Super Resolution of RGB Color Images Based On Group Sparse Representation

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Abstract- Images of poor resolution results due to number of phenomena like compression, motion of the object, motion of the sensor, environment disturbance, limitation of camera etc. Transforming such low resolution (LR) images into high resolution (HR) image is a research topic of great interest in recent times. Super Resolution(SR) is a promising technique which aims at recovering high resolution image from their low resolution counterparts. This paper focuses on RGB image super resolution algorithm, which constructs a coupled over complete dictionary and sparse coefficient represented by group sparsity of LR image patches and its corresponding HR patches. Group Orthogonal Matching Pursuit (GOMP) is used to obtain the sparse coefficient vector. Convolution of sparse vector and HR dictionary results in required HR image. The simulation results prove that the quality of the HR images resulting from the proposed algorithm is superior than those obtained from other SR techniques. Owing to the use of local sparse modeling, the proposed method is naturally robust to noise, hence this technique can super resolve noisy and blurred input in a more unified framework.

Keywords- Super-resolution, Group sparse representation, over-complete dictionary, Group Orthogonal Matching Pursuit.

I. INTRODUCTION

Super-resolution (SR) is a process to successfully bring out high-resolution (HR) image with more resolving power using one or more low-resolution (LR) images. Typical LR images generally contain artifacts, blur and noise. HR images accommodate increased pixel density and thereby providing more fine details about the original scene. Many applications such as surveillance, forensic and satellite imaging requires extreme zooming of a specific area, which in turn demands HR images. SR algorithms are classified into the three main types: Interpolation based method, Learning based method, and Reconstruction based method. In Interpolation based SR algorithms [1], [2], registration and non- uniform interpolation is done to produce an improved resolution output image. In Reconstruction based methods [3], HR image is obtained from multiple LR images of the same scene with sub-pixel overlaps. The new information contained in each LR image is exploited to obtain an HR image. If these scene motions are known or can be estimated within subpixel accuracy, the HR image can be obtained by combining LR images. In learning based algorithms [4], the correspondence between LR-HR images are mapped by learning a mapping function. The mapping function can be either implicit or explicit depending on the function which relates the LR-HR patch-pairs. In implicit learning [5], the correspondence between LR-HR patch-pairs are learned from local neighborhood of the image itself and are implied directly to the HR image space. Whereas, in explicit learning the mapping function is either learned by a regression operator or an over-complete dictionary.
The rest of paper is organized as, in Section 2, related work for super resolution using different algorithm is surveyed. In Section 3, the proposed method to construct SR image is described. Section 4 explains about performance of simulated results of the proposed work. Finally, Section 5 gives conclusion and future work directions of the work.

II. RELATED WORKS

Yongqin Zhang [6] proposed a common frame super-resolution structure modulator sparse representation improvement based on sparse image super-resolution effect. Ridge regression with multi-stage magnification is used for multi-scale redundancy program for an initial estimate of the high resolution image. Gradient preservation is used as a regularization term for sparse modeling. The author solved the issues by modeling numerical parameter estimation and sparse representation.

Kostadin Dabov[7] proposed a new method of enhancing sparse representation expressed in the transform domain . By a similar fragment obtained 2-D images into a 3-D enhancement data array. Collaborative filtering is to develop a special program to deal with these 3 -D groups. Author implement it with three consecutive steps : a group of a 3-D transform , the transform domain to reduce Spectrum, and 3-D inverse transform . The result is a three-dimensional estimate of jointly filter image block . Improvement of the method is obtained by collaborative Wiener filter.

Elad[8] et al. employed two paradigms for SR recovery. Firstly, the inverse SR problem is treated based on mathematical modeling and secondly it is modeled explicitly as an over-complete dictionary learned from random image patch-pairs. Initially the over-complete dictionary is learned by exploiting mathematical models such as wavelets, DCT, contourlets, and curvelets to construct an effective dictionary. Later, inspired from [7], the over-complete dictionary is trained from random raw patches extracted from the training samples and are learned by state-of-the-art K-singular value decomposition (K-SVD) algorithm. Sparse approximation of random image patches are carried out using orthogonal matching pursuit (OMP) algorithm. The K-SVD and generalized Principal Component Analysis (PCA) used in this method improves the efficiency and effectiveness of this approach.

Yang[9] et al. proposed super-resolved LR images by employing sparse representation model (SRM) as a prior to represent LR-HR patch-pairs. The correspondence between LR-HR patch-pairs is learned by training a coupled dictionary. In learning phase, it enforces the sparse representation of a LR image patch with respect to its LR dictionary and efficiently reconstructs the HR image patch from its corresponding HR dictionary in the HR image patch space. This modeling problem is optimized by a bi-level optimization technique, where the optimization includes a l^1 norm minimization problem as its constraints. The sparse coefficients consist of minimum non-zero elements and are multiplied to the dictionary to get the HR image which contain high frequency information.

Julien Mairal [10] proposed a method to bring together two different approaches of image restoration methods: 1) Learning the dictionary. 2) use of natural image of self-similarity. The author also proposed Sparse coding with greedy pursuit as a framework for combining these two methods. This is done through joint realization decomposing learned dictionary subset of cluster on similar signals, effectively restoring the original image at a reasonable speed and memory cost of the digital camera.

III. METHODOLOGY FOR SUPER RESOLUTION

The image super resolution is an ill posed problem. In this method, super resolution is the task of recovering original HR image ‘y’ from the given degraded image x,

$$x = Ly \qquad (1)$$

Where, L= Down sampling operator,

The proposed algorithm does super resolution in two steps, they are:

- Dictionary formation of grouped features: In this step, two dictionaries are created. Both the dictionary are learned using ODL algorithm[6]. One dictionary is generated for low resolution image and another dictionary is generated for high resolution image. Dictionary is trained using K-SVD.
- Estimation of sparse coefficient: In this step, sparse coefficient is produced and updated to obtain the patches of high resolution image.

The RGB bands are first extracted from the input CFA images. When the extracted bands are directly subjected to PCA based dimensionality reduction, the resulting aliasing degrade the quality of the final output. Hence each of the bands are subjected to Gabor filtering, which provide both good time and frequency resolution. PCA confines the information contain within each feature to minimum number of coefficients. Application of PCA to the gabor feature results in compact feature representation matrices for each of the three band. The LR image incorporates less information about the original scene, which is obtained by downsampling and upsampling by a scaling factor of 2. For upsampling, Bicubic interpolation is used. These steps reduce the detailed information present in the input image. This LR image is used for generate LR dictionary. High frequency image that represents HR image is obtained by subtracting the original image and interpolated image. This HR image patches are used in generating HR dictionary. Before creating the dictionaries, instead of processing the entire image, the features present in the image are extracted and processed. Here, Gabor filter with complex sinusoid is used for feature extraction and is be given as,

$$g(x, y; \lambda, \theta, \sigma, \gamma, \psi) = \exp(-\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2}) \exp(i(2\pi \frac{x'}{\lambda} + \psi)) \qquad (2)$$

Where,

$x' = x\cos\theta + y\sin\theta$ and

$y' = -x\sin\theta + y\cos\theta$

In the equation (2), λ symbolize the wavelength of the sinusoidal factor, θ represents the orientation of the stripes of a Gabor function, ψ denotes the phase offset, σ is the sigma/standard deviation of the Gaussian envelope and γ gives the spatial aspect ratio, and specifies the ellipticity of the support of the Gabor function.

In this algorithm, imaginary component of the Gabor filter with six orientation θ=0°, 30°, 60°, 90°, 120°, 150° is used. The output of the gabor filter represent the features of the RGB band separately. This six directional Gaussian kernel is convolved with each image in the interpolated LR image set I_i. Hence a set of feature images I_{ir}, I_{ig} and I_{ib} is obtained, which contain the edge present in the image and pattern information of the interpolated images. Now non-overlapping patches are collected from these feature image I_{ir},I_{ig} and I_{ib} and also from the high-pass filtered HR image set I_{hf} creating sets of patches ψ_r , ψ_g and ψ_b as γ_{rgb} , such that correspond to the same region as ψ_{hfj}, these corresponding patches from these different feature images are collected and concatenated to form a set of long feature vectors γ_i, one per patch, corresponding patches from the high-pass filtered image set ψ_{hf} are concatenated to form another set of feature vectors γ_{hf}.

Then , the elements of γ_{rgb} undergo PCA, which result in reducing the number of dimensions of the data and thus speeding up the dictionary training while preserving maximum of the energy and the PCA projection matrix is used for OMP. This leads to a low-dimensional set γ_{prgb}. Now the element of the set γ_{prgb} is used for creating low resolution dictionary. The HR patches obtained after PCA can be denoted as $\gamma_{phf.}$

A. Group Sparse Representation (GSR) Model for extracted features

In GSR model, the signals is approximated by a union of a few subspaces. Let $D \in \mathbb{R}^{n\times k}$ be an over-complete dictionary with K atoms. The K number of atoms are divided into N groups containing k atoms such that $K = Nk$. Let the over-complete dictionary with N atoms be defined as

$$D = [d_{1,1}, d_{2,1}, ..., d_{N,1}, d_{1,2}, ..., d_{N-1,k}, d_{N,k}]$$

In each atom, the first term in the subscript denotes the group index, and the second term denotes the index in the group. The linear combination of element of $x \in \mathbb{R}^n$ in terms of D can be given as:

$$\alpha = [\alpha_{1,1}, \alpha_{2,1}, ..., \alpha_{N,1}, \alpha_{1,2}, ..., \alpha_{N-1,k}, \alpha_{N,k}]^T$$

Where $\alpha \in \mathbb{R}^m$ is the group sparse vector, T denotes the transpose of matrix. Sparsity constraint α , can be required for following optimization problem

$$\min_{\alpha} \|\alpha\|_{2,0} \text{ subject to } x = D\alpha \qquad (3)$$

Where $\|\alpha\|_{2,0} = \sum_{i=1}^{k} \mathrm{I}(\|\alpha_k\|_2)$ and I(.) denotes indication function defined as

$$I(\|\alpha_j\|_2) = \begin{Bmatrix} 1, \text{ if } \|\alpha_j\| > 0 \\ 0, \text{ otherwise} \end{Bmatrix} \qquad (4)$$

The group sparse coefficient α is solved by satisfying the following problem:

$$\min_{\alpha} \|\alpha\|_{2,0} \text{ subject to } \|x - D\alpha\|_2^2 \le \varepsilon \qquad (5)$$

Where $\varepsilon \ge 0$ is error tolerance.

The above problem is the GSR problem. The solution for this problem is the Orthogonal Matching Pursuit (OMP), is an iterative and greedy algorithm. It maintains a set of atoms that are already picked, and adds a new atom at each iteration. The residual is then projected to a linear combination of all atoms in the active set, so that an orthogonal updated residual is obtained.

The detail of the OMP is summarized in Algorithm 1 as follows:

Algorithm 1 Group Orthogonal Matching Pursuit Algorithm

1: **Input** : an over-complete dictionary D with grouped atoms, image patch $p \in \mathbb{R}^n$ extracted from LR image.

2: **Initialization:** Residual $r = p$ and group index

3. Select the atom that maximizes the absolute value of the inner product with residue r and denote it as φ_p.

4.Compute the inner product $\mu = D^T r = [r^T d_{1,1}, r^T d_{2,1} .. r^T d_{N,k}]$

5. Divide μ into groups such that $[r^T d_{t,1}, r^T d_{t,2} .. r^T d_{t,k}]$, where $j = 1,2,..N$ and select the group with maximum correlation and update the group index $\Lambda = \Lambda \cup \{t\}$.

6. Update the residue r by

$r = p - D_\Lambda (D_\Lambda)^{\dagger} p$

7. Repeat until r is minimum.

6. **Output:** group sparse co-efficient α

B. Dictionary Formation

Dictionary can be defined as $D \in R^{n\times k}$, where K represents atoms. It is an coupled over complete dictionary so that (K>>n). Signal can be represented as $x \in D\alpha_0$ where $\alpha_0 \in R^K$ which has a very few number of non-zero entities. y is a high resolution patch. For low resolution image patch $x = Ly$, where L is a projection matrix. The patches of the high resolution image can be reconstructed from the patches of low resolution image, with the aid of dictionary. In general, various denoising algorithm proposed work by learning over complete dictionary using natural image as source. When forming a dictionary , each signal element is uniquely expressed as a linear combination of atomic dictionary. In some case where the dictionary is orthogonal, and the corresponding coefficients can be obtained by computing the inner products of the signal and the atoms.

Using sparse prior as regularization, the super resolution points out that each pair of HR and LR image patches will possess similar sparse representation with respect to the dictionaries D_h and D_l. To create a coupled dictionary image patch pairs, it should be sampled directly. This ensures that the corresponding HR and LR patch will be preserved. However, the resulting dictionary will be large and thus computationally expensive. The process of dictionary learning begins with an initial random dictionary. By maintaining the initial random dictionary as fixed value, sparse approximations of the training signals are computed. Following this, dictionary is trained by keeping the sparse coefficient remain fixed. This alternative process is repeated till we arrive at a specific approximation error. These algorithms have been derived for dictionary learning in a noisy sparse approximation environment.

A signal can be represented as Dα where D is an over complete dictionary. In this paper, with the aid of features extracted from the patches, dictionary is formed. Dictionary learning can be performed in two steps: i) keeping D fixed α is updated ii) α is fixed, D is updated. Dictionary can be learned from the training set. In some approaches [4], they uses already trained common dictionary. In this work, the dictionary can be trained using Online Dictionary Learning(ODL) [6] algorithm. In the algorithm, let the training can be X={x_1,x_2,…x_n}. For normalization, l_1 norm-minimization is used. The Dictionary, D can be obtained by

$$D = \min_{D\in R^{m*n}} (\sum_j \min_{\alpha} (\frac{1}{2}\|X - D\alpha\|_2^2 + \lambda\|\alpha\|_1)) \qquad (6)$$

Where the l_1 norm $\|\alpha\|_1$ is to enforce sparsity , and the l_2 norm is to remove scaling ambiguity.

1. Dictionary Training

In dictionary training, the optimization can be done as follows,

i. At the first, the element of D can be initialized using Gaussian random matrix.
ii. Then D is kept fixed, α is updated using,

$$\alpha = \arg\min_{\alpha\in R^n} \frac{1}{2}\|X - D\alpha\|_2^2 + \lambda\|Z\|_1 \qquad (7)$$

This can be solved efficiently.

iii. α is kept fixed, D is updated.

$$D = \arg\min_{D} \sum_{j} \left\| X - D\alpha \right\|_{2}^{2} \qquad (8)$$

iv) Iteration will be continued until convergence is reached.

The above algorithm is used to generate a coupled dictionary incorporates LR and HR dictionary. Low dimensional feature set γ_{prgbj} is the input for low resolution dictionary. High resolution feature set γ_{hfj}, is the input for high resolution dictionary. The low resolution dictionary D_l can be formed by,

$$D_l = \min_{D_l \in R^{m*n}} \left(\sum_{j} \min_{\alpha} \left(\frac{1}{2} \left\| \gamma_{prgb} - D_l \alpha \right\|_2^2 + \lambda \left\| \alpha \right\|_1 \right) \right) \qquad (9)$$

The equation (6) is the final low resolution dictionary which represents all the patches in the low resolution patch set γ_{prgb} through l_1 sparse combinations of its elements. The parameter λ is the sparsity constraint and is set to 1/8; larger the lambda more the sparsity in the ideal sparse representations of the elements of γ_{prgb} using D_l is enforced at the expense of RMS error.

Variant of Least Angle Regression(LARS) algorithm [4] is used to obtain a set of sparse vectors α satisfying(7) ,

$$\alpha_j = \arg\min_{\alpha_j \in R^n} \left(\frac{1}{2} \left\| \gamma_{prgb} - D_l \alpha_j \right\|_2^2 + \lambda \left\| \alpha_j \right\|_1 \right) \qquad (10)$$

This is the actual sparse representation of the final low resolution patch set γ_{prgb} using the atoms of the trained low resolution dictionary. As mentioned above, the λ essentially enforces sparsity in the solution.

Now we create the high resolution dictionary D_h via a pseudo-inverse approach, finding a D_h which satisfies,

$$D_h = \arg\min_{\alpha_j \in R^{m \times n}} \sum_{j} \left\| \gamma_{phf_j} - D_h \alpha_j \right\| \qquad (11)$$

This D_h is the final high resolution dictionary. It has correspondence with the low resolution dictionary such that if $D_l\alpha = \gamma l$ represents the concatenated PCAed feature vectors of some interpolated patch γl as an l_1 sparse combination of atoms from D_l, then $D_h\alpha \approx \gamma h$. Where γ_h is high resolution version of the patch. This is guaranteed for all patches in the final low resolution patch set γ_{prgb} and it is expected that it holds for any other patch encountered during SR σ as well. This is the crux of our approach.

C. Sparse Coefficient Estimation

Super resolved image is obtained by convolving the HR dictionary (D_h) and the sparse vector. Estimating a sparse multi-dimensional vector that satisfies a equations of a linear system which has a design matrix and extracted high-dimensional data is known as sparse approximation. A sparse approximation technique has several applications namely audio processing, image processing, document analysis and remote sensing. It is mainly used in the formation of dictionary and also for denoising purpose. It increases the accuracy of an image which is to be reconstructed and also used super resolution.

In many applications like computer vision, pattern recognition, surveillance and remote sensing etc, High-dimensional data's are used. In general, high-dimensional data are not uniformly distributed over the surrounding area in the image. HR data lie very close to a union of low-dimensional manifolds. Recovering high resolution information from low-resolution image structured data helps in significantly reducing the memory requirements of algorithms. Computational cost and the effect of noise in the high-dimensional data are reduced to increase the performance. Three primary tasks is mainly related to multi-manifold data are determined namely, clustering, dimensionality reduction and classification. In many areas, machine learning has proved a great advancement such applicability of extant algorithms are restricted by several challenges. First, manifolds are even converging and spatially close whereas existing methods works only when manifolds are detached sufficiently. Second, as a priori the most algorithms requires to know the dimensions of manifolds, while in many real-time problems such quantities are often unknown. However, the complexity of existing algorithms in effectively dealing with data nuisances, such as outliers, noise and lost entries, and also manifolds of different intrinsic dimensions.

1. Estimation Of Sparse Coefficient Vector

The dictionary based super-resolution problem requires: given a low-resolution image ***Y***, reconstruct a higher-resolution image ***X*** of the same scene. There are two constraints which are to be constructed in this work to solve this ill-posed problem: 1) constraint on reconstruction, which requires that the obtained ***X*** should be consistent with the input ***Y*** with respect to the image observation model; and 2) sparsity prior, where the high resolution patches can be sparsely represented by over complete coupled dictionary, and that its sparse representations can be reconstructed from the low resolution dictionary.

- *Reconstruction constraint:* The low-resolution image ***X*** is a down sampled version of the high-resolution image ***Y***:

$$\boldsymbol{X} = L\boldsymbol{Y} \quad (12)$$

Here, L the down sampling operator. Super-resolution remains severely ill-posed, for a given low-resolution input ***X***, many high-resolution images ***Y*** can satisfy the above reconstruction constraint. Thus, it can further regularize the problem via the following prior on small patches ***x*** of ***X***.

- *Sparsity prior:* The patches ***y*** present in the high-resolution image ***Y*** can be represented as a sparse linear combination of high resolution dictionary D_h trained from high-resolution patches sampled from training images,

$$y \approx D_h \alpha \quad (13)$$

Using Least Angle Regression(LARS) in dictionary training, α is obtained by,

$$\alpha = \arg\min_{\alpha \in R^n} \frac{1}{2} \left\| \tau_i - D_l \alpha \right\|_2^2 + \lambda \left\| \alpha \right\|_1 \quad (14)$$

Where λ is to enforce sparsity and favor solutions with slightly higher RMS error but greater sparsity in representation. Since the patches that are being represented sparsely in this step were likely not in the training set, λ is set to 1/16 for training. The sparse coefficient α obtained above is convolved with the high resolution dictionary D_h to obtain the super resolved image.

2. Obtaining Enhanced Patches

The super resolution of the low resolution image is obtained by enhancing the patches. So far we have trained a dictionary and found the sparse vector that recover image from the patches. Then convolve the high resolution dictionary D_h with the sparse vector α, to get high frequency only version patch set τ_{hf}. The patch set τ_{hf} consists only the high frequency patches, where the low frequency patches will be absent.

Iterating over each patch τ_i in γ_{prgb}, converting it to an improved patch τ_h, creates a new set of enhanced patches γ_h. Adding this improved patch with the original patch τ_i with the high frequency patches τ_{hf} creates a set γ_{hp}. Finally, the super resolved image from the set of enhanced patches γ_{hp} is retrieved by averaging in areas of overlap to create a final image. However, since all patches present in γ_{ip} are not present in γ_{prgb} due to pruning, we use patches from γ_{ip} whenever they are absent from γ_{hp} in the overlapping addition.

IV. RESULTS AND DISCUSSION

In this section, the experimental results are obtained by the proposed algorithm and various existing algorithms. The simulation is done using commercial computer system with the specification of Intel core™ i5-3210M @ 2.50 Ghz, installed memory of 4.00 GB RAM and 64-bit Microsoft windows OS processor installed with MATLAB R 2011a. Performance metrics used for comparing the methods are PSNR and training speed of dictionary.

We now compare the above algorithm with Yang et al. [9] and Elad et al [8] and show performance improvements both in PSNR and dictionary training speed. In this method, the general images are used for experiment.

Initially, the input image is down sampled to half size using Bicubic interpolation. Then the image rescaled is to 2x in the test set using various algorithms. The tests were performed using the latest version of Yang's code from [9], Elad's code from [8], and an implementation of the proposed algorithm in MATLAB using [11] for sparse minimization.

Table I summarizes the PSNR comparison of various algorithm. The proposed method shows the better performance in PSNR than the other algorithms. This makes the image suitable for applications that needs high frequency information without spending high cost in sensors.

It is noted that the proposed algorithm has more advantages and efficient than other algorithms, they are

- The proposed method uses non- overlapping patches from the image, thus the computational time of the method is decreased and pruning the patches gives some additional speed boost to the method.
- Gabor filter helps in processing only the features present in the images, therefore the memory required for processing image is less.

Table 1: PSNR Comparison (in dB) of the proposed algorithm with Bicubic, Yang et. al., Elad et. al.

Image	**Bicubic Algorithm**	**Yang Method**	**Elad Method**	**Proposed Method**
Lenna	**34.70**	**36.29**	**36.20**	**36.5**
Face	**34.83**	**35.62**	**35.56**	**35.61**
Baboon	**24.82**	**25.51**	**25.43**	**25.60**
Zebra	**30.63**	**33.12**	**33.23**	**33.58**
Foreman	**34.18**	**35.66**	**36.41**	**37.12**
Pepper	**34.82**	**36.24**	**36.42**	**36.75**
Barbara	**27.90**	**28.50**	**28.51**	**28.59**
Bridge	**26.58**	**27.54**	**27.53**	**27.79**
Flowers	**30.37**	**32.37**	**32.23**	**32.85**
Coastguard	**29.22**	**30.27**	**30.33**	**30.51**
Man	**29.25**	**30.50**	**30.42**	**30.79**
Fruits	**33.58**	**35.95**	**36.02**	**36.93**
comic	**26.02**	**27.80**	**27.64**	**28.25**

- The training is done real time using the ODL [10] algorithm by Mairal et al., which is faster than the algorithms used in [8, 9] and which can scale up the training patches to million in numbers.

V.CONCLUSION

In this paper, a new method for super resolution is proposed using group sparse representation. To form a dictionary K-SVD algorithm is used. With the help of dictionary, sparse coefficient is obtained. Sparse coefficient is formed by GOMP algorithm. Since, single image is used for dictionary formation, observing of the required data will be accurate compare to other methods. Computation time also very less. In sparse coefficient repetition of a value will not take place, so almost all HR data can be collected in sparse coefficient. When it is multiplied with dictionary all HR data will be obtained clearly.

SR approach based on sparse representation is applied for set of images called training set. The proposed SR technique involves the formation of a sparse dictionary. The HR result is obtained by sparse representation of input image using the dictionary. The dictionary learning phase involves the construction of LR and HR dictionaries from the LR and HR image patches respectively. The obtained dictionaries are jointly trained. The trained dictionaries help in efficient reconstruction of the SR image. Experimental result shows outcome of the proposed algorithm, for various images. Resulted values of PSNR have been improved from the existing algorithm.

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