

Solution to CEED Problem using Classical Approach of Lagrange's and PSO Algorithms

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Abstract- In this paper, the Classical Lagrange's method has been proposed to find the optimal solution for Combined Economic and Emission Dispatch (CEED) problems in comparison with Particle Swarm Optimization (PSO) based algorithm. The ultimate aim in the CEED problem is that scheduling of generators should operate with both minimum fuel costs and emission levels, simultaneously, while meeting the load demand and operational constraints. The problem of finding optimal solution for CEED problem is formulated as a multi-objective problem by considering the fuel cost and emission objectives of generating units and using price penalty factors a multi-objective problem is converted into a single objective function. The effectiveness of the proposed algorithm is tested on IEEE 30-bus system with various price penalty factors and is compared with particle swarm optimization method. The results presented show the superiority of the proposed method over PSO algorithm for this problem.

Keywords – Combined Economic Emission Dispatch, Lagrange's Method, Particle Swarm Optimization, Optimal Power Flow, MATLAB

I. INTRODUCTION

An electric power system consists of generation, transmission and distribution utilities enhance electrical power to the consumers. Economic dispatch (ED) is the determination of the optimal output power of generators to meet the system load and operate the generators at the lowest fuel cost. Energy management system is used to monitor, control and optimize the performance of the generators used in the electric utility grid. The main target of electric power utilities is to provide high quality reliable supply to the consumers at the lowest possible cost while operating along with meeting the limits and constraints imposed on the generating units. This necessitates to formulate the well known ED problem for finding the optimal combination of the output power of all online generating units that minimizes the total fuel cost, while satisfying all constraints.

A. CEED Problem

The optimum ED may not be the best in terms of the environment criteria. Gaseous pollutants from fossil fuel power plants will have very harmful ecological effects by the emissions; can be reduced by proper load allocation among the various generating units of the plants. But this load allocation may lead to increase in the operating cost of the generating units. So, it is necessary to find out a solution which gives a balanced result between emission and cost. This can be achieved by Combined Economic Emission Dispatch Problem.

B. Conventional Methods

Particle Swarm Optimization (PSO) is one of the modern heuristic algorithms. It has been found to be robust in solving continuous non-linear optimization problems [4]-[6]. The PSO technique can generate high-quality solutions within shorter calculation time and stable convergence characteristic than other stochastic methods [1]-[6].

In [7] it was proposed a fast novel modified price penalty factor method to solve CEED problem. In this paper, PSO method is applied for solving the OPF problem in the practical power system test case. In this paper it will be shown that Classical Lagrange's method provides best solution for CEED problem to PSO algorithm employed to solve CEED problem.

II. PROBLEM FORMULATION

The Lagrange's methods and algorithms are developed for the following case for CEED problem:

- using quadratic cost function

The PSO Algorithm is developed and will be shown that classical Lagrange's method provides best solution for CEED problem in comparison with PSO algorithm. IEEE 30 bus benchmark model is used to test the algorithms in Matlab environment.

A. ED Problem with a Quadratic Fuel Cost Objective Function

The formulation of an economic dispatch optimization problem can be written in the following way:

$$\text{Minimize } F_c = \sum_{i=1}^n F_i(P_i) = \sum_{i=1}^n (a_i P_i^2 + b_i P_i + c_i) \quad [$/h] \quad (1)$$

Where F_c Total Fuel Cost

$F_i(P_i)$ Fuel cost of the i^{th} generator

P_i Real power generation of a generator unit i

a_i, b_i, c_i Cost coefficients of generating for the unit i in [$$/MW^2h$], [$$/MWh$] and [$$/h$] respectively

n Number of generating units

Under the constraints

i. Power balance constraint

$$\sum_{i=1}^n P_i = P_G = P_D + P_L \quad [MW] \quad (2)$$

Where

P_G Total power generation of the system

P_D Total demand of the system

P_L Total transmission loss of the system

The transmission loss expressed as,

$$P_L = \sum_{i=1}^n \sum_{j=1}^n P_i B_{ij} P_j + \sum_{i=1}^n B_{0i} P_i + B_{00} \quad [MW] \quad (3)$$

Where

P_i Active power generation of unit i

P_j Active power generation of unit j

B_{ij}, B_{0i}, B_{00} Transmission loss coefficients

ii. Generator operational constraints

$$P_{i,\min} \leq P_i \leq P_{i,\max}, i = \overline{1, n} \quad [MW] \quad (4)$$

Where

$P_{i,\min}$ Minimum value of real power allowed at a generator i

$P_{i,\max}$ Maximum value of real power allowed at a generator i

B. Bi-Criteria Dispatch Problem with quadratic fuel cost and emission functions

The various pollutants like sulphur dioxide, nitrogen oxide and carbon dioxide are the major waste emissions from the thermal power plants. The problem for minimization of the quantity of the emissions is formulated by including the reduction of waste emissions as an additional objective function by the following equation

$$E_T = \sum_{i=1}^n (d_i P_i^2 + e_i P_i + f_i) \quad [\text{kg/h}] \quad (5)$$

The CEED problem is determined by the objective functions (1) and (5). A bi-objective optimization is converted into a single objective optimization (CEED) problem by introducing a price penalty factor h_i as follows:

$$F_T = \sum_{i=1}^n [(a_i P_i^2 + b_i P_i + c_i) + h_i (d_i P_i^2 + e_i P_i + f_i)] \quad \text{in } [\$/\text{h}] \quad (6)$$

Where

E_T Total emission,

d_i, e_i, f_i Emission coefficients of generating unit i in $[\text{kg}/\text{MW}^2\text{h}]$, $[\text{kg}/\text{MWh}]$ and $[\text{kg}/\text{h}]$ respectively

F_T CEED's fuel cost

The CEED problem is formulated by the criterion (6) and constraints (2) to (4).

h_i is determined by evaluating the average cost of each generator at its maximum output (P_i, max)

C. Formulation of various price penalty factors for CEED problem

The price penalty factor is used to convert bi-objective function into a single objective function in CEED problem. The comparative study of various price penalty factors such as Max-Max, Min-Min, Average and Common is proposed by [8]. In addition to that, this paper also proposes Min-Max and Max-Min price penalty factors. The impact of four types of price penalty factors such as Min-Max, Max-Max, Min-Min and Max-Min are considered in this paper to solve the CEED problem. The ratio between the minimum fuel cost and maximum emission is called —Min-Max— penalty factor is described as:

$$h_i = \frac{a_i P_{i,\text{min}}^2 + b_i P_{i,\text{min}} + c_i}{d_i P_{i,\text{max}}^2 + e_i P_{i,\text{max}} + f_i} \quad [\$/\text{kg}] \quad (7)$$

The second price penalty factor called —Max-Max is described as the ratio between the maximum fuel cost and maximum emission of the corresponding generator units as:

$$h_i = \frac{a_i P_{i,\text{max}}^2 + b_i P_{i,\text{max}} + c_i}{d_i P_{i,\text{max}}^2 + e_i P_{i,\text{max}} + f_i} \quad [\$/\text{kg}] \quad (8)$$

The third price penalty factor called —Min-Min is described as the ratio between the minimum fuel cost and minimum emission of the corresponding generator units as:

$$h_i = \frac{a_i P_{i,\text{min}}^2 + b_i P_{i,\text{min}} + c_i}{d_i P_{i,\text{min}}^2 + e_i P_{i,\text{min}} + f_i} \quad [\$/\text{kg}] \quad (9)$$

The last price penalty factor called —Max-Min is described as the ratio between the maximum fuel cost and minimum emission of the corresponding generator units as:

$$h_i = \frac{a_i P_{i,\text{max}}^2 + b_i P_{i,\text{max}} + c_i}{d_i P_{i,\text{min}}^2 + e_i P_{i,\text{min}} + f_i} \quad [\$/\text{kg}] \quad (10)$$

The role of all penalty factors is to transfer the physical meaning of the emission criterion from weight of the emission to the fuel cost for the emission. Comparison between the influences of the separate price factors over the optimization problem solution is done through application of the Lagrange's method for calculation of this solution.

III APPLICATIONS OF METHODS TO CEED PROBLEM

A. Lagrange's method for solution of the CEED problem

In mathematical optimization, the method of Lagrange's multipliers provides a strategy for finding the local maxima or minima of a function subject to the equality or inequality constraints. Method of Lagrange's is used to obtain the solution for the CEED problem with criterion given by the equation (6), subject to the constraints (2), (3) and (4). The problem is solved by introduction of a function of Lagrange based on a Lagrange's multiplier λ , as follows:

$$L = F_T + \lambda(P_D + P_L - \sum_{i=1}^n P_i) \quad (11)$$

$$L = \sum_{i=1}^n [(a_i P_i^2 + b_i P_i + c_i) + h_i (d_i P_i^2 + e_i P_i + f_i)] + \lambda(P_D + \sum_{i=1}^n \sum_{j=1}^n P_i B_{ij} P_j + \sum_{i=1}^n B_{0i} P_i + B_{00}) \quad (12)$$

The physical meaning of the Lagrange's multiplier λ is of a cost for fuel production. Then the physical meaning of the function of Lagrange is a cost for fuel production. The optimization problem (6), subject to the constraints (2), (3) and (4) is transferred to the problem for minimization of L according to $P_i = 1, n$, and maximization of L according λ , under the constraints (4).

The dispatch problem can be solved for different expressions of the penalty factors in a process of search of the best solution. Software developed for solution of the CEED problem using Lagrange's and simulation is done for IEEE30 bus case study with matlab programming.

B. CEED Solution Based On the Particle Swarm Optimization Algorithm

This section proposes Min-Max penalty factor to be used in addition to Max-Max one for formulation and solution of the CEED problem by the PSO algorithm. Comparison of the impact of the Min-Max and Max-Max penalty factors over the solution of the CEED problem is done on the basis of case study of IEEE 30 bus system.

Particle Swarm Optimization (PSO) is a population based stochastic optimization technique developed by [10], inspired by the social behavior of bird flocking or fish schooling. PSO is initialized with a group of random particles and then it searches for optima by updating the generation. In every iteration, each particle is updated by following two best values. The first one is the best solution it has achieved so far between all particles.

This value is called P^{best} . Another best value that is tracked by the PSO algorithm is the best value, obtained so far by any particle in the population. This best value is a global best one and is called G^{best} . After finding the two best values, the particle updates its velocity and positions with following equations (13) and (14).

$$V[] = \begin{bmatrix} \omega V[] + c_1 \text{rand} 1() * (P^{best} [] - P_{\text{present}} []) \\ + c_2 \text{rand} 2() (G^{best} [] - P_{\text{present}} []) \end{bmatrix} \quad (13)$$

$$P_{\text{present}} [] = P_{\text{present}} [] + V[] \quad (14)$$

Where

$V []$ Particles velocity

ω Inertia weight and is calculated as,

$$\omega = \omega^{\max} - \left(\frac{\omega^{\max} - \omega^{\min}}{\text{Iter}^{\max}} \right) \text{Iter}$$

$$\omega^{\min} = 0.4 \text{ and } \omega^{\max} = 0.9$$

$P_{\text{present}} []$ Position of the current particle ; $P^{best} []$ Best solution of the particle at each iteration

$G^{best} []$ Best solution out of all the best particle solutions ;

$\text{rand}1()$ and $\text{rand}2()$ Random number between (0,1) ; c_1 and c_2 Learning factors (usually $c_1=c_2=2$)

Iter Total number of iterations ; Iter^{\max} Maximum number of iterations

IV SIMULATION RESULTS

A. CEED Problem using Lagrange’s Algorithm Using Various Price Penalty factors

The proposed dispatch problem is solved for IEEE 30 bus system [9], Table- 1 represents the IEEE 30 bus system data [9]. The system has six generating units. The CEED problem is solved for the cases of various price penalty factors. The different power demand values are introduced. Various levels of disturbance in order to evaluate the solution for the considered power demands are:

$$P_D = [125; 150; 175; 200; 225; 250] \text{ MW.}$$

Table -1: IEEE 30 Bus System Data

Bus No.	Generator Limits[MW]		Fuel cost coefficients			Emission coefficients		
	Pmax	Pmin	ai [\$/MW2h]	bi [\$ /MWh]	ci[\$/h]	di[\$/kg 2h]	ei[\$/kgh]	fi[\$/h]
1	200	50	0.00375	2	0	0.0126	-0.9	22.983
2	80	20	0.0175	1.7	0	0.02	-0.1	25.313
5	50	15	0.0625	1	0	0.027	-0.01	25.505
8	35	10	0.00834	3.25	0	0.0291	-0.005	24.9
11	30	10	0.025	3	0	0.029	-0.004	24.7
13	40	12	0.025	3	0	0.0271	-0.0055	25.3
Transmission loss coefficients								
B01			B					
0.000003	0.000021	-0.00056	0.000218	0.000103	0.000009	-0.00001	0.000002	0.000027
			0.000103	0.000181	0.000004	-0.000015	0.000002	0.00003
0.000034	0.000015	0.000078	0.000009	0.000004	0.000417	-0.000131	-0.000153	-0.000107
			0.00001	-0.000015	-0.000131	0.000221	0.000094	0.00005
B00=0.000014			0.000002	0.000002	-0.000153	0.000094	0.000243	-0.000001
			0.000027	0.00003	-0.000107	0.00005	-0.000001	0.0003458

				[kg/h]		Iterations	
125	2.66	1.2499	308.1608	144.5304	377.1857	195	2.1216
150	3.02	2.0967	373.5001	162.2299	448.5151	153	1.5965
175	3.36	3.1119	443.967	190.4187	528.557	139	1.3095
200	3.70	4.3059	519.5033	228.1548	616.9529	129	1.2071
225	3.97	5.3865	601.1024	268.0758	713.0526	75	0.6671
250	4.19	6.4337	687.0737	310.2842	815.1852	85	0.8626

Table-3: The Bi-criteria dispatch problem solution using Max-Max price penalty factor

P _D [MW]	λ	P _L [MW]	F _c [\$/h]	E _r [Kg/h]	F _r [\$/h]	No.of Iterations	C _r [s]
125	3.24	1.2499	308.1608	144.5304	619.259	612	1.2622
150	4.39	2.1071	373.4835	162.2107	716.1636	413	0.4531
175	5.09	2.8502	447.4342	186.5649	835.6432	272	0.3182
200	5.64	3.5581	527.9257	214.427	969.912	247	0.2791
225	6.18	4.3444	612.0186	247.2931	1117.7	243	0.332
250	6.73	5.2153	699.5472	285.3648	1279.1	238	0.343

Table-4: The Bi-criteria dispatch problem solution using Min-Min price penalty factor

P _D [MW]	λ	P _L [MW]	F _c [\$/h]	E _r [Kg/h]	F _r [\$/h]	No.of Iterations	C _r [s]
125	4.13	1.1177	310.8005	149.3334	606.337	382	0.3509
150	4.79	1.3569	395.0747	171.2493	718.3321	277	0.4787
175	5.38	1.6571	486.0034	199.8198	845.5137	275	0.4875
200	5.97	2.0169	582.7623	235.5157	987.5126	277	0.5305
225	6.66	2.4492	686.3078	277.5522	1144.8	449	0.534
250	7.76	3.0511	800.2705	327.6648	1325.3	378	0.545

Table-5 :The Bi-criteria dispatch problem solution using Max-Min price penalty factor

P_D [MW]	λ	P_i [MW]	F_c [\$/h]	E_T [Kg/h]	F_T [\$ /h]	No.of Iterations	C_T [s]
125	7.1899	1.0204	317.8361	148.1861	1749.2	1140	2.423
150	9.3366	1.2397	408.3563	171.7818	1957.7	703	1.734
175	11.1257	1.5482	500.9697	202.0717	2213.5	763	0.834
200	13.3933	1.9221	596.6819	238.1609	2517.6	1359	2.632
225	16.9178	2.3907	702.0336	278.2113	2897.2	1540	4.367
250	22.2824	2.8874	818.3178	326.4815	3387.6	1850	3.532

The Solution of the bi-criteria economic dispatch problem for various penalty factors is shown in Table- 2, Table- 3, Table- 4, Table- 5, it consists of the real power outputs of the generators in MW, fuel cost in [\$/hr] and emission values in [kg/hr] for the IEEE 30 bus system. The initial lambda (λ^0) value is assumed as 8. The maximum number of Iterations, $Iter_{Max} = 2000$. The optimal solution is computed for the selected load demand.

Figure-1 shows the CEED problem fuel cost values for the various price penalty factors using Lagrange’s method for optimization. In Figure-1, x-axis represents power demand ranges from 125 to 250 [MW]. The y-axis represents the fuel cost values of the CEED problem. Figure-1 shows that the CEED fuel cost values are less when using Min-Max price penalty factor in comparison with the other price penalty factors for the solution of bi-criteria dispatch problem.



Figure- 1: CEED fuel cost values of the bi-criteria dispatch problem using various price penalty factors

B. Results of the multi-objective dispatch problem using PSO

PSO algorithm is used to solve the CEED problem for the IEEE 30 bus system .The various power demands (P_D) of 125 to 250 [MW] are considered. The calculations are carried out in the Matlab software environment. Most of the reference papers used Max-Max penalty factor to solve the CEED problem, but in addition to that Min-Max penalty factor as proposed in this paper is considered. The problem solution results in above conclude that the CEED problem based on quadratic criterion functions are less using Min-Max penalty factor in comparison to the Max-Max one.

Table-6: Bi-criteria dispatch Problem with valve point loading effect solution using various price penalty factors

P_D [MW]	250 283.4			
Penalty factor h_i	Min-Max		Max-Max	
Algorithm	Lagrange’s	PSO	Lagrange’s	PSO
P_i [MW]	6.4337	6.736	5.2153	5.539
F_c [\$/h]	687.0737	686.735	699.5472	694.519
E_T [Kg/h]	310.2842	317.206	285.3648	291.976
F_T [\$ /h]	815.1852	816.306	1279.1	1282.514
Number of Iterations	117	17	238	24
C_T [s]	0.1971	0.123	0.332	0.113

V. CONCLUSION

Lagrange's algorithm using Min-Max price penalty factor produces minimum fuel cost (F_C , F_T) and computation time (C_T) values in comparison with PSO, when using the Max-Max penalty factor, as given in Table-6. The transmission power loss (P_L) value is less in developed Lagrange's method [7] in comparison with the PSO algorithm.

This section of the paper compares the two used optimization approaches: the classical one-the (Lagrange's) and heuristic one-the random variable selection approach (PSO) on the basis of solution of the CEED problem with respect to the obtained solution and the computation time. The application of the Lagrange's and PSO methods to the CEED problem is described and the algorithms for calculations are given above respectively. The computational time of Lagrange's and PSO algorithms depends on the selection of the initial values of the Lagrange's variable (λ), and on the initial particle positions and velocity selections in the PSO algorithm. The results are given in Table -6.

The IEEE 30 bus system is considered to validate the solution results in MATLAB software environment. It concludes that Lagrange's algorithm provides better results for the CEED problem in comparison to the PSO algorithm.

REFERENCES

- [1] A. H. Mantawy, Y. L. Abdel-Magid, Integrating Genetic Algorithm, Tabu search, and Simulated Annealing for The Unit Commitment Problem, IEEE Transactions on Power Systems, Vol. 14, pp. 829-836, August 1999.
- [2] A. Bakirtzis, P. N. Biskas, C. E. Zoumas, and V. Petridis, Optimal Power Flow By Enhanced Genetic Algorithm, IEEE Transactions on Power Systems, Vol.17, No.2, pp.229-236, May 2002.
- [3] K .P. Wong and J. Yuryevich, Evolutionary Programming Based Algorithm for Environmentally Constrained Economic Dispatch, IEEE Trans.Power Syst., Vol.13, No.2, pp. 301, May 1998.
- [4] P. J. Angeline, Using Selection to Improve PSO, in Proc. IEEE International conference on Evolutionary. Computations, pp. 84–89, May 1998.
- [5] J. Kennedy and R. Eberhart, PSO, Proc. IEEE Int. Conf. Neural Networks, Vol. IV, pp. 1942–1948, 1995. [6] R. Labdani, L. Slimani and T. Bouktir, Particle Swarm Optimization Applied To the Economic Dispatch Problem, Journal of Electrical Systems, Vol.2, Issue 2, June 2006.
- [6] Gnanadass, R. NOV 2005. Optimal power dispatch and pricing for deregulated power industry. PhD Thesis, Pondicherry University, India.
- [7] Balamurugan, R., and Subramanian, S. 2007. A Simplified Re-cursive Approach to Combined Economic Emission Dispatch. Electric Power Components and Systems, Vol. 36, No. 1, pp. 17-27.
- [8] Ming, Z., Xiaotong, L., Fan, Y., and Kuo, T. 2010. The multi-objective optimization model of energy-efficient scheduling based on PSO algorithm. Asia-Pacific, Power and Energy Engineering Conference , Chengdu, pp. 1 – 4.
- [9] Kennedy, J., and Eberhart, R. 1995. Particle Swarm Optimization. Proceedings of the IEEE international conference of neural networks (ICNN'95), Vol.IV, pp.1942-1948.