_2^2 - \lambda m_2 - \Delta)}, \quad A_3 = \frac{-Gr\lambda^2 e^{m_3}}{(e^{m_3} - e^{m_4})\left(m_3^2 - \lambda m_3 - (\Delta + i\omega)\right)},$$

$$A_4 = \frac{-Gr\lambda^2 e^{m_4}}{(e^{m_3} - e^{m_4})\left(m_4^2 - \lambda m_4 - (\Delta + i\omega)\right)}, A_5 = \frac{-Gr\lambda^2 e^{m_5}}{(e^{m_5} - e^{m_6})\left(m_5^2 - \lambda m_5 - (\Delta - i\omega)\right)},$$

$$A_6 = \frac{-Gr\lambda^2 e^{m_6}}{(e^{m_5} - e^{m_6})\left(m_6^2 - \lambda m_6 - (\Delta - i\omega)\right)},$$

$$B_1 = -\frac{e^{n_2} + A_1(e^{n_2} - e^{m_2}) + A_2(e^{n_2} - e^{m_1})}{(e^{n_2} - e^{n_1})}, B_2 = \frac{e^{n_1} + A_1(e^{n_1} - e^{m_2}) + A_2(e^{n_1} - e^{m_1})}{(e^{n_2} - e^{n_1})},$$

$$B_3 = -\frac{e^{n_4} + A_3(e^{n_4} - e^{m_4}) + A_4(e^{n_4} - e^{m_3})}{(e^{n_4} - e^{n_3})}, B_4 = \frac{e^{n_3} + A_3(e^{n_3} - e^{m_4}) + A_4(e^{n_3} - e^{m_3})}{(e^{n_4} - e^{n_3})},$$

$$B_5 = -\frac{e^{n_6} + A_5(e^{n_6} - e^{m_6}) + A_6(e^{n_6} - e^{m_5})}{(e^{n_6} - e^{n_5})}, B_6 = \frac{e^{n_5} + A_5(e^{n_5} - e^{m_6}) + A_6(e^{n_5} - e^{m_5})}{(e^{n_6} - e^{n_5})}.$$

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