Genetic Algorithm for Optimizing Noiseless, Non-Lubricated Helical Gear Pair

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Abstract - This research is motivated by optimal designing of bevel gear pair for transmitting more power between shafts in the machine tool without noise and without lubricating oil. In this research, the parameters such as power, efficiency, weight and centre distance have been optimized since the parameters are the central part of the gear problem. The sub problem of reducing the noise, running without lubricants, standardizing the parameters, satisfying the bending and compressive stress had been considered as constraints. Thus the formulated problem became a nonlinear NP Hard problem with number of problem specific (approximate) heuristic procedures. In consequence, a need arises for a more structured design intended to provide solutions which are feasible and stable for most of the helical gear design problems. As a result, genetic algorithm has been used as the optimization approach and their relative performances by varying the parameters are compared by sensitivity analysis to reduce the computational time. The optimized genetic result had been used for helical gear manufacturing and the trial results were compared with the standard helical gear. It was found that the performance has been at par with the standard gear and also the optimized gear pair having 8% reduction of gear size and approx. 2 % increase in efficiency.

Keywords: Genetic Algorithm; Helical gear; NP hard; Optimization;

I. INTRODUCTION

The basis of this research is the problem of eliminating the lubricating oils which creates a grubby environment and reducing the size and weight of the power transmitting element (gears) to make the compact noiseless automated machines in the manufacturing firms. The helical gears are commonly used for power transmission with less noise in most of the machine tools. These gears are available in the standard sizes and the same have been used in the machine tools which lead to increase in the size and weight of the computer controlled machine. In succession, it leads to usage of high torque prime movers and advanced complex digital control mechanisms. In order to surmount these problems, gears of optimized standard sizes may be used based on the application and loading conditions. There is no absolute restriction on using smaller gears with different materials instead of larger cast iron gears for the machine tools. However, this is considered undesirable in practice. Besides, though the optimized gears are satisfying the stress constraints, it is desirable to design the gears in such a way that little or no noise and vibration will be produced during power transmission. If not, lubricating oils might be required to reduce the noise and yet again lead to messy environment. In most cases, gears used are comparatively larger for given load conditions due to design standardization. Indeed, complete standardized power transmission system may be larger and results in more transmission losses. Generally the gear design problem can be divided into three main parts:

(i) Design the gear for standards.
(ii) Constraints such as factor of safety, stress, etc need be satisfied.
(iii) Optimize the design.

Because of the conflicting constraints and complexity in the optimization, this problem became non-linear and NP Hard. The results obtained from one part are then given as input to next part. Depending on the methods employed, several iterations may need to be carried out to obtain a reasonably optimal design. Genetic algorithm is one among the evolutionary algorithm which can identify the optimized results from the multi-objective and multi-constrained problems. So in this research, genetic algorithm has been used to optimize the helical gear design parameters for running without lubrication and less noise.

This paper is organised as follows. Section 2 provides the details about the current research in the field of gear design. Section 3 describes the mathematical programming formulation of the problem. Section 4 discusses heuristic solution approaches and defines the characteristics. Sections 5 present the experimental implementation and the simulation results followed by the conclusion in the section 6.

II. LITERATURE SURVEY

The literature on gear design problems in general is extensive and too broad so some of the major research works are covered in this section. The relation between the sliding velocities with line of action having impact in design of gears [18]. It also proved that the sliding velocity became zero at pitch line and increases when the conjugated teeth contact line, travels away from the pitch line on both directions [18]. Heat is generated by sliding friction of teeth surfaces and is proportional to the distribution of contact pressure and sliding velocity. Variations in temperature along the line of action are calculated by Blok's contact temperature theory. So in this research work, change in temperature also calculated for designing.

The encroachments of computers lead to lot of researches in developing the computer programming for gear design. A computer program had been developed by the researchers in order to design a required type of gear under a specified set of working conditions [12]. A new computer-aided method for automated gearbox design had been portrayed [11]. An interactive physical programming had formulated in order to optimize a three-stage spur gear reduction unit [9]. An expert system for designing and manufacturing a gearbox unit had also illustrated by the researchers [2]. So this research also focused in developing an interactive programming module for helical gear based on the given application.

Application of genetic algorithm can be used for optimization problems [8]. A non-dominated sorting GA had used to solve a multi-objective optimization of a multi-speed gearbox. A generalized optimal design of two-stage and three-stage spur gear reduction units in a formulation with multiple objectives [4, 20]. The benefits of the particle swarm searches in resolving different engineering designs were also revealed [14]. Two advanced optimization algorithms known as particle swarm optimization (PSO) and simulated annealing (SA) are brought into play for minimizing the weight of a spur gear train [16]. GA had utilized to minimize the volume of two stage helical gear train [6]. A complete automated optimal design of a two-stage helical gear reducer using a two-phase evolutionary algorithm also developed by the researchers [17, 19]. A study had been carried out for minimizing the centre distance of a helical gear using American Gear Manufacturers Association (AGMA) procedures [1, 10]. So GA has been used to optimize the helical gear parameters in this research.

III. MATHEMATICAL PROGRAMMING FORMULATION

In this section, the problem of designing the helical gear for an application which can run without lubricating oil and without or with less noise is formulated as a nonlinear mixed integer programming problem. Gears are the friction wheels having teeth's on its circumference and are used to transmit power between the shafts by mating with each other. Gears are used to amplify or reduce the speed, power, torque, etc. Some of the commonly used gears are Spur gear, Helical gear, Herringbone gear, Bevel gear, Worm gear, Rack, and many more. Helical gears are the far most common type of gear used in machine tools which can rotate with comparatively less noise. So the helical gear is taken in this research to optimize and the objectives given in the following sections along with the constraints considered for the designing.

A. *Data and Decision Variables*

The following conventions for specifying the data and decision variables for the problem are used. Parameters with parenthesis and without parenthesis represent the design and actual value respectively.

B. *Fitness Function Formulation*

Using the notation defined above, the formulation of the helical gear design problem considered in this paper is given below and the equations [7]. The multi-objective function used to identify the best helical gear parameters are given in the Equation 1.

$$COF = \{[(F_1/\text{max. } F_1) + (F_2/\text{ max. } F_2)] / 2\} + \{[(F_3 /\text{min. } F_3) + (F_4/\text{min. } F_4)] / 2\}/2 \qquad (1)$$

Whereas,

F_1 is the maximization function for power transmission and is given in the Equation 2.

$$F_1 = P \quad \text{where, } P^{(L)} \leq P \leq P^{(U)} \qquad (2)$$

'P' is Power transmitting capacity and lies between lower and upper limit.

F_2 is the maximization function for efficiency calculation and is given in the Equation 3.

$$F_2 = 100 - P_L \qquad (3)$$

'P_L' = Power loss which is calculated by the Equation 4.

$$P_L = \frac{50f}{\text{Cos}\Phi} \times \frac{(H_s^2 + H_t^2)}{(H_s + H_t)} \quad (4)$$

'H_s' = Specific sliding velocity at start of approach action
'H_t' = Specific sliding velocity at end of recess action
'f' = Coefficient of friction
'Φ' = Pressure angle in degrees
'H_s' and 'H_t' are calculated by the Equations 5 & 6 respectively.

$$H_t = \frac{(i+1)}{i} \times \sqrt{\left(\left[\frac{r_0}{r}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \quad (5)$$

$$H_s = (i+1) \times \sqrt{\left(\left[\frac{R_0}{R}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \quad (6)$$

Whereas,
'R' & 'R_o' = Pitch and Outside circle radius of gear in mm.
'r' & 'r_o' = Pitch and Outside circle radius of pinion in mm
R_0 = R + one addendum

One addendum for 20^o full depth involutes system = one module = m.

$$r_o = r + m = \frac{d_1}{2} + m$$

$$R_o = R + m = \frac{d_2}{2} + m$$

F_3 is the minimization function for calculating gear weight and is given in the Equation 7. The weight optimization of gear using GA [20].

$$F_3 = \{[\frac{\pi}{4} \times d_1^2 \times b \times \rho] + [\frac{\pi}{4} \times d_2^2 \times b \times \rho]\} \quad (7)$$

'd_1', 'd_2' = Pitch circle diameter of pinion and gear in mm
'b' = Thickness of pinion and gear in mm
'ρ' = Density of the material in kg/mm^3

F_4 is minimization function for calculating center distance and is given in Equation 8.

$$F_4 = \frac{(d_1 + d_2)}{2} = \frac{m}{2}(Z_1 + Z_2) \quad (8)$$

Where, 'Z_1', 'Z_2' = Number of teeth in pinion and gear respectively.

It is worth noting several characteristics about this formulation are given as follows.

(i) There are usually many different combinations of parameters which give the same objective function value. But the feasible design is one which differs by obeying design for manufacturing and assembly concepts. So while generating the values for the variables, it have been limited to only the standard values.

(ii) Because of the combinatorial optimization of multi-objectives, the solution space is discontinuous. An improved feasible solution occurs only when the best values for all the objective functions obtained.

(iii) The problem involves a multiple objective function which looks only at the optimal gear parameter values and may not provide the feasible solution. So the manufacturing and the practical constrains need to be satisfied. The constraints considered are given in the following section.

C. Constraints Formulation

In this research, the optimization is done for the gears which are rotating without lubricating oil, so more heat energy will be developed on the gear surface. The surface temperature of the gear wheels should be kept within the allowable limits while optimizing the design, because the gear life and the lubrication depend mainly on the amount of heat generated. Also the noise generated should be within the allowable limits.

Gear Surface Temperature - The maximum contact temperature is obtained by Equation 9.

$$\theta_{B\,max} = \theta_M + \theta_{fl\,max} \tag{9}$$

Whereas, 'θ_M' is the tooth temperature,

'$\theta_{fl\text{-}max}$' is maximum flash temperature along the line-of-action, which is calculated by Blok's relation given in the Equation 10.

$$\theta_{fl} = 31.62\ K\ \mu_m\ (X_r\ W_n / \sqrt{(b_H)})\ x\ \{(|V_{r1} - V_{r2}|) / [(B_{M1}\sqrt{(V_{r1})}) - (B_{M2}\sqrt{(V_{r2})})]\} \tag{10}$$

Whereas, 'K' is the Hertzian distribution of frictional heat = 0.8;

'μ_m' is the Mean coefficient of friction; X_r is the Load sharing factor;

'W_n' is the Normal unit load;

'B_{M1}' & 'B_{M2}' is the Thermal contact coefficients of the pinion and wheel;

'V_{r1}' & 'V_{r2}' is the Rolling tangential velocities in m/s of the pinion and wheel;

Gear Noise Calculation - Noises in the gear trains are due to vibration and transmission error caused by change in tooth topology, shaft deflections and mesh stiffness variation. Transmission error along the line of action is given in the Equation 11.

$$TE = R_b\ \{\delta_2 - (i\ \delta_1)\} \tag{11}$$

Whereas, 'R_b' is gear base radius;

'δ_1', 'δ_2' are the angular rotation of pinion and gear;

D. Design Formulation

The formulation helical gear design procedure is a formidable task which should considers all the parameters involved [1]. The major items to be taken care for a feasible design are given below.

Bending Stress - During mating of gear teethes, the top surface of the gear teeth will subject to bending stress and the condition for the bending stress is given in the Equation 12.

$$\sigma_b \leq [\sigma_b]_{al} \tag{12}$$

Whereas, '$[\sigma_b]_{al}$' = Allowable bending stress in N/mm^2.

'σ_b' = Induced bending stress in N/mm^2 and is given in Equation 13.

$$\sigma_b = \frac{(i+1)}{(a\ m\ b\ y)} \times [M_t] \tag{13}$$

Where, 'i' = Gear ratio / Speed ratio

'a' = Center distance between gear and pinion

'y' = Form factor

'$[M_t]$' = Design twisting moment in N-mm, and is given in Equation 14.

$$[Mt] = M_t \times k \times k_d \tag{14}$$

'M_t' = Normal twisting moment transmitted by the pinion in N-mm

'K' & 'k_d' are the Load Concentration factor and Dynamic load factor

Compressive Stress- During mating of the gears, compressive stresses are created at the bottom surface of teethes of the mating gears. The compressive stress function is given in the Equation 15.

$$\sigma_c \leq [\sigma_c]_{al} \tag{15}$$

Where, '$[\sigma_c]_{al}$' = Allowable crushing stress in N/mm^2.

'σ_c' = Induced crushing stress in N/mm^2 and is given in Equation 16.

$$\sigma_c = 0.74\left(\frac{i+1}{a}\right) \times \sqrt{\left[\left(\frac{i+1}{ib}\right) \times E \times [M_t]\right]} \tag{16}$$

Where, 'E' = Young's Modulus of the gear material in N/mm^2

Module - Module plays a major role in the gear design and in manufacturing. The condition for module is given in the Equation 17.

$$m \geq m_{min} \tag{17}$$

'm_{min}' is the minimum module and is given in the Equation 18.

$$m_{min} = 1.26 \times \sqrt[3]{\frac{[M_t]}{(y\,\sigma_b \Psi_m Z_1)}} \tag{18}$$

'Ψ_m' is the ratio between the gear pair thickness and module.

The obtained module value should be standardized to the 'R' series values which are stored in the database.

Center Distance between pinion and gear (a) - The size of the gear wheels decides the centre distance and the formula for calculating the centre distance is given in the Equation 19 and the condition to be satisfied is given in the Equation 20.

$$a = \frac{(d_1 + d_2)}{2} = \frac{m}{2}\,[Z_1 + Z_2] \tag{19}$$

$$a \geq a_{min} \tag{20}$$

The minimum center distance can be calculated from the Equation 21.

$$a_{min} = (i + 1)\,\sqrt[3]{\left[\left(\frac{0.74}{[\sigma_c]}\right)^2 \times \left(\frac{E[M_t]}{i\Psi}\right)\right]} \tag{21}$$

'Ψ' is the ratio between the gear pair thickness and center distance.

IV. SOLUTION METHODOLOGY

The optimization of gear design problems has a very difficult structure and with non-linear mathematical models, it is hard to solve. Complex NP-hard problem virtue the usage of heuristic approaches. As most of the researchers proved that the heuristic procedures can construct feasible solutions by satisfying the intuitive rules or constraints. Among the heuristic procedures, Genetic algorithm (GA) has used adaptive search techniques to identify the best optimal solution[13]. GA operates with the chromosomal formats, so the process of converting the real world data into genetic chromosomal format is encoding.

A. *Encoding*

Naturally, the effectiveness of this approach depends on whether or not the good characteristics of one generation of strings can be passed along to successive generations [5,8]. So in this research, the strings are comprised of actual values rather than binary digits based on the Radcliffe (1991) theory for generalizing the coding of strings. This type of direct representation reduces the separate heuristic for decoding process and thereby reduces the computational complexity and time. Furthermore, in binary encoding the first part of the parent string before cross-over is preserved in the child [15]. So the solution space shrivels around a specified region and converges to a local spike instead of global spike. This problem can be overcome by the method of direct representation. In one GA application [3], it appears possible that no part of the parent solution is present in the child after it has been decoded. The problem of the decoding heuristic had been avoided in GA, which moves from infeasible to feasible solutions [3]. The sample encoding is shown in the figure 1. The next stage after encoding is the population generation with the available encoded values.

For Power = 31.21 KW; Module = 20.00 mm; No. of Teeth = 24; Thickness = 5.73 mm
GA Encoded Parent - 31212000240573

Figure 1: Randomly Generated Parent

B. *Initial Population Generation*

Initial parents are generated with the population size of 100 and parent size of 14 strings and a sample set of five parents are shown in the figure 2. The generated parents in the population are allowed for cross-over to produce next generation.

Parent 1 /1 30412022180473, Parent 2 /1 30063612180483,

Parent 3 /1 30074423220774, Parent 4 /1 31273053240628,
Parent 5 /1 31294636160507,

Figure 2: Five Set of Randomly Generated Parents

C. Crossover

The probability of the strings being reproduced is set in this research by conducting sensitivity analysis. The crossover of set of strings swapped between the parents occurs at the randomly selected crossover point. For example, given the following crossover operation in the Figure 3, with the random crossover point after third cell.

Parent 1: 30412022180473
Parent 2 : 30063612180483
Child 1 : 3041202218 04830
Child 2: 30063612180473

Figure 3: Crossover Operation

D. Mutation

Mutation is the process to generate new parameter numbers in randomly selected cells to avoid stagnation. In this research, 10 % of the generated child have allowed for mutation. Then the fitness function value has been calculated for the mutant child. The same procedure has to be continued till the termination conditions.

E. Termination Conditions

GA terminates if all strings have the same fitness value or if a stipulated number of generations has been reached.

V. COMPUTATIONAL EXPERIMENTS

The design and the results of a series of computational tests used to evaluate the algorithms presented in the following sections.

A. Test problem

As GA is a meta-heuristic, the procedure is not dependent on the particular problem structure. But the performance and the computational time can be improved by doing some local modification in GA parameters like sequential cross-over, parent sorting, etc. For experimentation purpose, power transmitting capability, module, tooth thickness and number of teeth's had been considered for optimization. The range of the parameters can vary in large range and lead to more time. So as an alternative the parameter or decoding range of the strings have been set based on the application and is given in the Table 1, because the problem is less constrained, may produce better solutions in less computational time.

Table 1: Primary Spur Gear Parameters

Power	: 30 to 45 Kw
Module	: 01 to 22 mm
Tooth Thickness	: 08 to 100 mm
Number of Teeth	: 12 to 50

The remaining parameters such as gear ratio, material property, young's modulus, etc had set to constant values. The next section describes the computational performance of the algorithm.

B. Genetic Programming

Heuristic GA presented in this paper was programmed in Visual basic software and run in 50 MHz IBM personal computer by the second-named author. Bentley (1999) conducted a series of computational tests problems. The problems were generated using random number within the range given in the table 1 and those problems called as the initial population. This population were intended to be representative of the size of gear experienced in practice for manufacturing. The number of different parents in the population formulates the search space of 11,53,680 solutions. So the initial population size has been set with fifty parents in this research after conducting sensitivity analysis. The relative fitness function values of the parents were classified as best, satisfactory and worst parents. Normalized fitness function value varied from 0 to over 1. In all cases, the smallest value was assumed to be the worst parent and vice-versa. By randomly generating the parents will be a subset of the possible combinations of the solutions. Then the parents were allowed for sequential crossover operation and mutation operation to generate a new generation with best properties in it till the termination conditions achieved. In each generation, the crossover and mutation points were randomly generated for each pair of position strings. The final solution produced by the algorithm was taken as the best of the solutions.

C. Result

In general the best of five trials can be considered as the best and each trial consisted of 100 iterations for each of the test problems. Each trial was based on a different position string containing randomly generated position numbers. The best candidate solution obtained after satisfying the stopping conditions for experimentation is given as follows. A gear pair had been made of 40 Ni 2 Cr 1 Mo 28 (Cr-Mo) series steel and surface hardened up to 56 HRC with the density and young's modulus values as 8.9×10^{-6} kg/mm^3 and 2.15×10^5 N/mm^2 respectively. Tooth flanks were ground to achieve AGMA grade A4. Gears can run at high speed of 32 m/s without lubrication continuously for 100 hrs with the average air and surface temperature of 180°C and 420°C respectively. The maximum power transmitting capability was 31.27 Kw. In comparison with the traditional design, the gear size reduced to 13% and efficiency increase to 2 % with negligible increase in noise factor.

VI. CONCLUSIONS AND FUTURE WORK

The research came from a problem of designing helical gear of various sizes based on application to run under non-lubricated environment with less noise. The problem was formulated as a non-linear NP hard problem, so GA has been used to optimize the gear parameters. GA performance have also enhanced by examining it through sensitivity analysis. Thereby the devised method can identify the best optimal solution in the desired range of input parameters. In the methodological viewpoint, optimization had been carried out along with feasibility checking to produce solution with ease in manufacturing. As expected the results of experimental tests were satisfactory and outperformed compared to the traditional method. Further work is required to establish the optimal designing with more than four primary parameters. The interaction between the gear and gear trains in the gear box problems is more complex and the optimization required for those problems.

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