Active Vibration Control of Smart Beam under Random Vibrations

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Abstract- Thepresent paper discusses active vibration control of a cantilever beam subjected to random base excitation. The smart materials used are piezoelectric patches for both as sensor and actuator, and actuation based on the surface strains. Finite element model of the smart beam is obtained using Euler-Bernoulli formulation. Positive Position Feedback (PPF) controller is proposed for active damping to control the vibrations at all the resonant modes using piezoelectric sensor voltage feedback.Experiments are carried out on a smart cantilever beam and the closed loop responsesare studied using three collocated sensor-actuator pairsat different locations and with different controller gains. It is observed that the selection of the sensor-actuator pair helps in controlling the modes significantly.Generally,larger gains are required for controlling the higher modes. For a particular chosen collocated sensor-actuator pair, higher damping ratios are achieved for lower modes; lower damping ratios are obtained for higher modes even with higher gains.

Key words- Smart Beam, Active Vibration Control, PZT patch, PPF controller

I. INTRODUCTION

Structural vibration controlis widely researched topic in the past few decades. Piezoelectric materials have been applied in structural vibration control as wellas in structural acoustics because of their advantages of fast response, large force output and the fact that they generate no magnetic field in the conversion of electricalenergy into mechanical motion. Passive, semi-active, active and hybrid vibration control methods[1] are studied by many researchers. Application of smart materials in vibration control is advantageous because of lesser weight, high frequency bandwidth and low cost. Active vibration control system[1] consists of the sensors, actuators and a control system to achieve the desired reduction in the vibrational amplitudes of the structure. Smart materials are easily embedded into the host structure, have high level of integration and produce desirable response.

Gohand Caughey[2] introduced a Positive position feedback (PPF) concept to control vibration of large flexible space structures. It was applied byfeeding the structural position coordinate directly to the compensator and theproduct of the compensator and a scalar gain positively back to the structure. Fanson and Caughey[3]have first demonstrated Positive Position Feedback (PPF) method in 1990. This method is insensitive to spillover and is unaffected by sensor and actuator dynamics.McEver and Leo[4]pointedout that a PPF controller can be formulated as an output feedback controller andcontrol design algorithms for output feedback systems can be used to design PPFcontroller. PPF control was implemented for single-mode vibration suppressionand for multi-mode vibration suppression, and the robustness of PPF controllerto uncertainty in frequency was studied by Song et.al.[5]. Adaptive PPF methodology [6-8] was also introduced to handle the mass uncertainty and is demonstrated using experiments.Saurabh Kumar et.al.[9] used a PID controller to supress the vibration level of a cantilever beam. They also concluded that sensor/actuator pair is more effective when placed near the base. K Ramesh Kumar and S Narayanan[10] used a LQR controller for vibration control. They also considered optimal placement of collocated sensor/actuator pair. S. NimaMahmoodi and Mehdi Ahmadian[11] used a modified PPF controller wherein a first order compensator provides damping and second order compensator is used for vibration suppression. Zhi-chengQiu et.al.[12] used an accelerometer located at tip of beam as sensor and piezo-patches located near the fixed end as actuators for vibration control of the beam.

In this paper, an attempt has been made to design a real-time PPF controller using Matlab/Simulink environment. Finite element model of a smart beam with piezoelectric patches is formulated and the piezo sensor voltage transfer functions are derived for different sensor-actuator configurations. In section II, the generalized formulation for base excitation problems is presented. A positive position feedback controller is designed to suppress the vibration of the structures. Experiments are carried out to evaluate the open loop and closed loop responses for random input base excitation. The experimental results are discussed in section III. Parallel PPF controllers are used to suppress the multi modes of the structure. Selection of the sensor-actuator pairs to control particular mode is also studied.

II. GENERALFORMULATION

A. *Beam equations -*

The dynamic equation of the beam can be written as,

$$[\mathrm{M}]\{\ddot{\mathrm{w}}_{\mathrm{b}}(\mathrm{t})\}+[\mathrm{C}]\{\dot{\mathrm{w}}_{\mathrm{b}}(\mathrm{t})\}+[\mathrm{K}]\{\mathrm{w}_{\mathrm{b}}(\mathrm{t})\}=\{0\} \tag{1}$$

where, $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices.

For base excitation problem[13],Eq. (1) can be written as,

$$\begin{bmatrix} M_{cc} & M_{cu} \\ M_{uc} & M_{uu} \end{bmatrix}\begin{Bmatrix} \ddot{w}_c(t) \\ \ddot{w}_u(t) \end{Bmatrix}+\begin{bmatrix} C_{cc} & C_{cu} \\ C_{uc} & C_{uu} \end{bmatrix}\begin{Bmatrix} \dot{w}_c(t) \\ \dot{w}_u(t) \end{Bmatrix}+\begin{bmatrix} K_{cc} & K_{cu} \\ K_{uc} & K_{uu} \end{bmatrix}\begin{Bmatrix} w_c(t) \\ w_u(t) \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2}$$

where

$$w_b(x,t) = \begin{Bmatrix} w_c(x,t) \\ w_u(x,t) \end{Bmatrix} \tag{3}$$

$w_c(x,t)$ is the constrained DOF and $w_u(x,t)$ is the unconstrained DOF.The suffix 'c' refers to constrained and 'u' refers to unconstrained. The unconstrained displacements can be decomposed into pseudo-static and dynamic parts as,

$$\{w_u(x,t)\} = \{w_s(x,t)\} + \{w_d(x,t)\} \tag{4}$$

The pseudo-static displacements, $w_s(x,t)$ may be obtained from Eq. (2) by excluding the first two terms on the left-hand side of the equation and by replacing $w_u(x,t)$ by $w_s(x,t)$. $[K_s]$ is the constant matrix.

$$\{w_s(x,t)\} = -[K_{uu}]^{-1}[K_{uc}]\{w_c(x,t)\} = [K_s]\{w_c(x,t)\} \tag{5}$$

$$\{w_u(x,t)\} = [K_s]\{w_c(x,t)\} + \{w_d(x,t)\} \tag{6}$$

Substituting Eq. (6) in Eq. (2), we get

$$\begin{aligned}&[M_{uu}]\{\ddot{w}_d(t)\}+[C_{uu}]\{\dot{w}_d(t)\}+[K_{uu}]\{w_d(t)\}=-\{[M_{uc}]+[M_{uu}][K_s]\}\{\ddot{w}_c(t)\}\\&-\{[C_{uc}]+[C_{uu}][K_s]\}\{\dot{w}_c(t)\}-\{[K_{uc}]+[K_{uu}][K_s]\}\{w_c(t)\}\end{aligned} \tag{7}$$

For input base acceleration problems of the beam with piezoelectric patches[14],Eq. (7) becomes,

$$\begin{aligned}&[M_{uu}]\{\ddot{w}_d(t)\}+[C_{uu}]\{\dot{w}_d(t)\}+[K_{uu}]\{w_d(t)\}\cong-\{[M_{uc}]+[M_{uu}][K_s]\}\{\ddot{w}_c(t)\}+F_vV_a(t)\\&=F_du(t)+[K_{em}]V_a(t)\end{aligned} \tag{8}$$

where, $\{F_d\}=-\{[M_{uc}]+[M_{uu}][K_s]\}, u(t)=\{\ddot{w}_c(t)\}, [K_{em}]$ is the electro-mechanical coupling matrix and $\{V_a(t)\}$ is the applied voltage to the actuator.

From modal analysis approach[15,16],

$$\{w_d(t)\} = [P]\{q_m(t)\} \tag{9}$$

where[P] is the modal matrix and $\{q_m(t)\}$ is the modal response vector.
Using Eqn (9) and Pre-multiplying Eqn. (8) by $[P]^T$, we get

$$[P]^T[M_{uu}][P]\{\ddot{q}_m(t)\}+[P]^T[C_{uu}][P]\{\dot{q}_m(t)\}+[P]^T[K_{uu}][P]\{q_m(t)\} = [P]^T\{F_d\}\{u(t)\}+[P]^T[K_{em}]V_a(t) \quad (10)$$

$$[M_m]\{\ddot{q}_m(t)\}+[C_m]\{\dot{q}_m(t)\}+[K_m]\{q_m(t)\}=[F_{dm}]\{u(t)\}+[F_{vm}]\{V_a(t)\} \quad (11)$$

$$\{\ddot{q}_m(t)\}+[M_m]^{-1}[C_m]\{\dot{q}_m(t)\}+[M_m]^{-1}[K_m]\{q_m(t)\} = [M_m]^{-1}[F_{dm}]\{u(t)\}+[M_m]^{-1}[F_{vm}]\{V_a(t)\} \quad (12)$$

$$\{\ddot{q}_m(t)\}+[\hat{C}_m]\{\dot{q}_m(t)\}+[\hat{K}_m]\{q_m(t)\} = [\hat{F}_{dm}]\{u(t)\}+[\hat{F}_{vm}]\{V_a(t)\} \quad (13)$$

Where $[\hat{C}_m]$ is the modal damping matrix, $[\hat{K}_m]$ is the modal stiffness matrix, $[\hat{F}_{dm}]$ is the modal force matrix and $[\hat{F}_{vm}]$ is the modal voltage matrix. The modal damping matrix is defined for 3 modes as,

$$[\hat{C}_m] = \begin{bmatrix} 2\zeta_1\omega_{n1} & 0 & 0 \\ 0 & 2\zeta_2\omega_{n2} & 0 \\ 0 & 0 & 2\zeta_3\omega_{n3} \end{bmatrix} \quad (14)$$

A modal damping ratio (ζ) of 2% is assumed for all the modes in simulation. Eqn.(13) is used to develop a state space model for the beam. Considering the state vectors as,

$$\{z_{1(t)}\} = \{q_m(t)\} \quad (15)$$

$$\{z_2(t)\} = \{\dot{q}_m(t)\} \quad (16)$$

The modal transfer function can be evaluated as,

$$[H_m(s)] = [C_a][sI-A]^{-1}[B]+[D] \quad (17)$$

where [A] is the state matrix, [B] is the input matrix and [D] is the feed forward matrix. The overall transfer functions for N modes at a particular location i can be found by the following summation,

$$[H(s)]_i = \sum_{k=1}^{N}\{\phi\}_{ik}\left[(H_m(s))_k\right] \quad (18)$$

$$\{W_d(s)\}_i = [H(s)]_i\{U(s)\} \quad (19)$$

The piezoelectric sensors are used as strain rate sensors. The sensor voltage output, $V_s(t)$ of the jth piezoelectric patch can be obtained as,

$$V_s^j(t) = G_c\frac{dC_q(t)}{dt} \quad (20)$$

WhereG$_c$is the Gain of the charge amplifier. Charge developedC$_q$is given by

$$C_q(t) = \frac{1}{2}\int_{s_j}[B_1]e_{31}dS\{w_d(t)\} = [K_v^j]\{w_d(t)\} \quad (21)$$

Where S – Surface, e_{31} - piezoelectric constant, $[B_1]$ - Strain matrix

$$V_s^j(t) = G_c[K_v^j]\{\dot{w}_d(t)\} \tag{22}$$

$$V_s^j(s) = sG_c[K_v^j]\{W_d(s)\} = sG_c[K_v^j]H(s)\{U(s)\} = \hat{H}(s)U(s) \tag{23}$$

From Eq. (23), the overall voltage responses of the piezoelectric sensors in the frequency band of interest at different locations can be obtained.

B. Positive Position Feedback (PPF) Controller

In Positive Position Feedback (PPF), the position state of the system is positively sent to the actuator and position state of the actuator is positively sent to the system. A PPF controller is designed as follows,

$$V_s(s) = G_a\eta(s) + \hat{H}(s)U(s) \tag{24}$$

$$(s^2 + 2\xi_f\omega_f s + \omega_f^2)\eta(s) = K_{PPF}V_s(s) \tag{25}$$

where,η is the filter coordinate, ω_f is the filter frequency, ξ_f is the filter damping ratio, U(s) is the external base acceleration, K_{PPF} is the gain of the controller, G_a is the gain of the high voltage power amplifier for the piezo actuator.

For a single mode control, the sensor voltage transfer function will become as,

$$V_s(s) = \frac{G_a}{s^2 + 2\xi_1\omega_{n_1}s + \omega_{n_1}^2}\eta(s) + \frac{\hat{K}_1}{s^2 + 2\xi_1\omega_{n_1}s + \omega_{n_1}^2}U(s) \tag{26}$$

$$V_s(s) = \left(\frac{G_a}{s^2 + 2\xi_1\omega_{n_1}s + \omega_{n_1}^2}\right)\left(\frac{K_{PPF}}{s^2 + 2\xi_f\omega_f s + \omega_f^2}\right)V_s(s) + \frac{\hat{K}_1}{s^2 + 2\xi_1\omega_{n_1}s + \omega_{n_1}^2}U(s) \tag{27}$$

where, $\hat{K}_1$ is the constant. The closed loop transfer function can be obtained as,

$$\frac{V_s(s)}{U(s)} = \left[\frac{(s^2 + 2\xi_f\omega_f s + \omega_f^2)(s^2 + 2\xi_1\omega_{n_1}s + \omega_{n_1}^2)}{(s^2 + 2\xi_f\omega_f s + \omega_f^2)(s^2 + 2\xi_1\omega_{n_1}s + \omega_{n_1}^2) - G_aK_{PPF}}\right]\left[\frac{\hat{K}_1}{s^2 + 2\xi_1\omega_{n_1}s + \omega_{n_1}^2}\right] \tag{28}$$

Transfer function is stable if and only if $K_{PPF}G_a < \omega_f^2\omega_{n_1}^2$. When the controller and the structure have the same natural frequency, the PPF controller in this case results in an increase in the damping term, which is called active damping.

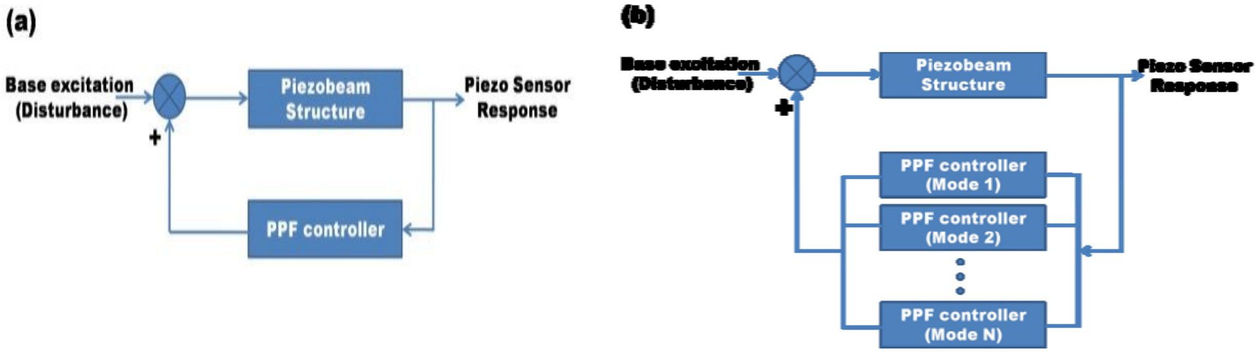

Fig. 1 Schematic of PPF Controller for (a) Single Mode (b) Multi – Mode Control

Fig. 1(a) shows the schematic of the PPF controller for controlling the individual modes whereas to control 'N' number of modes, 'N' numbers of PPF controllers are placed in parallel as shown in Fig. 1(b).The overall sensor voltage response of the structure for a given base acceleration disturbance is passed through three different parallel PPF controllers. Three collocated sensor actuator pairs (S1/A1, S2/A2 & S3/A3) of piezoelectric (PZT-5H) patches (0.05 m x 0.025 m x 0.001 m) are bonded using araldite on an aluminum beam (0.27 m x 0.025 m x 0.003 m) in bimorph configuration. The starting location of the 3 pairs of piezo patches are 0.02, 0.09 and 0.16 m respectively from beam fixed edge. The PPF controller is realized in Matlab/Simulink® Real Time Windows Target® environment with National Instruments® (PCI 6229) data acquisition card. The beam is also instrumented with the accelerometers to measure the structural responses as shown in Fig. 2 and Fig. 3. A constant base acceleration power spectral density of 0.0001 g^2/Hz in the frequency range of 10 Hz to 500 Hz is generated with help of shaker table and associated control system. The experiments are carriedout to measure the amplitude reduction and increase in the modal damping ratios for different gain values with and without the controllers.

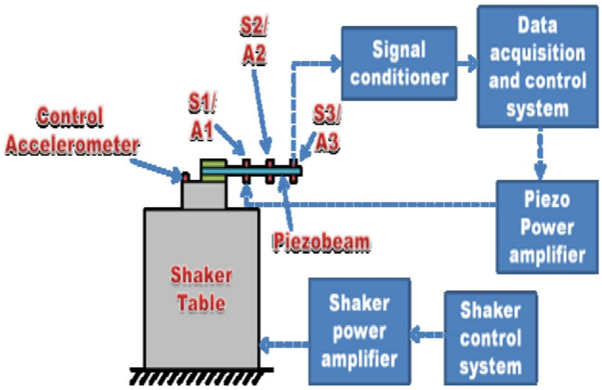

Fig. 2 Active Vibration Control scheme

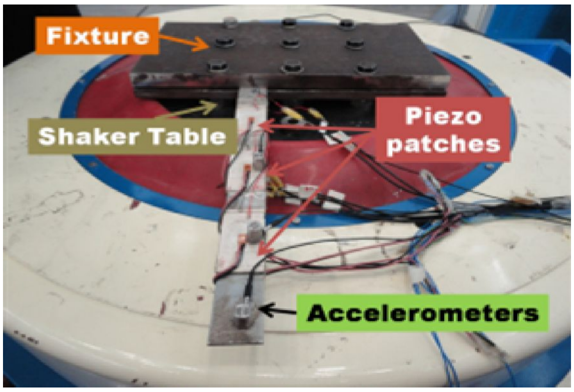

Fig. 3 Test setup photograph

III. RESULTS AND DISCUSSION

The modal parameters of the cantilever beam with piezo patches without control are obtained by exciting the beam at its base with random excitation. The piezo sensor voltage measured is converted to frequency domain to calculate the frequency and modal damping ratio using half power method. Table 1 shows the comparison of the predicted and experimental frequencies. It has been observed that the frequency estimation is with 2% error upto first two modes. Fig. 4 shows a good comparison of the predicted and measured acceleration frequency response function (FRF) at the tip of the beam.

Table 1: Comparison of natural frequencies, piezo sensor response (V/g) per unit base acceleration with S1-A1 pair and modal damping ratios for individual mode control at different gains

Mode	Frequency (Hz)		Uncontrolled		Controlled			
					Gain1		Gain2	
	Pred	Expt	V/g	ξ (%)	V/g	ξ (%)	V/g	ξ (%)
1	21.72	21.50	362.8	1.698	329.9	1.896	265.1	2.308
2	144.37	147.00	15.92	2.084	12.23	2.558	11.44	2.796
3	395.79	433.00	5.792	1.759	5.123	1.992	4.658	2.253

A. *Single Mode Control*

The first three resonant frequencies of the cantilever beam are controlled with S1-A1 sensor-actuator pair. The typical gain values (K_{PPF}) for the first mode are 100 and 200 respectively. Fig. 5 shows a typical piezo voltage FRFat sensor S1 for two different gains of the controller. It has been observed from Table 1 that a 27 % reduction is obtained in the sensor voltage of the first mode with 36 % increase in the modal damping ratio. The required actuator voltages to suppress the vibrations at the first mode are shown in Fig. 6. It is noticed that the control voltages are increasing with the gains. From Fig. 7, it is observed that larger gains are required to control the higher modes compared to the gains required to control the lower modes of the beam. Moreover, higher damping ratios are achieved with the smaller gains for the lower modes.

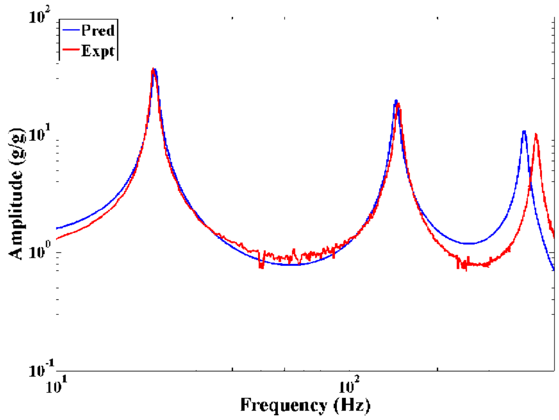

Fig. 4 Comparison of the predicted and experimental acceleration FRF at the tip of the smart cantilever beam

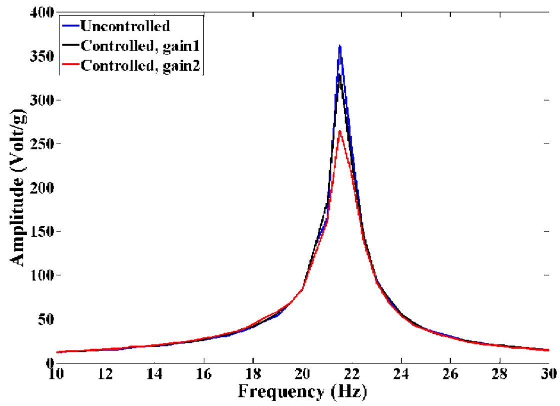

Fig. 5 Comparison of the uncontrolled and controlled piezo sensor response measured at 1st mode for S1-A1 pair at different gains

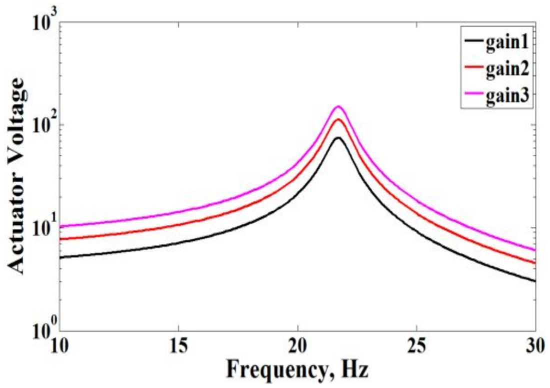

Fig. 6 Comparison of the piezo actuator voltages to control 1st mode for S1-A1 pair at different gains

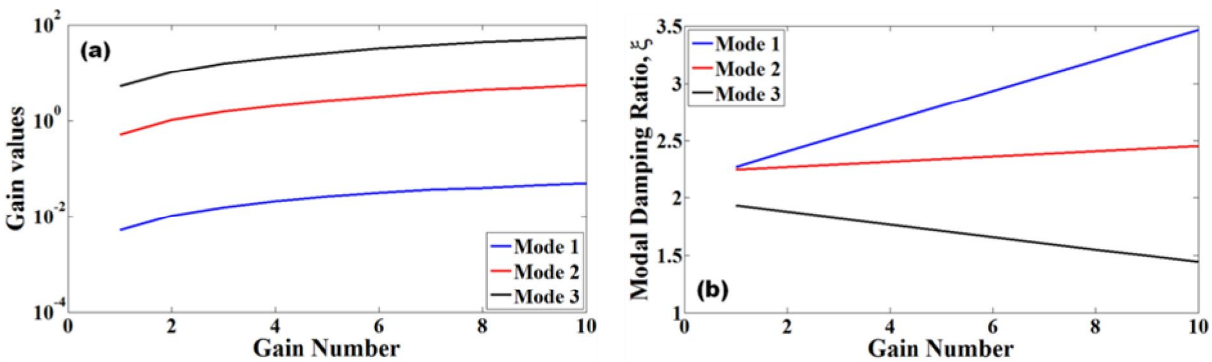

Fig. 7 (a) Gain values and (b) measured ξ for the first three modes with S1-A1 pair

B. Multi Mode Control

All the three modes of the cantilever beam are controlled simultaneously with the help of three parallel PPF controllers. The gain values of the PPF controllers are kept same as mentioned in the previous paragraph. The random input base acceleration disturbance from 10 Hz to 500 Hz frequency range is used to excite all the three modes of interest. The piezo sensor measures beam response due to random base accelerationand the sensor voltage is passed through three parallel PPF controllers. Each PPF controller enhances damping at each mode and the actuator voltages are calculated for the different sensor-actuator pairs.Fig. 8 shows the comparison of the piezo sensor frequency response functions at sensor (S1) with and without controllers. Table 2 shows piezo sensor root-mean-square voltage amplitudes at different frequency ranges for different gains and for different sensor-actuator pairs whereas the achieved modal damping ratios are shown in Table 3. For larger gains, the overall achieved amplitude reduction of the beam response is 50 % of the uncontrolled response. Fig. 9 shows that higher modal damping ratios are obtained for the first mode with S1-A1 pair compared to other two modes. Hence, S1-A1 pair is the most suited pair to control the first mode of the beam. It is noticed that there is a marginal increase in the first and third mode damping ratios whereas the modal damping ratio of the second mode is increased significantly for S2-A2 pair as shown in Fig. 10. There is a marginal increase in the modal damping ratio of the third mode with the increase in the gains for S3-A3 pair as shown in Fig. 11.

Table 2: Comparison of piezo sensor voltage (V_{rms}) amplitudes for all modes control for different gains at different sensor-actuator pairs

Freq range, Hz	No control	Controlled								
		S1-A1 pair			S2-A2 pair			S3-A3 pair		
		G1	G2	G3	G1	G2	G3	G1	G2	G3
10-30	3.90	3.59	2.36	2.15	2.18	2.17	2.11	2.16	2.15	2.16
120-170	0.48	0.44	0.41	0.34	0.33	0.32	0.29	0.34	0.34	0.34
380-500	0.28	0.26	0.26	0.24	0.22	0.22	0.22	0.23	0.23	0.23
10-500	3.96	3.66	2.45	2.22	2.24	2.23	2.17	2.22	2.22	2.22

Note: G1, G2, G3 corresponds to gains

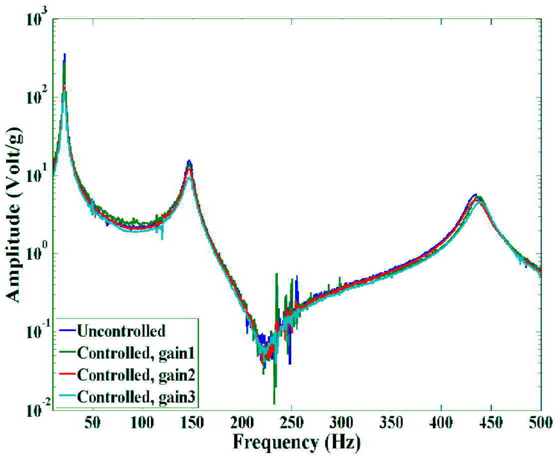

Fig. 8 Comparison of uncontrolled and controlled piezo sensor (S1) voltage FRF for S1-A1 pair for different gains

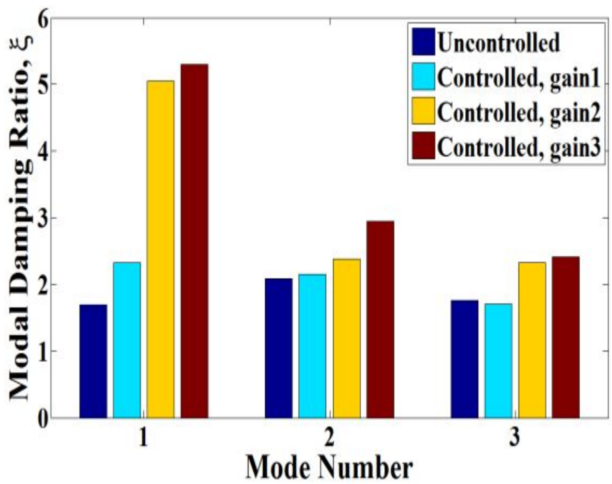

Fig. 9 Measured modal damping ratios of the three modes of the smart cantilever beam for S1-A1 pair for different gains

Table 2: Comparison of measured modal damping ratios for all modes control for different gains at different sensor-actuator pairs

Mode	No control	Controlled								
		S1-A1 pair			S2-A2 pair			S3-A3 pair		
		G1	G2	G3	G1	G2	G3	G1	G2	G3
1	1.70	2.32	5.04	5.29	4.65	4.58	5.09	4.49	4.63	4.53
2	2.08	2.15	2.38	2.95	3.09	3.51	4.11	2.83	2.83	2.66
3	1.76	1.70	2.32	2.41	2.33	2.45	2.44	2.16	2.06	2.13

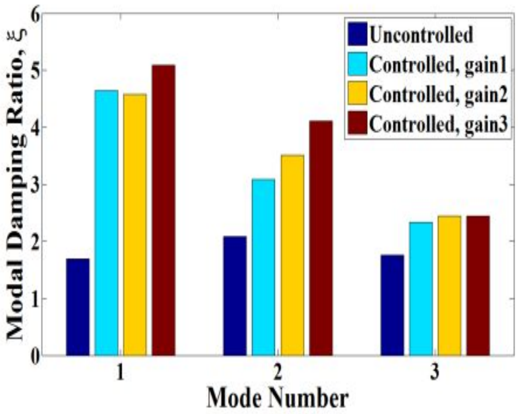

Fig. 10 Measured modal damping ratio of the three modes of the smart cantilever beam for S2-A2

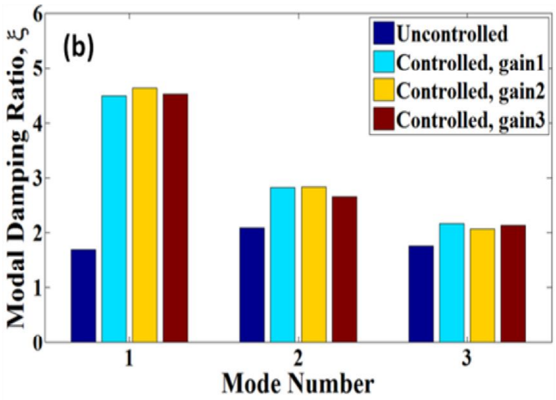

Fig.11 Measured modal damping ratios of the three modes of the smart cantilever beam for S3-A3

IV. CONCLUSIONS

First three modes of a base excited smart cantilever beam are independently controlled using single loopPPF controllers.To control all the three modes simultaneously, PPF controllers are used in parallel configuration. Selection of the sensor-actuator pair plays a significant role in controlling the modes.In general, it is seen that larger gains are required for higher modes. During a single mode control, it is observed that with S1/A1 collocated pair, higher damping ratios are achieved for lower modes at higher gains. There is a case for studying the effect of location of these patches on the damping during vibration control to find more effective locations.

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