# Computing and comparing the minimum distance between two Bézier & interval Bézier curves

Megha Jain
*Department of Applied Mathematics and Computational Science Shri G.S.Institute of Technology and Science, 23, M.Visvesvaraya Marg, Indore 452003 Madhya Pradesh, India*

Ashok Ganguly
*Department of Applied Mathematics and Computational Science Shri G.S.Institute of Technology and Science, 23, M.Visvesvaraya Marg, Indore 452003 Madhya Pradesh, India*

**Abstract - In this paper we compute the minimum distance between two interval Bézier curves, we then compare it with the case of minimum distance between two Bézier curves by using sweeping sphere clipping method. We want to stress that approximated good result also came from interval Bézier curve and it is capable to find minimum distance with higher efficiency.**

**Keywords: Bézier curve, Interval arithmetic, Interval Bézier curve, Sweeping sphere clipping method**

## I. INTRODUCTION

The minimum distance problem is to find the nearest point pair $(p,q)$ such that $p \in o_1$, $q \in o_2$ and that the distance between $p$ and $q$ is minimum .It is an important problem in many fields such as geometric modeling[9,12], Computer graphics [3,7],and computer vision[9,12,7,6]. Further the minimum distance information and location where the minimum distance occurs are very useful in the location design of a bridge or undersea tunnel XD Chen et.al [2] computed the minimum distance between two Bézier curves by using a sweeping sphere clipping method and established the efficiency and robustness of the method [2].

However, in the past decade there has been considerable interest in approximating curves and surfaces that arise in CAD/CAM applications. Sederberg et.al. [1,10] introduced a new representation form of parametric curves- the interval Bézier curves -that can transfer a complete description of approximation errors along with the curves to applications in other systems. The work of Sederberg inspired Hu.et.al. [1, 4, 5] and Tuohy et.al. [1, 11], to divert attention to interval forms of geometric objects and rounded interval arithmetic to deal with the problem of robustness. The series of works by Hu et.al. indicate the numerical stability in geometric computations, and thus enhance the robustness of current CAD/CAM systems.

In this paper, we discuss the problem of minimum distance between interval Bézier curves by using sweeping sphere clipping method, as done by Chen et.al. [2] and compare it with the computed minimum distance between two Bézier curves. We found that approximate good result also came from interval Bézier curve and it leads to higher efficiency.

The organization of this paper is as follows. We first briefly review the definitions of interval arithmetic and interval Bézier curves, and then we describe two algorithms in interval Bézier curves (in rectangular domain). Finally in the last section we compared the minimum distance obtained for Bézier curves and interval Bézier curves.

## II. ALGORITHM OF THE SWEEPING SPHERE CLIPPING METHOD FOR TWO INTERVAL BÉZIER CURVES:

The minimum distance computation problem between curves and surfaces may be divided into five cases. i.e. point-curve, point –surface, curve-curve, curve-surface, surface-surface. Here we consider curve-curve case. Suppose we are given two curves $C_1(u)$ and $C_2(v)$. When the nearest points are both inner points of the curves, then we have the corresponding equation system

$$\begin{cases} S_u(u,v)=0, \\ S_v(u,v)=0, \end{cases} \tag{1}$$

Where $S(u,v)$ is equal to $(C_1(u)-C_2(v))^2$, which is the squared distance between $C_1(u)$ and $C_2(v)$. The algorithm for sweeping sphere clipping method is as follows. It mainly contains two steps. The first step is to compute Bézier form of $S(u,v)$ from Eq. (1). The second step is to compute the minimum value d of $S(u,v)$ as well as its parameter pair $(u,v)$.

*Step 1:*

Let $B_n^i(u)$ and $B_m^j(v)$ are nth-degree and mth-degree Bézier basis functions, and that two Interval Bézier curves are

$$[C_1](u)=\sum_{i=0}^{n}[p_i]B_n^i(u), \qquad u\in[0,1]$$

And

$$[C_2](v)=\sum_{j=0}^{m}[q_j]B_m^j(v), \qquad v\in[0,1]$$

Where $[p_i]$ and $[q_j]$ are control points of the two curves in $\Re^p$ space. The key technique of the sweeping sphere clipping method is to judge whether a curve is outside of a sweeping sphere. It seems difficult to solve it directly by the information of the control points. We overcome this problem by analyzing the objective squared distance function.

$$S(u,v)=\left(\sum_{i=0}^{n}[p_i]B_n^i(u)-\sum_{j=0}^{m}[q_j]B_m^j(v)\right)^2$$

*2.1. Computing $S(u,v)$ in interval Bézier curve form*

Let $C_i^j$ denotes the binomial coefficient $\begin{pmatrix} j \\ i \end{pmatrix}$, $\theta=\max\{0,r-n\}$, $\upsilon=\min\{r,n\}$, $\sigma=\max\{0,k-m\}$, $\varsigma=\min\{k,m\}$ and

$$[A_r]=\sum_{i=\theta}^{\upsilon}([p_i].[p_{r-i}])C_n^iC_n^{r-i}/C_{2n}^r$$

$$[B_k]=\sum_{j=\sigma}^{\varsigma}([q_j].[q_{k-j}])C_m^jC_m^{k-j}/C_{2m}^k.$$

Where $r=0,1,........2n$ , $k=0,1,........,2m,$ and "." Denotes the inner product between two vectors in $\Re^p$ space. We obtain

$$\left(\sum_{i=0}^{n}[p_i]B_n^i(u)\right)^2=\sum_{r=0}^{2n}[A_r]B_{2n}^r(u)=\sum_{r=0}^{2n}[A_r]B_{2n}^r(u)\sum_{k=0}^{2m}B_{2m}^k(v),$$

$$\left(\sum_{j=0}^{m}[q_j]B_m^j(v)\right)^2=\sum_{k=0}^{2m}[B_k]B_{2m}^k(v)=\sum_{k=0}^{2m}[B_k]B_{2m}^k(v)\sum_{r=0}^{2n}B_{2n}^r(u)$$

It can be verified that

$$\sum_{i=0}^{n}[p_i]B_n^i(u)=\sum_{r=0}^{2n}\left(\sum_{i=\theta}^{\upsilon}[p_i]C_n^iC_n^{r-i}/C_{2n}^r\right)B_{2n}^r(u)$$

*and*

$$\sum_{j=0}^{m}[q_j]B_m^j(v)=\sum_{k=0}^{2m}\left(\sum_{j=\sigma}^{\varsigma}[q_j]C_m^jC_m^{k-j}/C_{2m}^k\right)B_{2m}^k(v).$$

We have

$$\sum_{i=0}^{n}[p_i]B_n^i(u).\sum_{j=0}^{m}[q_j]B_m^j(v)=\sum_{r=0}^{2n}\sum_{k=0}^{2m}[C_{r,k}]B_{2n}^r(u)B_{2m}^k(v),$$

Where

$$[C_{r,k}]=\left(\sum_{i=\theta}^{\upsilon}([p_i]C_n^iC_n^{r-i}/C_{2n}^r\right).\left(\sum_{j=\sigma}^{\varsigma}([q_j]C_m^jC_m^{k-j}/C_{2m}^k\right).$$

Thus, we can turn $S(u,v)$ into an interval Bézier form and obtain

$$S(u,v)=\sum_{r=0}^{2n}\sum_{k=0}^{2m}[D_{r,k}]B_{2n}^r(u)B_{2m}^k(v),$$

Where $[D_{r,k}]=[A_r]+[B_k]-2[C_{r,k}].$

*Step 2:*

Algorithm 1: Algorithm for computing the minimum value of a surface $S(u,v)$ .

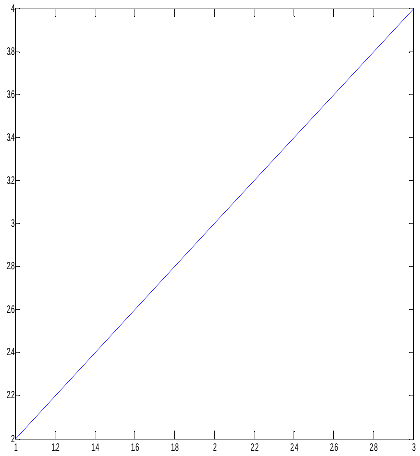
1. The current minimum value denoted by $\alpha$ ;
2. Compute the minimum value d of $S(u,v)$, record the parameter pair $(u,v)$;
3. If d is smaller than $\alpha$ , then update $\alpha = d$ and its parameter pair as well; goto step 5;
4. Update $\alpha = \min\{\alpha, S(0,0), S(1,0), S(0,1), S(1,1)\}$ ;
5. End of algorithm 1: return the minimum value d and its parameter pair as well.

Algorithm 2. Algorithm for computing the minimum distance between two interval Bézier curves.

1. Compute $S(u,v)$ in an interval Bézier form:
2. Set $\alpha$ as $\min\{S(0,0), S(1,0), S(0,1), S(1,1)\}$;
3. End of algorithm 2: use algorithm 1 to compute the minimum value d of $S(u,v)$ and the parameter pair $(u,v)$ where the minimum value occurs;

## III. COMPARISON OF BÉZIER CURVE WITH INTERVAL BÉZIER CURVE & CONCLUSION:

For a set of values (0,0),(1,0),(0,1),(1,1) we have calculated $S(u,v)$ and found that, from Bézier curve we got a straight line and similarly for interval Bézier curve we got a straight line and that is the minimum distance. Thus, the sweeping sphere clipping method also works for interval Bézier and due to inherent robustness in interval Bézier curve in solid models it increase numerical stability in geometric computations. We should pursue minimum distance problem with interval Bézier curves.



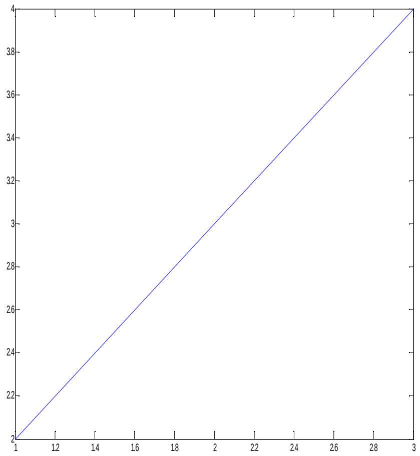




Fig 1: Illustration of minimum distance Between two Bézier curves

Fig 2: Illustration of minimum distance between two interval Bézier curves

All programming are implemented in MATLAB.

## REFERENCES

[1] Chen Falai, Wenping L. Degree reduction of interval Bézier curves. Computer Aided Design 2000(32)571-582.
[2] Chen XD, Chen L,Wang Y,Xu G, Yong JH,Paul JC. Computing the minimum distance between two Bézier curves. Journal of computational and applied mathematics.2009 (229, 1) 194-301.
[3] D. Johnson, E. Cohen, A framework for efficient minimum distance computations,Proceedings of IEEE International Conference on Robotics & Automation, 1998, pp. 3678_3684.
[4] Hu C-Y, Maekawa T,Sherbrooke EC,Patrikalakis NM. Robust interval algorithm for curve intersections. Computer Aided Design 1996; 28(6/7):495-506.
[5] Hu C-Y, Maekawa T,Patrikalakis NM, Ye X. Robust interval algorithm for surface intersections.Computer Aided Design 1997;29(9):617-27.
[6] J. Pegna, F. Wolter, Surface curve design by orthogonal projection of space curves onto free-form surfaces, Journal of Mechanical Design 118 (1996) 42_52.
[7] L. Piegl, W. Tiller, Parametrization for surface fitting in reverse engineering, Computer-Aided Design 33 (2001) 593_603.
[8] N. Patrikalakis, T. Maekawa, Shape Interrogation for Computer Aided Design and Manufacturing, Springer-Verlag, New York, 2001.
[9] S. Hu, J. Wallner, A second order algorithm for orthogonal projection onto curves and surfaces, Computer Aided Geometric Design 22 (2005) 251_260.
[10] Sederberg TW,Farouki RT. Approximation by interval bezier curves.IEEE comput graph Appl 1992;15(2):87-95.
[11] Tuohy ST,Maekawa T,Shen G,Patrikalakis NM. Approximation of measured data with interval B-splines. Computer Aided Design. 1997; 29(11):791-9.
[12] Y. Ma, W. Hewitt, Point inversion and projection for NURBS curve and surface: Control polygon approach, Computer Aided Geometric Design 20 (2003).