Column Optimization using a Direct Search Method

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Abstract: Columns are an important structural component in any given framed structure, and contribute substantially towards its total cost. The idea of column optimization therefore plays an important role in bringing down the overall cost of structure. The study presents one of the direct search methods - Complex Iterative Method - for minimum cost design of reinforced concrete columns. Cost of an RCC column constitutes the cost of concrete as well as steel, which are function of its cross-sectional dimensions and corresponding steel area. In the current study only two independent design variables, namely ratio of depth of neutral axis to depth of column 'k' and percentage area of longitudinal reinforcement 'p' have been considered. Remaining design variables were made dependent on these two variables. Complex Iterative Method, being a gradient free method was adopted to test its efficiency. Various columns were considered for design with encouraging results. The paper illustrates application of the method with an example problem.

Keywords: Optimization, Complex Iterative Method, RCC Column

I. INTRODUCTION

The aim of devising better design solutions while satisfying safety and performance constraints at least cost, is not a new one. From time immemorial an engineer has investigated several alternatives and chosen the best among these. A design process has always been a process of choice in which the designer's intuition and experience played an important role. But the cost and social importance of structures built nowadays has increased so much that it has become impossible to rely exclusively on the intuition and experience of engineers. The need to create objective methods of effective optimum design has been the primary cause of the rapid development of structural optimization techniques (Majid, 1974). Bringing optimization techniques openly and consistently into design practice makes it possible for a designer to produce optimal structures.

Columns are an important structural component in any given framed structure, and contribute substantially towards its total cost. The concept of column optimization thus plays a significant role in reducing the overall cost of structure. This paper presents a simple and easy to apply technique for obtaining optimum column design parameters.

II. FORMULATION OF OPTIMIZATION PROBLEM

The cost of a reinforced concrete column consists of the cost of concrete and steel. This can be obtained using the following relationship:

$$C = C_{st} V_{st} + C_c V_c \tag{1}$$

where
C = total cost of column
C_{st} = cost of steel per unit volume of steel
V_{st} = volume of steel in the column
C_c = cost of concrete (including formwork) per unit volume of concrete
V_c = volume of concrete in the column

Dividing Eq. 1 by C_c, we get

$$\frac{C}{C_c}=\frac{C_{st}V_{st}}{C_c}+V_c \qquad (2)$$

Putting $V_c = V_G - V_{st}$, where V_G = gross volume of column Eq. 2 becomes

$$\frac{C}{C_c}=\frac{C_{st}V_{st}}{C_c}+(V_G - V_{st})$$

$$» \qquad \frac{C}{C_c}=\left(\frac{C_{st}}{C_c}-1\right)V_{st}+V_G \qquad (3)$$

Taking objective function $Z=\frac{C}{C_c}$ and cost ratio $\alpha=\frac{C_{st}}{C_c}$, Eq. 3 becomes

$$Z=(\alpha-1)V_{st}+V_G \qquad (4)$$

Since C_c is a constant parameter for a given place, the objective function $Z=\frac{C}{C_c}$ represents total cost of column which we need to minimize.

Following constraints (IS 456: 2000) were considered while formulating the optimization problem:

i) Constraints for axial load capacity and moment capacity of the column
ii) Constraints for minimum and maximum longitudinal reinforcement in the column
iii) Constraint for minimum number of longitudinal bars
iv) Constraint for maximum peripheral distance between longitudinal bars
v) Constraint for minimum and maximum width and maximum depth of column

Without violating any of the constraints, column optimization problem consists in determination of depth D_C, width b_C, percentage area of longitudinal reinforcement 'p' and lateral tie spacing s_C such that the cost of column is minimized. Mathematically, this optimization problem can be stated as

$$\text{Minimize}\, Z=(\alpha-1)V_{st}+V_G$$

Subject to
$$0.36 f_{ck}\, b_C\, k\, D_C+\sum_{i=1}^{n}(f_{si}-f_{ci})\frac{p_i\, b_C\, D_C}{100}\geq P \qquad (5)$$

$$0.36 f_{ck}\, b_C\, k\, D_C^2\,(0.5-0.416\,k)+\sum_{i=1}^{n}(f_{si}-f_{ci})\left(\frac{p_i}{100\, f_{ck}}\right)\left(\frac{y_i}{D_C}\right)\geq M \qquad (6)$$

$$0.8\leq p\leq 4.0 \qquad (7)$$

$$\frac{p\, b_C\, D_C}{25\,\pi\,\phi^2}\geq 4 \qquad (8)$$

$$d_p\leq 300 \qquad (9)$$

$$b_C\geq \text{Max.}(b_{B1}, b_{B2}, b_{B3}, b_{B4}) \qquad (10)$$

$$D_{C_j}\leq D_{C_{j-1}} \qquad (11)$$

$$b_{C_j}\leq b_{C_{j-1}} \qquad (12)$$

III. METHOD OF SOLUTION

The optimization problem was solved using Complex Iterative Method. The method essentially consisted in evaluating the objective function at r (≥ s+1, where s = number of independent design variables) feasible vertices of a complex (closed figure) and iteratively moving towards the optimum point by successive modifications. In each step the vertex X_L which yielded the largest value of objective function (worst vertex) was replaced by a new vertex X_N along the line joining the worst vertex and centroid of the remaining vertices. It was importantly assured that the new vertex did not violate any of the constraints and gave a smaller objective function value than the worst vertex. The new vertex was obtained as

$$X_N = X_O + \beta(X_O - X_L) \tag{13}$$

where β > 0

X_O = centroid of all vertices except X_L

X_L = worst vertex

When the reflected point X_N violated any of the constraints, it was moved half way towards the centroid by reducing β-value by half, until it became feasible. In this way, the complex was rolled over and over towards the minimum, remaining within the feasible space. The process was stopped when the deviation of function value at the vertices from the centroid became sufficiently small ($\beta < 0.001$). A graphical representation of the method is shown in Figure 1.

In the present study, value of s (number of independent design variables) was taken as two. Ratio of depth of neutral axis to depth of column 'k' and percentage area of longitudinal reinforcement 'p' were taken as independent design variables. Remaining design variables like cross-sectional dimensions D_C and b_C were obtained from these two values using their relationship with each other (Eq. 5 and Eq. 6). Number of longitudinal steel bars in the column was taken as 16; equally divided on four sides.

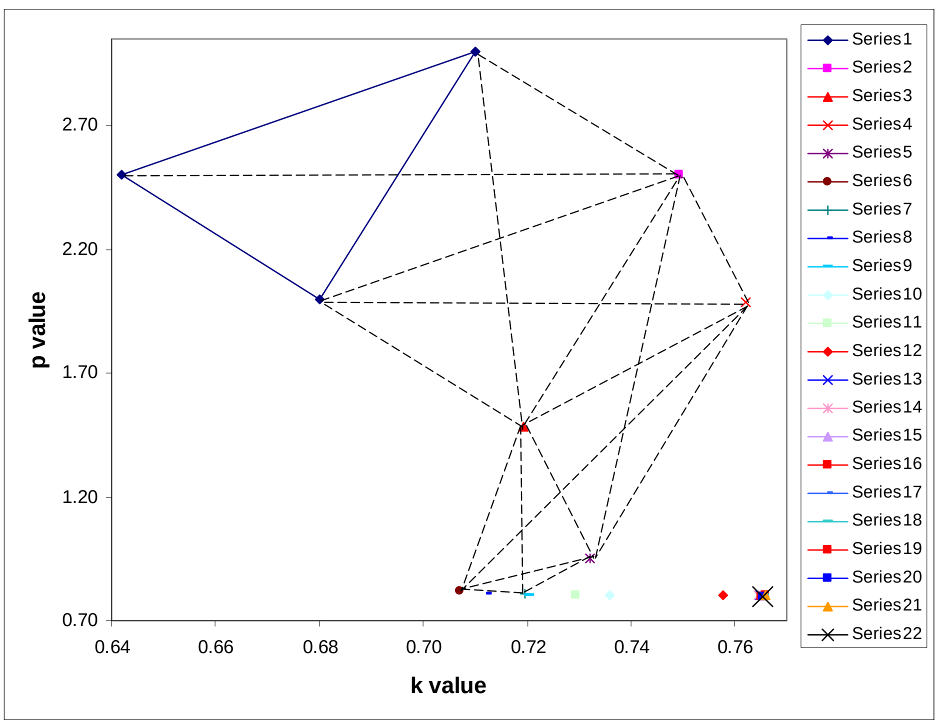

Figure 1. Graphical representation of "Complex Iterative Method" as applied to column optimization

IV. SELECTION OF INITIAL POINTS AND OTHER PARAMETERS

The number of vertices (r) in the complex was taken as 3, i.e. equal to s+1 (s =2). Thus 3 feasible starting designs required to initiate the optimization process were selected by considering conventional column design corresponding to 3 different values of percentage of longitudinal steel, namely 2.1, 2.5 and 3.0 (lying within the permissible range of 0.8 to 4.0). Certain other parameters like cost ratio (α), grades of concrete and steel (f_{ck} and f_y), column height (h) and column loads were given at the beginning.

V. EXAMPLE

The given set of loads for the column was taken as: Axial load = 1011 kN and Moment = 137 kN-m. Column height was 3.5 m. Grades of concrete and steel were taken as M30 and Fe415 respectively. The cost ratio was taken as 85. The optimum design parameters viz. percentage of longitudinal steel, cross-sectional dimensions and spacing of lateral ties, were determined and are presented in Table 1. Number of iterative cycles required for optimization has also been indicated in the same table. Percentage of longitudinal steel decreased from 2.0 % to 0.8 % and objective function (Z) reduced from 1.67338 to 1.06969, indicating a substantial optimization level of 36.1 %.

VI. CONCLUSIONS

1. The optimization results obtained by the use of Complex Iterative Method (Direct Search Method) for design of reinforced concrete columns were very encouraging and accordingly its suitability stands proved.
2 Although both reduction in steel area and cross-sectional area of column contributes towards optimization of reinforced concrete columns, the reduction in steel area plays a greater role in optimization as compared to reduction in cross-sectional area of columns.

Table-1 Comparison of conventional and optimum design values

Parameter	Conventional design	Optimum design
Longitudinal steel (p)	2. 0 %	0.8 %
Depth (D_C)	480 mm	900 mm
Width (b_C)	310 mm	150 mm
Tie spacing (8 mm dia. bar)	192 mm	150 mm
Objective function (Z)	1.67338	1.06969
Number of iterative cycles	-	48

VII. LIST OF SYMBOLS

A_{st_i} = area of reinforcement in the i^{th} row

$b_{B_1}, b_{B_2}, b_{B_3}, b_{B_4}$ = width of four beams, namely B_1, B_2, B_3 and B_4 that are attached to the given column C_j

b_C = width of column

b_{C_j} = width of column C_j

$b_{C_{j-1}}$ = width of column C_{j-1}, which lies immediately beneath column C_j

d_p = maximum peripheral distance among longitudinal bars of the column

D_C = depth of column

D_{C_j} = depth of column C_j

$D_{C_{j-1}}$ = depth of column C_{j-1}, which lies immediately beneath column C_j

f_{ci} = stress in concrete at the level of i^{th} row of reinforcement

f_{ck} = Characteristic strength of concrete

f_{si} = stress in the i^{th} row of reinforcement, compression being positive and tension being negative

$k\,D_C$ = depth of neutral axis from extreme compression fiber

M = actual value of bending moment as applied on the column

n = number of rows of reinforcement

p = percentage area of longitudinal reinforcement

p_i = percentage area of steel in the i^{th} row of reinforcement $\left(p_i = \frac{100\,A_{st_i}}{b_C\,D_C} \right)$

P = actual value of axial load as applied on the column

y_i = distance of the i^{th} row of reinforcement steel, measured from the centroid of the section. It is positive towards the highly compressed edge and negative towards the least compressed edge.

ϕ = diameter of longitudinal bar

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