Effects of overtaking Disturbances on the Motion of Cylindrical Hydromagnetic Shock waves in a Rotating Gas

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Abstract - Effects of overtaking disturbances behind the flow, on the motion of cylindrical hydromagnetic shock waves in an ideal gas in presence of an axial magnetic field (H_{z0} constant), have been included to improve the accuracy of CCW[1-3] approach. Assuming an initial density distribution $\rho_G = \rho^1 r^w$ where, ρ^1 is the density at the axis of symmetry and w is a constant, the analytical expressions for flow variables have been deduced. Their numerical estimates, only at permissible shock front locations, have been computed and compared with the results describing so called Free propagation.

Key words : Hydromagnetic shock waves, Effects of Overtaking Desturbances; $\rho_G = \rho^1 r^w$, (Hz_0) constant axial Magnetic field.

I. INTRODUCTION

Having accepted the correctness of CCW[1-3] method claimed by Whitham[4], Kumar and his coworkers[5-7] have extended this approach for simultaneous study of propagation of weak and strong shocks through uniform and non-uniform madia. However, Roscizewski[8] and Oshima[9] et.al. have formulated the error involved in using CCW description by integrating along two neighboring overtaking characteristics and forms the difference of the two integrals. Encouraged by Yousaf's[10-12] remark about including the eod behind the flow on the motion of shock waves, very recently, Kumar et.al.[13-16] have shown that the Free Propagation description is qualitatively adequate but needs considerably high corrections as a result of inclusion of the Effects of Overtaking Disturbances.

In this paper, the e.o.d. behind the flow have been included to improve the accuracy of CCW description for the motion of diverging cylindrical shock waves in an ideal gas subjected to solid body rotation in presence of an axial magnetic field ($H_{z0} = \text{constant}$). For an initial density distribution $\rho_G = \rho^1 r^W$, the modified forms of analytical expressions for flow variables have been derived. Their numerical estimates only at psfl, inview of arbitrary choice of initial density distribution. So that $p_0 \rho_0^{-\gamma}$ = constant, have been computed Finally, the corrections required for flow variables at defferent locations of shock fronts with β^2, ξ and M_0^2 have been given through Figures.

2. Basic Equations : Under the assumption that the gas is infinitely electrically conducting, inviscid and non-heat conducting, the basic equations for the flow behind the shock front are[17]

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{p}{r} + \frac{\mu \partial H^2}{\rho\, \partial r} - \frac{V^2}{r} = 0$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)(v, r) = 0$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = 0$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + a^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{r}\right) = 0$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\left(\frac{\partial u}{\partial r} + H\frac{u}{r}\right) = 0 \text{ ---------------- } (1)$$

where r is a radial co-ordinats, u and v are the radial and azimuthal components of the particle velocity p, ρ, H and μ denote respectively the pressure, the density the axial magnetic field and the magnetic permeability of the gas and $a_0^2 = \gamma p_0/\rho_0$ and $v = \Omega_0^{\,2}(\gamma)$ being the adiabatic index of the gas and $\Omega_0^{\,2}$ is the angular velicity.

3. Boundary Conditions : The magnetic hydrodynamic shock conditions can be written in terms of a single parameter $\xi = \rho_1/\rho_0$ as

$$\rho_1 = \rho_0\xi, \qquad H_1 = H_0\xi, u = \frac{\xi - 1}{\xi}U$$

$$U^2 = \frac{2\xi}{(\gamma + 1) - (\gamma - 1)\xi}\left[a_0^{\,2} + \frac{b_0^{\,2}}{2}\{(2 - \gamma)\xi + \gamma\}\right.$$

$$p_1 = p_0 + \frac{2\rho_0(\xi - 1)}{(\gamma + 1) - (\gamma - 1)\xi}\left[a_0^{\,2} + \frac{(\gamma - 1)}{4}b_0^{\,2}(\xi - 1)\right] \text{ -------- } (2)$$

where 0 and 1, respectively stand for the states immediately ahead and behind the shock front, U is the shock velocity, a_0 is the sound speed $(\gamma p_0/\rho_0)^{1/2}$ and b_0 is the alfven speed $(\mu H_0^{\,2}/\rho_0)^{1/2}$

Weak Shock : For very weak shock, we take the parameter ξ as

$$\rho_1/\rho_0 = \xi = 1 + \varepsilon$$

where ε is another parameter which is negligible in comparison with unity, i.e. ε <<1,

Now we consider, the two cases of weak and strong magnetic fields.

Case 1 :- For weak magnetic field $b_0^{\,2} \ll a_0^{\,2}, i.e., \mu H_0^{\,2}/\gamma p_0 \ll 1$ under this condition the boundary condition (2) for very weak shock reduce to

$$\rho_1 = \rho(1 + \varepsilon), H_1 = H_0(1 + \varepsilon), v_1 = \varepsilon a_0$$

$$U = \left(1 + \frac{\gamma + 1}{4}\varepsilon\right)a_0, \qquad p_1 = p_0(1 + \gamma\varepsilon) \text{ ----------- } (4)$$

Case 2 :- For storng magnetic fields $b_0^{\,2} \gg a_0^{\,2}, i.e., \mu H_0^{\,2}/\gamma p_0 \gg 1$ by using this condition and equation (3), the boundary condition (2) become

$$\rho_1 = \rho_0(1 + \varepsilon), H_1 = H_0(1 + \varepsilon), v_1 = \varepsilon b_0$$

$$U = \left(1 + \frac{3}{4}\varepsilon\right) b_0, \qquad p_1 = p_0(1 + \gamma\varepsilon) \text{ - - - - - - - - - - - }(5)$$

Strong Shock :- In the limiting case of a strong stock p_1/p_0 is large. In the magnetic case this achieved in two ways.

Case 1:- The puraly non-magnetic way when $\xi = \left(\frac{\gamma+1}{\gamma-1}\right)$ is small

Case 2:- When $b_0^2 >> a_0^2$ or when $\mu H_0^{\ 2} >> \gamma p_0$ i.e. magnetic pressure is very much greater than the gas pressure in the equilibrium state. In terms of ξ, the boundary condition reduce to

$$\rho_1 = \rho_0\varepsilon, \qquad H_1 = H_0\varepsilon, \qquad u = \frac{\xi - 1}{\xi} U$$

$$\frac{p_1}{p_0} = K(\varepsilon)\left(\frac{U}{a_0}\right)^2 + L$$

where

$$K(\varepsilon) = \frac{\gamma(\gamma - 1)(\xi - 1)^2}{2\xi\{(2 - \gamma)\xi + \gamma}$$

$$L = \frac{(\gamma + 1) - (\gamma - 1)}{(\gamma + 1) - (\gamma - 1)\xi} \text{ - - - - - - - - - - }(6)$$

Characteristic Equations :- The characteristic from of the system of equation (1) is easily obtained by forming a linear combination of first and third equation of system of equation (1) in only one direction in (r,t) plane, can be written as

$$dp + \mu HdH - \rho a du - \rho_c \frac{v^2}{u + a} + \rho^2 \frac{u}{u + a}\frac{dr}{r} = 0 \text{ - - - - - - - - - - }(7)$$

In order to estimate the strength of overtaking disturbances independent C, characteristic is considered. The differential equation valid across a C_+ disturbance is written by replacing C by C. in equation (7), we get.

$$dp + \mu HdH - \rho a du - \rho_c \frac{v^2}{u - a} + \rho^2 \frac{u}{u - a}\frac{dr}{r} = 0 \text{ - - - - - - - - - - }(8)$$

Analytical Relations state for Flow Variables :- Assuming the initial density distribution of the form

$$\rho_0 = \rho^1 r^W \text{ - - - - - - - - - - - - - - - }(9)$$

The equilibrium state of the gas is assumed to be specified by the condition

$$\frac{\partial}{\partial_t} = 0 = u \text{ -------------- } (10)$$

and Hz_0 = constant

using (10) the first equation of the system of equation (1) the hydrostatic equilibrium condition previling in front of the stock can be written as

$$\frac{1}{\rho_0}\frac{dp_0}{dr} - \Omega o^2 r = 0 \text{ -------------------- } (11)$$

The integration of equation (11) we get

$$p_0 = \frac{K + \Omega_0{}^2 \rho^2 r^{W-2}}{W+2} \text{ ----------- } (12)$$

and

$$a_0 = \sqrt{\frac{\gamma^{1/2} K}{\rho^2}}\left[r^{-W/2} + \frac{1}{2}\frac{\rho^2 \Omega_0{}^2}{K(W+2)} r^{W+2/2}\right] \text{ ------------ } (13)$$

where K is Constant of integration.

Weak Shock with Weak Magnetic Field (WSWMF) :- Substituting the shock condition (4) into equation (7) and using the equation (11), we get

$$d\varepsilon + \frac{1}{2}\left(1 - \frac{\mu Ho^2}{2\gamma p_0}\right)\left(\frac{dp_0}{p_0} + \frac{da_0}{a_0} + \frac{dr}{r}\right)\varepsilon = 0 \text{ ------------ } (14)$$

The above equation will remain valid, only when $\mu H_0{}^2/\gamma p_0 \ll 1$ and using this equation into (4) we get

$$du_- = \varepsilon da_0 + a_0\left\{-\frac{1}{2}\left(1 - \frac{\mu Ho^2}{2\gamma p_0}\right)\frac{dp_0}{p_0} - \frac{1}{2}\left(1 - \frac{\mu Ho^2}{2\gamma p_0}\right)\frac{da_0}{a_0} - \frac{1}{2}\left(1 - \frac{\mu Ho^2}{2\gamma p_0}\right)\frac{dr}{r}\right\}\varepsilon \text{ -- } (15)$$

on substituting the shock condition (4) into equation (8) and using the equation (11) we get

$$d\varepsilon + \frac{\gamma p_0}{\mu H_0{}^2}\left(1 - \frac{2}{r}\right)\frac{dp_0}{p_0} - \frac{da_0}{a_0} - \frac{dr}{r}\Big\}\varepsilon \text{ -------------- } (16)$$

and using this equation into (4), we get

$$du_- = \varepsilon da_0 + a_0\left[-\frac{\gamma p_0}{\mu H_0{}^2}\left\{\left(1 - \frac{2}{r}\right)\frac{dp_0}{p_0} - \frac{da_0}{a_0} - \frac{dr}{r}\right\}\right]\varepsilon \text{ --------- } (17)$$

Now in presence of bath C- and C+ disturbance, the fluid velocity behind the shock will be related as

$$du_- + du_+ = z\,da_0 + a_0\,dz \quad - - - - - - - - - - - - - - (18)$$

from equation (15), (17) and (18) substituting the values of p_0, a_0, $d\,p_0/p_0$ and $d\,a_0/a_0$ on the integration we get

$$z = K^{11} r^{B4} \exp(\Delta^*) \quad - - - - - - - - - - - - - - (19)$$

K^{11} is integration Constant.

Weak Shock with Strong Magnetic Field (WSSMF) :- Substituting the shock condition (5) into equation (7) we get and using this equation (5), we get

$$dz + \frac{1}{2}\left(1 - \frac{\gamma p_0}{2\mu H_0^2}\right)\left(\frac{\gamma dp_0}{\mu H_0^2} + \frac{db_0}{b_0} + \frac{dr}{r}\right) = 0 \quad - - - - - - - - (20)$$

and using this equation (5) we get

$$du_- = z\,db_0 + b_0\left\{-\frac{1}{2}\frac{\gamma dp_0}{\mu H_0^2} - \frac{1}{2}\frac{db_0}{b_0} - \frac{dr}{r} + \frac{(w+2)}{8\mu H_0^2}\gamma p_0\frac{dr}{r}\right\} z \quad - - - - - (21)$$

on substituting the stock condition (5) into (8) we get

$$dz + \frac{\mu H_0^2}{\gamma p_0}\left\{-\frac{(2-\gamma)}{\mu H_0^2} db_0 - \frac{db_0}{b_0} - \frac{dr}{r}\right\} z \quad - - - - - - - - - - - - (22)$$

$$du_+ = z\,db_0 + b_0\left\{\frac{(2-\gamma)}{\gamma p_0} db_0 + \frac{\mu H_0^2}{\gamma p_0}\frac{db_0}{b_0} + \frac{\mu H_0^2}{\gamma p_0}\frac{dr}{r}\right\} z \quad - - - - - - (23)$$

Now in persence of both C- and C+ disturbances, the fluid Velocity behind the shock will be related as

$$du_- + du_+ = z\,db_0 + b_0\,dz \quad - - - - - - - - - - - - - - (24)$$

from equation (21),(23) and (24) and substituting the values of p_0, a_0, $d\,p_0/p_0$ and $d\,b_0/b_0$ on integration we get

$$z = K^{11^*} r^{B4} \exp(\Delta^*) \quad - - - - - - - - - - - - - - (25)$$

strong Shock (SS) :- Substituting the shock condition (6) into equation (7), we get

$$dU^2 + \frac{B_1}{r} U^2 dr - \alpha\Omega_0^2 r\,dr = 0 \quad - - - - - - - - - - - - (26)$$

where

$$B_1 = B/A, \quad C_1 = C/A$$

$$A = \left[\frac{\chi(\xi)}{\gamma} + \frac{1}{2}(\xi - 1)\sqrt{\frac{\chi(\xi)}{\xi}}\right]$$

$$B = \left[\frac{\chi(\xi)w}{\gamma} + \frac{\chi(\xi)(\xi-1)}{(\xi-1)+\{\xi\chi(\xi)\}^{1/2}}\right]$$

$$C = \frac{\xi\sqrt{\xi}\chi(\xi)}{(\xi - 1) + \sqrt{\xi}\chi(\xi)} + L$$

equation (26) can be Simplified as

$$du = -\frac{B_1}{2U}\frac{U^2}{r}dr + \frac{C_1\Omega_0{}^2 r}{2U}dr \text{ - - - - - - - - - - - - - (27)}$$

Now for C disturbance generated by the shock, the fluid velocity increment using (27), into equation (6) may be expressed as

$$dU - \frac{\xi - 1}{\xi}\frac{1}{2U}\left\{-B_1U^2\frac{dr}{r} + C_1{}^*\Omega_0{}^2rdr\right\} \text{ - - - - - - - - - - - - (28)}$$

on substituting the shock condition (6) into equation (8) we get

$$dU^2 = -B_1{}^*\frac{U^2}{r}dr + C_1{}^*\Omega_0{}^2rdr \text{ - - - - - - - - - - - - - - (29)}$$

where

$B_1{}^* = B^*/A^*,$ $\qquad C_1{}^* = C^*/A^*$

$$A^* - \left\{\frac{\chi(\xi)}{\gamma} - \frac{(\xi-1)}{2}\sqrt{\frac{\chi(\xi)}{\xi}}\right\}$$

$$B^* = \left\{\frac{\chi(\xi-1)}{[(\xi-1)+(\xi\chi(\xi))^{1/2}]} - \frac{\chi(\xi)w}{\gamma}\right\}$$

$$C^* = \left\{\frac{\xi\sqrt{\xi}\,\chi(\xi)}{(\xi-1)+\sqrt{\xi}\,\chi(\xi)} + L\right\}$$

equation (29) Can be simplified as

$$dU = \frac{(\xi - 1)}{\xi}\frac{1}{2U}\left\{-B_1U^2\frac{dr}{r} - C_1{}^*\Omega_0{}^2rdr\right\} \text{ - - - - - - - - - - - - (30)}$$

Now, for C disturbance generated by the shock, the fluid Velocity increment using (30), into equation (6), may be expressed as

$$dU_+ = \frac{\xi - 1}{\xi}\left[\frac{1}{2\Pi}\left\{-B_1 U^2 \frac{dr}{r} - C_1^* \Omega_0^2 r dr\right\}\right] \text{------------} (31)$$

In presence of both C- and C+ disturbances, the fluid Velocity increment behind the shock will be related as

$$dU_- + dU_+ = \frac{\xi - 1}{\xi} dU \quad \text{------------------} (32)$$

using equation (23), (31) and (32) we get,

$$U^2 = \left[K^{111^*} r^{-(B+B_1^*)} + \frac{(C - C_1^*)\Omega 0^2 r^2}{B + B_1^* + 2}\right] \quad \text{-----------} (33)$$

where K_{111}^* is a constant of integration.
The flow variable expressions can be written by substituting (19), (25) and (33) respetively in (4), (5) and (6).

II. RESULTS AND DISCUSSIONS

WSWMF :- Taping ε = 0.125 at r = 0.140 for γ =1.5, B^2 = 0.40, 0.45, 0.50, W = 0.0002, 0.002, Ω_0^2 = 0.50, 10.20, WSSMF : Taking ε = 0.125 at r = 0.140, for γ = 1.5, 1.3, W = 0.0002, 0.002, Ω_0^2 = 0.5, 1.0, 2.0, SS : Taking $U/a_0 = 20$ at r = 0.40, for γ = 1.5, W = 0.0002, Ω_0^2 = 10.0, ξ = 1.5 (ii) $U/a_0 = 20$ at r = 0.04, for γ = 1.5, W = 0.0002, Ω_0^2 = 15.0, ξ = 1.5 (iii) $U/a_0 = 20$ at r = 0.04, for γ = 1.5, W = 0.002, Ω_0^2 = 20.0, ξ = 1.5 (iv) $U/a_0 = 20$ at r = 0.04, for γ = 1.5, W = 0.002, Ω_0^2 = 10.0, ξ = 1.5 (v) $U/a_0 = 20$ at r = 0.04, for γ = 1.5, W = 0.002, Ω_0^2 = 10.0, ξ = 3.0 (vi) $U/a_0 = 20$ at r = 0.04, for γ = 1.5, W = 0.002, Ω_0^2 = 10.0, ξ = 0.5, the numerical estimates of flow variables have been computed only at permissible propagation distance (r) and shown through Figures. 1-10 together with results describing Free Propagation. It is observed on comparison that those numerical values of flow variables representing so called Free Propagation with the values of flow variables that include the e.o.d. results over all correction percentages from - 819.40 to 0-2.73, -32.70 to 05.60, -1707.33 to 0-4.54 respectively for WSWMF, WSSNF and SS.

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