Characteristics Investigation of Squeeze Film Damper using Triangular Element

B. Venkata Narayana
Research Scholar, Department of Mechanical Engineering
S.V.U. College of Engineering, Tirupati – 517502 India

Dr. V Diwakar Reddy
Associate Professor, Department of Mechanical Engineering
S.V.U. College of Engineering, Tirupati – 517502 India

Dr. G. Krishnaiah
Professor, Department of Mechanical Engineering
S.V.U. College of Engineering, Tirupati – 517502 India

Abstract- A **comprehensive method of theoretical analysis for finite Squeeze Film Damper using finite element method is presented. The modified Reynolds equation in the film is solved using the finite element discretization. A Galerkin variational method is applied for solving modified Reynolds equation to determine the pressures as well as load carrying capacities. The pressure distribution and load carrying capacity of the Squeeze Film Damper is obtained for different length to diameter ratios and eccentricity ratios for half film condition.**

Keywords – Finite Element Method, Galerkin Method, Modified Reynolds Equation, Squeeze Film Damper

I. INTRODUCTION

Squeeze Film Damper (SFD) is considered to be one of the most important technological developments to occur in rotating machinery during the last four decades. These dampers can improve rotor stability, attenuate vibration levels and reduce transmissibility. The squeeze film dampers allow a maximum amount of flexibility and predictability in the level of damping. It is used as an additional damper for the control of vibrations. A significant number of theoretical and experimental works have been published since the 1960s as the interest in the application of the kind of damper grew strong.

A squeeze film damper is basically combines a bearing (generally a ball or roller, although different types of bearing, such as tilting-pad fluid bearings can be adopted) and a damping oil film so as to determine a support provided with a suitable damping action. An illustration of this device is given in Figure 1.

The shaft **P** of the rotor is supported by a roller bearing **J**, while an anti-rotation pin **T** prevents Open the outer race of the bearing from rotating. The journal, represented by the whole of **P** and **J**, is free to move in the fixed housing or bearing **B** that is integral to the frame **S**, with a suitable radial clearance. The annulus is fed with lubricant, at a given pressure that realizes by the journal, that is due to the residual unbalance of the rotor, causes a dynamic pressure distribution to originate in the squeezed oil film and consequent damping of the same motions can be obtained.

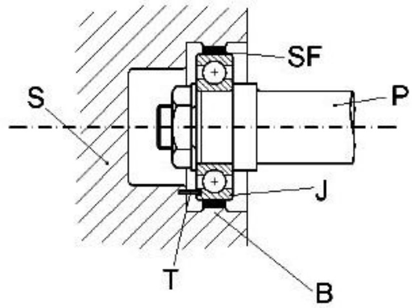

Figure 1. Typical representation of SFD

The parameters like squeezing film pressures, load carrying capacity and dynamic characteristics depend upon the squeeze-out of the lubrication.

II. MATHEMATICAL MODELLING

A. Geometric Parameters and Operation –

The operational characteristics of a SFD can be categorized into static and dynamic aspects. Static characteristics include a damper's load carrying capacity, temperature, power loss, and the amount of oil it requires during operation. The damper's load carrying capacity is often measured using eccentricity ratio that relates directly to its minimum film thickness. An SFD's dynamic performance is characterized by its stiffness and damping properties. These properties interact with the rotor system determines a machine's overall vibrational behavior.

The main objective of this paper is to provide a general understanding of the basics that governs the damper's static operational characteristics.

Before discussing the static operational aspects of SFD, some basic geometric parameters need to be defined. Figure 2 shows SFD geometry for the axial length of L. The bearing center and journal center is represented by the 'O_b' and 'O_j' respectively. The center distance between O_b and O_j is the eccentricity 'e'. The angular coordinate for bearing film 'θ' is measured from the line of center of journal and bearing as shown in figure. The radial clearance, $c=R_b - R_j$, allows the journal to operate at some eccentric position defined by distance e and altitude angle 'α'. The altitude angle is always measured with respect to the direction of the applied load 'W' and the line of centers. For a fixed geometry, the line of centers establishes the minimum film thickness locations.

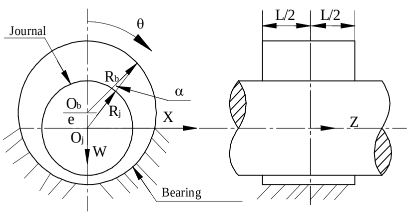

Figure 2. Geometry of the Squeeze Film Damper

The film thickness 'h' in squeeze film damper can be defined as the difference in the radial co-ordinates at the outer and inner surfaces for a fixed angle. It is an important parameter in the damper design. All of these geometric design parameters can significantly affect SFDs static and dynamic characteristics. For example, tighter clearances lead to greater load carrying capacity and higher stiffness.

A typical pressure distribution for half film condition is shown in Figure 3. The pressure distribution is symmetric along the load line of the bearing. It also shows that the pressures are decreasing with increase of film thickness. No pressure is developed in the upper half of the bearing because of the diverging clearance and the relatively low oil supply pressure. This condition, which exists in most fixed geometry bearings, causes the film to cavitate and restrict the pressure in the film to the vapor pressure of the lubricant.

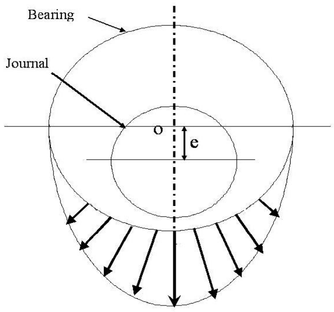

Figure 3. Typical Pressure Distribution for Half Film SFD

B. Oil-film Pressure Model –

In most of the squeeze films, the squeeze film pressure is developed due to viscosity of lubricants. But the effect of squeeze film pressure will become more and more significant as the squeeze velocity or film thickness increases. The load carrying capacity is obtained by integrating the pressures. In the present work, the optimum bearing pressures and load carrying capacities are obtained through simultaneous numerical solution of the modified Reynolds equation (MRE). The MRE is solved by the finite element method using Galerkin approximation. The analysis is applied on finite length bearing for different length to diameter 'L/D' and eccentricity ratios.

The SFD pressure distribution is governed by the classical Reynolds Equation (RE) under the following assumptions.

- The flow is laminar
- The lubricant is an incompressible Newtonian fluid with a constant uniform density 'ρ' and viscosity 'μ' throughout the film
- Gravity and inertia forces are negligible
- The pressure across the film is constant
- There is no slip between the lubricant and bearing surface
- The bearing surfaces have no axial velocities
- The film is held in between the annular space isothermally

By considering the above assumptions in thin film lubrication, the complete Reynolds Equation is

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)=6\left\{\frac{\partial}{\partial x}(Uh)+\frac{\partial}{\partial z}(Vh)+2(w_h-w_o)\right\} \quad (1)$$

Where U and V are sliding velocities
and w_h-w_o= Squeezing term, can be written as

$$\frac{\partial h}{\partial t} \quad (2)$$

h = Film Thickness = c(1+ε cosθ)

The complete formulation of Reynolds equation of thin film lubrication contains squeeze film term and sliding velocities on its right hand side. Due to the nonrotating journal in SFD, the sliding velocities become zero. By disregarding the sliding velocities in complete Reynolds Equation, a Modified Reynolds equation for finite SFD is

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)=12\frac{\partial h}{\partial t} \quad (3)$$

It is convenient to analyze the problem by converting the dimensional equation in to non-dimensional equation. The nondimensional parameters are as given below.

$\theta = x/R$ and $\bar{z} = 2z/L$

$$\bar{p} = \left(\frac{c}{R}\right)^2 \frac{p}{\mu\left(\frac{d\in}{dt}\right)} \qquad (4)$$

With the substitution of non-dimensional parameters in (2), which reduces to

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right)+\nu^2\frac{\partial}{\partial\bar{z}}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{z}}\right)=12\cos\theta \qquad (5)$$

Where $\nu = D/L$

The above non-dimensional modified Reynolds equation is valid for incompressible isothermal and isoviscous laminar flow.

C. Finite Element Method Formulation –

The Finite Element Method is a numerical technique for finding an approximate solution for partial differential equations (PDE) as well as of integral equations. The solution approach is based on rendering the PDE into an approximating system of ordinary differential equations, which are then solved using standard numerical techniques. The modified Reynolds equation is solved numerically by using a finite element method.

The domain of the film is divided into linear triangular elements. Each element is having three nodes. An approximate solution for governing differential equation over the element is obtained by using the Galerkin approximation method. Galerkin methods are equally ubiquitous in the solution of partial differential equations and in fact form the basis for the finite element method. Since the aim of Galerkin's method is the production of linear system of equations, we build its matrix form which can be used to compute the solution by a computer program.

D. Boundary Conditions:

The SFD is assumed as a continuous film from θ= 0 to θ = 2π, measured from the position of maximum film thickness. Negative pressures are developed in the unloaded half portion of the bearing, which is neglected as it involves effect of cavitation etc which is beyond the purview of this paper. Therefore the limits of loading for the film are then taken as θ=π/2 to θ=3π/2. In the axial direction, the ends of the bearings are exposed to the ambient pressure.

$$\bar{p} = 0 \text{ at } \bar{z} = +\frac{1}{2}$$
$$\bar{p} = 0 \text{ at } \bar{z} = -\frac{1}{2}$$
$$\bar{p} = 0 \text{ at } \theta = \frac{\pi}{2} \qquad (6)$$
$$\bar{p} = 0 \text{ at } \theta = \frac{3\pi}{2}$$

Let

$$L(\bar{p})=\frac{\partial}{\partial\theta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right)+\nu^2\frac{\partial}{\partial\bar{z}}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{z}}\right) \qquad (7)$$

and f=12 cosθ

Then, the modified Reynolds equation becomes as

$$L(\overline{p}) = f$$

$$L(\overline{p}) - f = 0$$

Here, $\overline{p}$ is symmetric, then

$$J(\overline{p}) = L(\overline{p}) - 2f$$

For such a differential equation, the relevant functional can be readily and directly written as follows

$$J(\overline{p}) = \iint \left(L(\overline{p})\overline{p} - 2f\overline{p} \right) d\theta d\overline{z} \qquad (8)$$

After applying the Green's theorem and boundary conditions, the functional equation is reduced to

$$J(\overline{p}) = -\iint \overline{h}^3 \left[\left(\frac{\partial \overline{p}}{\partial \theta} \right)^2 + v^2 \left(\frac{\partial \overline{p}}{\partial \overline{z}} \right)^2 \right] d\theta d\overline{z} - 24 \iint \overline{p} \cos\theta \, d\theta d\overline{z} \qquad (9)$$

According to the variational principle the required solution is obtained when the value of $J(\overline{p})$ is stationary with respect to the nodal point pressure $\overline{p}_i$,

$$\frac{\partial J(\overline{p})}{\partial \overline{p}_i} = \sum_{e=1}^{E} \frac{\partial J_e(\overline{p})}{\partial \overline{p}_i} = 0 \qquad (10)$$

After solving for triangular element, the above expression reduces to

$$\overline{h}^3 C \, \overline{p}_e = -16 \, \Delta \cos\theta \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad (11)$$

Where $\overline{h}^3 C$, is the stiffness matrix of the element e, Δ is the area of the triangular element and $\overline{h}$ is the average thickness of the element e.

From the element stiffness matrix, we get the global stiffness matrix and the overall linear system equations:

$$\overset{n \times n}{K} \; \overset{n \times 1}{P} = -16 \overset{n \times 1}{F}_{\cos\theta} \qquad (12)$$

or

$$\overset{n \times 1}{P} = -16 K^{-1} F_{\cos\theta} \qquad (13)$$

Where

$$K = \sum_{e=1}^{E} (\overline{h}^3 C)^e$$

and

$$F_{\cos\theta} = \sum_{e=1}^{E} \begin{bmatrix} \Delta \cos\theta \\ \Delta \cos\theta \\ \Delta \cos\theta \end{bmatrix}^e \qquad (14)$$

The above equations merely mean that the overall stiffness matrix and the overall column force matrix formed respectively by the element stiffness matrix and the element column force matrix.

$$p = \left[\overline{p_1} \quad \overline{p_2} \quad \overline{p_3} \quad .... \quad \overline{p_n} \right]^T \qquad (15)$$

E. Calculation of Load Carrying Capacity –

Once the nodal pressures on the damper are known, the squeeze film characteristic such as load carrying capacity on the bearing can be obtained by integrating the pressures over the surface area of the film. The load carrying capacity can be calculated as

$$W = -2R \int_0^{L/2} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} p \cos\theta \, d\theta \, dz \qquad (16)$$

where $\pi/2$ and $3\pi/2$ are the positions of the start and end of the noncavitated pressure region respectively.

Introduce the non-dimensional quantity,

$$\overline{W} = \frac{WC^2}{\mu L R^3 \frac{d\in}{dt}} \qquad (17)$$

The expression (16) reduces to

$$\overline{W} = -\frac{1}{3}(F_{Cos\theta})^T P \qquad (18)$$

Substituting equation (13) into equation (18)

$$\overline{W} = \frac{16}{3}(F_{Cos\theta})^T K^{-1} F_{\cos\theta} \qquad (19)$$

III. RESULTS AND DISCUSSIONS

Results are presented in Figures 4 – 8. The half film condition was considered for determining the squeeze film pressures and squeeze film static characteristics like load carrying capacity.

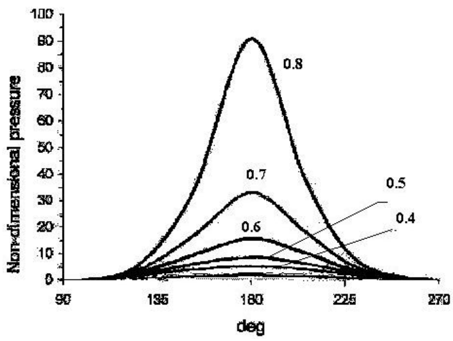

Figure 4. Non-dimensional Pressure Distribution for L/D=0.5 at different ε

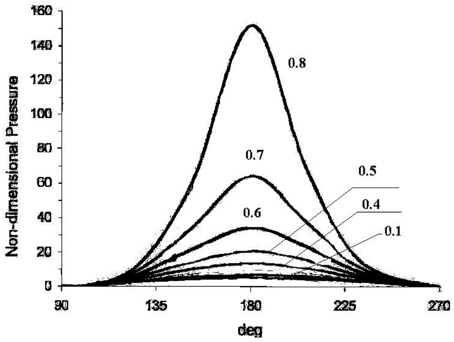

Figure 5. Non-dimensional Pressure Distribution for L/D=1.0 at different ε

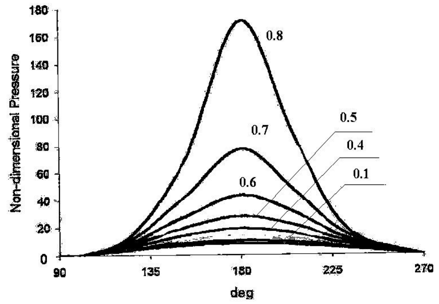

Figure 6. Non-dimensional Pressure Distribution for L/D=1.5 at different ε

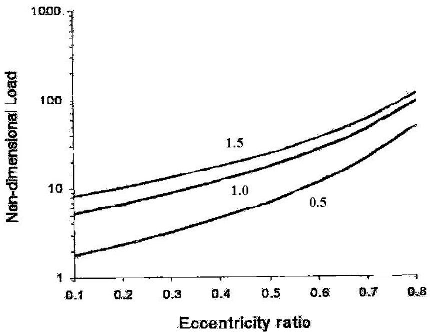

Figure 7. Non-dimensional Load Carrying Capacity versus ε for different L/D ratios

Figure 4 to Figure 6 shows the non-dimensional film pressure $\overline{P}$ versus circumferential coordinate θ (deg) in the mid plane $\overline{Z}$=0 for L/D =0.5,1.0, and 1.5 respectively. The results are presented at different eccentricity ratios (ε). It can be observed from the figure that the pressure distribution is symmetric at θ=180° for all eccentricity and L/D ratios.

The pressures are increased with increasing value of eccentricity and L/D ratio. The figures shows that the values of non-dimensional pressures at ∈=0.1, 0.2, and 0.3 are very low for all the L/D ratios. Therefore, for better performance the damper should be designed above 0.3 eccentricities.

Figure 7 shows the non-dimensional load carrying capacity versus eccentricity ratios for different L/D ratios. The load carrying capacity increases with increase in eccentricity.

IV. CONCLUSION

A theoretical analysis has been presented for the finite length squeeze film damper. The analysis is based on Modified Reynolds Equation by finite element method.

Non-dimensional pressure distribution and load carrying capacity for a squeeze film damper at different L/D ratios from 0.5, 1.0, and 1.5 for all eccentricity ratios from 0.1 to 0.8 were determined.

REFERENCES

[1] A.Z Szeri, A.A Raimondi and A Girson Duarte, "Linear force Coefficients for Squeeze film dampers," *ASME journal of Lubrication Technology*, Vol 105, pp 326-334, July 1983.

[2] G.L Arauz and L. A. San Andres, "Experimental pressures and Film forces in Squeeze film damper," *ASME journal of Tribology*, Vol 115, No 1, pp: 134-140, January 1993.

[3] Lin.J. R , "Squeeze film characteristics of finite Journal bearings couple stress fluid model," *Tribology Internaltional*, Vol. 31, No 4,, pp 201-207, 1998.

[4] Dargaiah, K kamalam p and Prabhu B.S,"Finite Element method for computing Dynamic coefficients of Multi-lobe bearings," *STLE/ASME Tribology conference*, October 14-16,1991

[5] O.C Zienkiewicz, "*The finite Element method*," Tata-McGraw Hill Co,1978

[6] Pinkus, O. and Sternlicht,B, "*Theory of Hydrodynamic lubrication*," Mc Graw Hill Co Ltd, 1961

[7] S.Mohan and E.J Hahn, "Design of Squeeze film Damper supports for rigid motors," *ASME journal of Engineering for Industry*, Vol 96, Series B, No 3, pp 977-982, August 1974.

[8] Chu F., and Holmes R., "The Effect of Squeeze Film Damper Parameters on the Unbalance Response and Stability of a Flexible Rotor," *ASME J. Engineering for Gas Turbines and Power, Vol.120, 140-148, 1998.*

[9] L. Della Pietra and G. Adiletta, "The Squeeze Film Damper over Four Decades of Investigations. Part I: Characteristics and Operating Features," *The Shock and Vibration Digest*, Vol.34, No.1, pp. 3-26, January 2002.