Free Vibration Analysis of FGM Plates

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Abstract - Free vibration response of functionally graded material plate (FGM) is presented using different shear deformation theories. Governing equation of the plate is obtained Hamilton's principle. A MATLAB code is developed incorporating polynomial radial basis function (RBF) base meshfree method to obtain the solution . Numerical results related to vibration of FGM plates are presented . Different theories taken here are compared based on the effect of grading index and span to thickness ratio.

Keywords – Meshless ,rbf, fgm ,plate, vibration

I. Introduction

Functionally graded materials (FGMs) considered as a special composite material for their gradually changing the material properties in one or more directions. For withstanding very high thermal gradients, aerospace and aircraft industries implement FGM frequently. In recent years, the analyzing of vibration of FGM plates attracted the researchers frequently. A simple four variable plate theory for free vibration analysis of functionally graded plates was invented by Thai and Kim [1].The solution technique to examined exact solution for the free vibration analysis of plates lighted by Hashemi et al. [2]. By using higher order shear deformation theory, Singh et al. [3] employed free vibration analysis of composite plates. Ahmadian and Zangeneh [4] carried finite element method for vibration analysis of rectangular plates. Peng et al. [5] worked on free vibration analysis of folded plate by mesh-free method using FSDT. Xiang et al. [6] employed meshless radial point method for the free vibration analysis of isotropic plates. Combination of finite differences and radial basis functions for the free vibration analysis of plate was introduced by Roque et al. [7]. Yang and Shen [8] carried the TSDT with von Karman to analysis the vibration characteristics and transient response of initially stressed functionally graded plates in thermal environments.

II. Mathematical formulation

A rectangular shape plate of edge length a, b along x, y axes respectively and thickness h is the thickness along z axis whose mid plane is coinciding with x-y plane of the coordinate system is considered. The diagram of rectangular shaped functionally graded material (FGM) plate in rectangular coordinate system is shown in Figure 1.

The homogenization technique considered in this work is the law of mixtures, which provides the following elastic properties at each material layer. The top surface of the plate is ceramic rich and the bottom surface is metal rich.

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^n \tag{1}$$

where n is exponent governing the material properties along the thickness direction known as volume fraction exponent or grading index,

The volume fraction of the metal phase is obtained by

$$V_m(z) = 1 - V_c(z) \tag{2}$$

The material property gradation through the thickness of the plate is assumed to have the following form

$$E(z)=\left[E_c-E_m\right]\left(\frac{2z+h}{2h}\right)^n+E_m \qquad , \qquad \rho(z)=\left[\rho_c-\rho_m\right]\left(\frac{2z+h}{2h}\right)^n+\rho_m \tag{3}$$

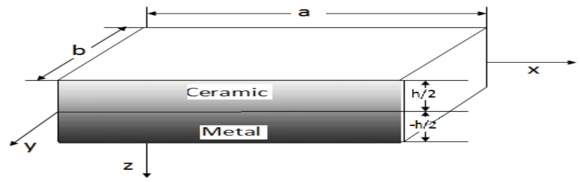

Fig 1. Geometry of rectangular FGM plate in rectangular coordinate system

Here E and denote the modulus of elasticity and density of FGM structure, while these parameters come with subscript m or c represent the material properties for pure metal and pure ceramic plate respectively., h is the thickness of the plate, E_m and E_c are the corresponding Young's modulus of elasticity of metal and ceramic and z is the thickness coordinate.

The displacement field at any point in the plate is expressed as Singh and Shukla [9]:

$$\begin{aligned} U &= u_0(x,y)-z\frac{\partial w_0(x,y)}{\partial x}+f(\mathrm{z})\phi_x(x,y) \\ V &= v_0(x,y)-z\frac{\partial w_0(x,y)}{\partial y}+f(\mathrm{z})\phi_y(x,y) \\ W &= w_0\left(x,y\right) \end{aligned} \tag{4}$$

Where, transverse shear function $f(\mathrm{z})$ is as under.

Notation	$f(\mathrm{z})$	Proposed/Used by
The[1]	$\mathrm{z}.\left[1-\frac{4}{3\mathrm{h}^2}\mathrm{z}^2\right]$	Levinsion[10],Reddy[11]
The[2]	$\mathrm{z.m}^{\left(-2\left(z/h\right)^2\right.}$, m = 3	Mantari et al. [12]
The[3]	$\mathrm{z.e}^{\left(-2\left(z/h\right)^2\right.}$	Karama et al. [13]
The[4]	$\mathrm{z}.\left[\frac{5}{4}-\frac{5}{3\mathrm{h}^2}\mathrm{z}^2\right]$	Kruszewski [14], Panc [15] Reissner [16]

The governing differential equations of plate are obtained using Hamilton's principle and expressed as Singh et al. [17]:

$$\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\frac{\partial^2 u_0}{\partial\tau^2}-I_1\frac{\partial^3 w_0}{\partial x\partial\tau^2}+I_3\frac{\partial^2\phi_x}{\partial\tau^2} \tag{5}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=I_0\frac{\partial^2 v_0}{\partial\tau^2}-I_1\frac{\partial^3 w_0}{\partial y\partial\tau^2}+I_3\frac{\partial^2\phi_y}{\partial\tau^2} \tag{6}$$

$$\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_y}{\partial y^2}+2.\frac{\partial^2 M_{xy}}{\partial x\partial y}=I_0\frac{\partial^2 w_0}{\partial t^2}+I_1\left(\frac{\partial^3 u}{\partial x\partial\tau^2}+\frac{\partial^3 v}{\partial y\partial\tau^2}\right)-I_2\left(\frac{\partial^4 w}{\partial x^2\partial\tau^2}+\frac{\partial^4 w}{\partial y^2\partial\tau^2}\right)+I_4\left(\frac{\partial^3\phi_x}{\partial x\partial\tau^2}+\frac{\partial^3\phi_y}{\partial y\partial\tau^2}\right) \tag{7}$$

$$\frac{\partial M^f_{xx}}{\partial x}+\frac{\partial M^f_{xy}}{\partial y}-Q^f_x=I_3\frac{\partial^2 u_0}{\partial\tau^2}-I_4\frac{\partial^3 w_0}{\partial x\partial\tau^2}+I_5\frac{\partial^2\phi_x}{\partial\tau^2} \tag{8}$$

$$\frac{\partial M_{xy}^{f}}{\partial x}+\frac{\partial M_{yy}^{f}}{\partial y}-Q_{y}^{f}=I_{3}\frac{\partial^{2}v_{0}}{\partial\tau^{2}}-I_{4}\frac{\partial^{3}w_{0}}{\partial y\partial\tau^{2}}+I_{5}\frac{\partial^{2}\phi_{y}}{\partial\tau^{2}}$$

(9)

The force and moment resultants in the plate and plate stiffness coefficients are expressed as Singh and Shukla [18]:

$$N_{ij},\ M_{ij},\ M_{ij}^{f}=\int_{-h/2}^{+h/2}(\sigma_{ij},\ z\sigma_{ij},\ f(z)\sigma_{ij})\,dz \tag{10}$$

$$Q_{x}^{f},\ Q_{y}^{f}=\int_{-h/2}^{+h/2}\left(\sigma_{xz},\ \sigma_{yz}\right)\left(\frac{\partial f}{\partial z}\right)dz$$

(10)

$$A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}=\int_{-h/2}^{h/2}\left\{\left(\left(\left[Q_{ij}^{c}-Q_{ij}^{m}\right]\right)\left(\frac{2z+h}{2h}\right)^{n}+Q_{ij}^{m}\right)\left(1,z,z^{2},f(z),z\,f(z),f^{2}(z)\right)\right\}dz \quad i,\ j=1,\ 2,\ 6$$

(11)

$$A_{ij}=\int_{-h/2}^{h/2}\left\{\left(\left(\left[Q_{ij}^{c}-Q_{ij}^{m}\right]\right)\left(\frac{2z+h}{2h}\right)^{n}+Q_{ij}^{m}\right)\left(\frac{\partial f(z)}{\partial z}\right)^{2}\right\}dz$$

i, j = 4, 5 (12)

$$I_{0},I_{1},I_{2},I_{3},I_{4},I_{5}=\int_{h_{1}}^{h_{2}}\rho(z)\left(1,z,z^{2},f(z),zf(z),f^{2}(z)\right)dz \tag{13}$$

The boundary conditions for an arbitrary edge with simply supported conditions are as follows:

$$\begin{aligned}x=0,a: v=0;\phi_{y}=0;w=0;M_{xx}=0;N_{xx}=0\\ y=0,b: u=0;\phi_{x}=0;w=0;M_{yy}=0;N_{yy}=0\end{aligned} \tag{14}$$

III. SOLUTION METHODOLOGY

The governing differential equations (5-9) are expressed in terms of displacement functions. Radial basis function based formulation works on the principle of interpolation of scattered data over entire domain. A 2D rectangular domain having NB boundary nodes and ND interior nodes is shown in Figure-2.

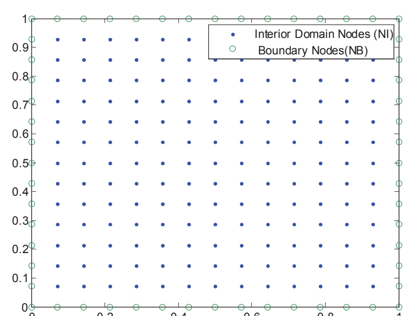

Fig 2. An arbitrary two dimensional domains

The variable $u_0, v_0, w_0, \phi_x \; and \; \phi_y$ can be interpolated in form of radial distance between nodes. The solution of the linear governing differential equations (5-9) is assumed in terms of polynomial radial basis function for nodes 1:N, as;

$$u_0, v_o, w_o, \phi_x, \phi_y = \sum_{j=1}^{N} (\alpha_j^u, \alpha_j^v, \alpha_j^w, \alpha_j^{\phi_x}, \alpha_j^{\phi_y}) \; g\left(\left\|X - X_j\right\|, m\right)$$

Where, N is total numbers of nodes which is equal to summation of boundary nodes NB and domain interior nodes ND. $g\left(\left\|X - X_j\right\|, m\right)$ is polynomial radial basis function expressed as $g = r^m$, $\alpha_j^u, \alpha_j^v, \alpha_j^w, \alpha_j^{\phi_x}, \alpha_j^{\phi_y}$ $\alpha_j^u, \alpha_j^v, \alpha_j^w, \alpha_j^{\phi_x}, \alpha_j^{\phi_y}$ are unknown coefficients. $\left\|X - X_j\right\|$ is the radial distance between two nodes.

Where, $r = \left\|X - X_j\right\| = \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2}$ and m is shape parameter. The value of 'm' taken here is 5.

Polynomial radial basis function becomes singular, when r = 0 i.e. for zero distance. In order to eliminate the singularity, an infinitesimally small value is added into the r^2 or zero distance. Mathematically it is explained as; $r^2 = r^2 + \mu^2$, when r = 0 or i = j; μ^2 is small numerical value of the order 10-^{10}.

Free Vibration Analysis, t The discretized governing equations for vibration analysis can be written as:

$$\left[\begin{bmatrix}[K]_L \\ [K]_B\end{bmatrix}_{5N\times 5N} + \omega^2 \begin{bmatrix}[M] \\ 0\end{bmatrix}_{5N\times 5N}\right] \{\delta\}_{5N\times 1} = 0 \tag{15}$$

or $([K] + \lambda[M])\{\delta\}=0$ (16)

Using standard eigen value solver for equation (16) , the frequency is calculated as:

[V,D] = eig([K], [M]); frequency (ω) = $\sqrt{D}$ (17)

IV. Numerical Results and Discussions:

In order to demonstrate the accuracy and applicability of present formulation, a RBF based meshless code in MATLAB is developed following the analysis procedure as discussed above. Several examples have been analyzed and the computed results are compared with the published results. Based on convergence study, a 15×15 node is used throughout the study. The material properties of FGMs have been taken as follows:

Ceramic $E_c = 380\,GPa, \nu_c = 0.3, \rho_c = 3800$, Aluminum (Al) $E_m = 70\,GPa,$ $\nu_m = 0.3,$ $\rho_m = 2702$ -

The dimensionless natural frequency parameter is defined as: $\Omega = \left(\frac{\omega a^2}{h}\right)\left(\frac{\rho_c}{E_c}\right)^{(1/2)}$

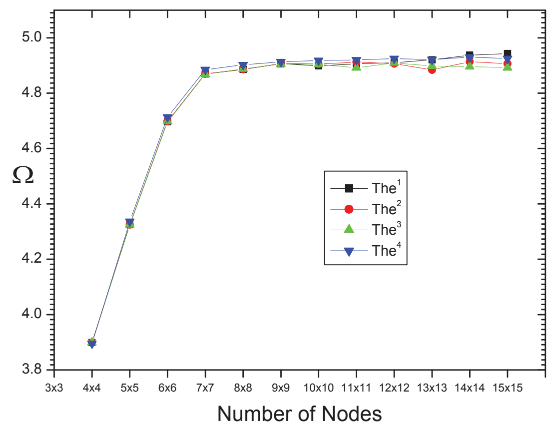

Fig. 3 Convergence study for free vibrations of simply supported FGM plate (a/h = 10)

Table 1 Effect of span to thickness ration on fundamental Ω of a square FGM plate

'n'	Abdelkader et al [19]	Hosseini et al [20]	Zhao et al [21]	Present					
	a/h								
	10			10	20	30	40	50	100
0.5	4.9000	4.9207	4.8209	4.9068	5.0231	5.0455	5.0497	5.0543	5.0642
1	4.4166	4.4545	4.3474	4.4425	4.5304	4.5501	4.5567	4.5603	4.5687
2	4.0057	4.0063	3.9474	4.0210	4.1234	4.1430	4.1491	4.1512	4.1573
5	3.7660	3.7837	3.7218	3.7809	3.8944	3.9212	3.9295	3.9330	3.9396

In order to show the accuracy and efficiency of the present solution methodology, detailed convergence studies for simply supported isotropic plate (a/h=10) is carried out. The convergences of the frequency parameters for different theories are shown in Fig. 3. It can be seen that convergence all the theories are achieved within 1 % at 15×15 nodes.

Table 1 lists the fundamental frequency parameter of square FGM plates for different values of the span to thickness ratios (a/h = 10 and 20) and with various ceramic volume fraction exponent 'n'. The results are compared with the

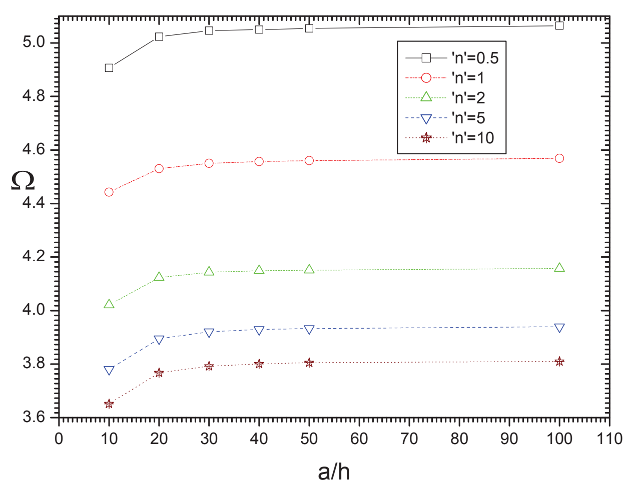

Fig 4. Effect of span to thickness ration on fundamental Ω of a square FGM plate

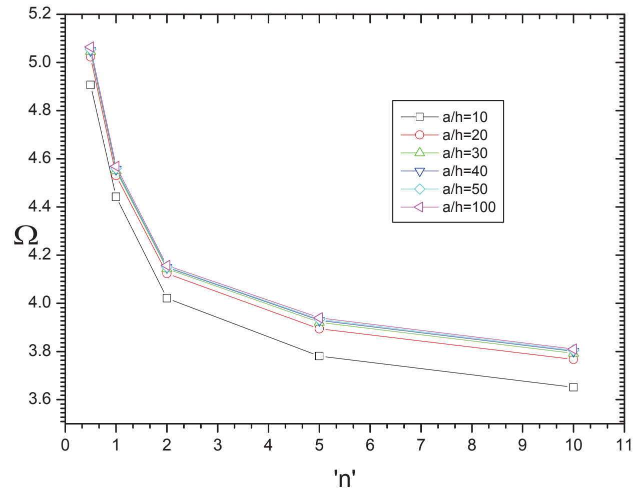

Fig 5. Effect of grading index 'n' on fundamental Ω of a square FGM plate

results due to Abdelkader et al [19], Hosseini et al [20] and Zhao et al [21]. It is clear that the results are in good agreement with available results and the maximum discrepancy is less than 2%. The results are depicted in Fig. 4 and Fig. 5 to see the nature. It can be seen that effect of span to thickness ratio decreases for a/h $\geq$ 30.
Fig 5. shows that effect of grading index 'n' is prominent for lesser values of 'n' and decreases as 'n' increases.
Table 2 shows the results for all 8 modes of frequency parameter.
Further, to compare different theories taken here, results are obtained at a/h=10 and grading index 'n'=5. The results obtained are shown in **Table 3** and same is depicted in **Fig. 6** for better comparison. It is observed that effect is more visible for thick plates while it nullifies as plate becomes thinner.

Table 2 Effect of span to thickness ration on Ω of a square FGM plate for 8 modes

		'n'				
	Modes	0.5	1	2	5	10
a/h=10	1	4.9068	4.4425	4.0210	3.7809	3.6514
	2	9.5110	10.0618	9.5687	8.8897	7.0813
	3	11.7282	10.5868	9.5901	10.2316	8.5210
	4	11.7282	10.5868	10.3586	10.9026	8.6036
	5	14.6272	12.3449	14.5695	12.3985	11.4650
	6	16.9636	16.1189	14.5924	12.5322	11.8552
	7	17.4514	16.4682	14.8174	14.0766	12.3540
	8	18.1089	19.6233	17.5455	14.9379	13.0500
a/h=100	1	5.0642	4.5687	4.1573	3.9396	3.8100
	2	12.5849	11.4304	10.3973	9.8805	9.5570
	3	12.7564	11.4402	10.4308	9.8837	9.5606
	4	19.5479	18.6087	16.9364	16.0389	15.4986
	5	24.9711	22.7381	20.7224	19.6280	18.9874
	6	25.6854	22.8981	20.8818	19.7617	19.1128
	7	32.5328	30.4065	27.7080	26.2346	25.3435
	8	33.5781	30.4835	27.7512	26.7675	25.5815

Table 3 Comparison of different theories for fundamental Ω of a square FGM plate (a/h=10, 'n'=5)

a/h	Theories			
	The[1]	The[2]	The[3]	The[4]
10	3.7728	3.7809	3.7758	3.8016
20	3.8979	3.8944	3.8963	3.9059
30	3.9213	3.9212	3.9201	3.9246
40	3.9303	3.9295	3.9293	3.9323
50	3.9342	3.9330	3.9340	3.9375
100	3.9406	3.9396	3.9387	3.9429

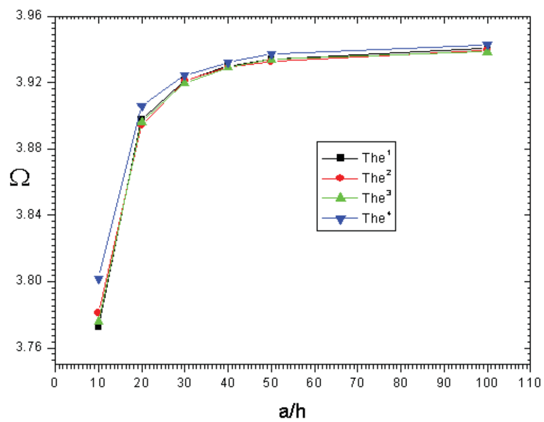

Fig 6. Comparison of different theories for fundamental Ω of a square FGM plate (a/h=10, 'n'=5)

IV.CONCLUSION

Free vibration response of functionally graded material plate (FGM) is presented using different shear deformation theories. . It is observed that effect on frequency parameter is more visible for thick plates while it nullifies as plate becomes thinner for different theories. The effect of span to thickness ratio decreases for a/h $\geq$ 30. The effect of grading index 'n' is prominent for lesser values of 'n' and decreases as 'n' increases

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