A Study to Compute the Numerical Solution of Stochastic Models in Reliability Engineering

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Abstract - In this Paper , a mathematical solution by using numerical technique to stochastic partial differential equation in reliability theory is being explained. This method is based upon finite difference methods to resolve and emerge the transient state of Markovian system in reliability theory .In reliability engineering the repair rates and failure rates are difficult to pursue on the basis of different parameters and demand in reliability theory .We use finite difference method for understand and get better valuation as per data prediction. The method Lagrange's formula to interpolate the missing value of repair rates of the system which is required for analysis on the basis of different analytical techniques and computations. Results thus obtained are found to the efficient for studying the transient state behavior of the system.

Keywords: Markov models, Stochastic Process,Transient Solution, Finite Difference Method ,Lagrange's Method, Differential Equations.

I. INTRODUCTION

The complexity in planning, designing and operation of the systems in the fast growing scenario of today's world make the professional engineers and system managers more and more responsible. The failure of these can often cause effects which range from inconvenience and irritation to a severe impact on society and on its environment. The people expect that the products and systems they purchase should be reliable and safe. A question, which arises, is 'how reliable or how safe will the system be during its future operating life'? This question can be answered, in part, by the use of quantitative reliability appears to have had its inception during the Second world war, and continue today, required by the size and complexity of modern systems.

The need of developing systems that are relatively less prone to failure has always engaged engineers. On the other hand their reliability is also a question of concern. In the past, however, this reliability generally has been achieved from the subjective and qualitative experience of design and operating engineers. The skeptics of modern reliability evaluation techniques often refer to this method of assessing the reliability of a system as engineering judgment. It is a fallacy, however, to suggest that engineering judgment is displaced by quantitative reliability evaluation since as much, if not more, engineering judgment is required in its use. In addition to providing a set of numerical indices, reliability evaluation can be used to indicate how a system may fail, the consequences of failures and also to provide information to the enable engineers and system managers to relate the quality of their system to economics and capital investment, In so doing it can lead to better and more economical designs, and a much improved knowledge of the operation and behavior of a system.

So, Reliability is defined as the probability that a component, device, equipment or system will satisfactorily perform its intended function for a specified period of time under specified operating conditions. Let X(t) be a binary vitiate which take two value 1 and 0 respectively when the system operates and does not operate at time t. Then reliability of the system at time ‘t’ is given by

$$R(t)=P[x(u)=1, \quad 0 \le u \le t \mid X(0)=1] \qquad (1)$$

Mathematically , if t is the life time of the system , then the reliability of the system at time ‘t’ is given by

$$R(t) = \int_t^{\infty} f(t)\,dt = 1-F(t) \qquad (2)$$

Where f(.) and F(.) are probability density function and cumulative distribution function of the random variable T.

Today the reliability evaluation techniques have wide applications in domestic appliances, automobiles, communication economic effect when they fail. Thus it is evident that engineers and mathematicians should have some awareness of the basic concepts associated with the application of reliability theory.

II. PROBLEM DEFINITION

The world reliable and reliability are used in social, political, business and technical field for expressing a faith/trust in a person, firm or equipment. The reliability analysis of an industry can help the management in taking timely decision for its smooth functioning. In 1960, first text book on reliability by Dummer and Griffen appeared in literature. Since then a number of research papers have been published in the field of reliability (1960, 1963, 1964, 1969, 1978). Professor R. S. Verma and his students Mohan, Garg and Sangal (1962) at Delhi University initiated the study of reliability technology. Singh (1976) first time used reliability technology to analyse the working of production system. Dhillon and Singh (1981) discussed the basic theory of reliability in their book entitled “Engineering reliability-new technique and applications”. The availability of working systems used in daily life was calculated by Singh and Aggarwal (1979). He along with his co-workers studied the behavior of process industries. Yang and Dhillon (1996) calculated the availability of a robot with safety system. The need and application of reliability technology in the process industry was discussed by Michelsen (1998). Kumar et al. (1989, 1990, and 1992) calculated the availability for number of systems in process industries. Gupta et al. (2005) discussed the reliability and availability analysis of serial processes of butter oil plant and behavior analysis of the cement industry. Agnihotri et al. (2008) have studied the reliability analysis of boiler used in readymade garment industry.

III. AVAILABILITY ANALYSIS METHOD USING MARKOV MODEL AND SUPPLIMENTARY VARIABLE TECHNIQUE

In order to find the availability of a system one has to form a system of linear differential equations using mnemonic rule. This rule states that the derivative of the probability of every state is equal to the sum of all probability flows which come from other states to the given state minus the sum of all probability flows which go out from the given state to the other states. The differential equations thus derived are known as the Chapman-Kolmogorov differential equations.

The calculation of the availability of a system with elements exhibiting dependent failures and involving repair or standby operation is, in general, complicated and several approaches have been suggested to carry out the computations. In case the failure and repair rates are variable then it loses its markovian property. By Introducing supplementary variables, the non marlovian character of the system is changed to makovian. Any Markov model is defined by a set of probabilities Pij which define the probability of transition from any state ‘i’ to another ‘j’. This transition probability depends only on states ‘i’ and ‘j, and is independent of all previous states except the last one, i.e., state ‘i’.

The failure rate $\lambda(y)$ and the repair rate $\beta(x)$ are variable. In order to illustrate the availability of the system, let us consider a system consisting of a single repairable component „A‟ with a variable failure and repair rate $\beta(x)$ and respectively.

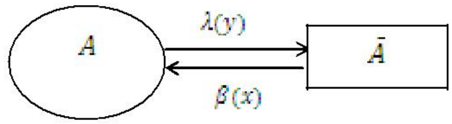


Fig. : Transition diagram of two components

For this system, the possible states are:

(i) State A(t) where the system is in state A at time t

(ii) State $\bar{A}(x,y,t)$ where the system is in state $\bar{A}$ at time t and has an elapsed failure time 'y' and elapsed repair time 'x'.

If the system remains in state A there can be two possibilities as:

(i) That system is in state ' A ' at time *t* and no failure occurs in the interval (t + . Probability of such a state is given by P_A(t)(1- **).**

(ii) That system is in state Δ at time *t* and the repair is carried out in the interval (t, t+Δt) Probability of such a state is given by $P_{\bar{A}}(x,y,t)(\beta(x)\Delta t)$. Following the mnemonic rule and transition diagram (fig.1). The resulting equation for the system in state A can be written as:

$$P_A(t+\Delta t) = (I-\lambda(y)\Delta t)\,P_A(t) + \int [(\beta(x).\Delta t).\,P_{\bar{A}}(x,y,t)]\,dx\,dy$$

$$\frac{P_A(t+\Delta t)-P_A(t)}{\Delta t} = -\lambda(y)P_A(t) + \int [(\beta(x))\,P_{\bar{A}}(x,y,t)]\,dx\,dy$$

taking limit as $\Delta t \rightarrow 0$ we have

$$\frac{d}{dt}P_A(t) = -\lambda(y)P_A(t) + H_0(t)$$

where $H_0(t) = \int [(\beta(x).\Delta t).\,P_{\bar{A}}(x,y,t)]\,dx\,dy$ (3)

Similarly, when the system is in state '$\bar{A}(x,y,t)$' the equation will be

$$\frac{P_{\bar{A}}(x,y,t+\Delta t)-P_{\bar{A}}(x,y,t)}{\Delta t} + \frac{P_{\bar{A}}(x+\Delta x,y,t)-P_{\bar{A}}(x,y,t)}{\Delta x} + \frac{P_{\bar{A}}(x,y+\Delta y,t)-P_{\bar{A}}(x,y,t)}{\Delta y} = -\beta(x)\,P_{\bar{A}}(x,y,t) + \lambda(y)\,P_A(t))$$

taking limit as $\Delta t \rightarrow 0, \Delta x \rightarrow 0, \Delta y \rightarrow 0$, we have

$$[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\frac{\partial}{\partial y}]P_{\bar{A}}(x,y,t) = -\beta(x)\,P_{\bar{A}}(x,y,t) + \lambda(y)\,P_A(t)) \quad (4)$$

Initial Conditions: As elapsed failure and repair time are zero initially and the system is completely in working state, the initial conditions thus becomes:

$P_{\bar{A}}(x, y, 0) = 0$ (5)

$P_A(0)=0$ (6)

Boundary condition: Since a system is in the failed state with failure rate $\lambda(y)$ but repair has not been done at that time, so the boundary condition is:

$P_{\bar{A}}(0, y, t) = \lambda(y) P_A(t)$ (7)

IV. SOLUTION OF EQUATIONS

Equation (3) is a first order ordinary differential equation and equation (4) is a linear partial differential equation which constitutes a Chapman Kolmogorov differential equations. In order to find the reliability of the system ,the governing equations (3-4) will be solved along with the initial and boundary conditions. Equation (3) is ordinary differential equation, so can be integrated directly whereas the equation (4) is a partial differential equation which can be solved by using Lagrange's method and thus we have

$$P_A(t) = e^{-\int \lambda(y) dt}\left[1 + \int e^{-\int \lambda(y) dt} H_0(t)\, dt\right] \quad (6)$$

$$P_{\bar{A}}(x, y, t) = e^{-\int \beta(x) dx}[\lambda(y - x) P_A(t - x) + \int \lambda(y) P_A(t) e^{\int \beta(x) dx} dx] \quad (7)$$

The time dependent Availability A(t) of the system is next computed as:

$$A(t) = P_A(t) \quad (8)$$

Special Case: When all the transition rates, that is failure and repair are constant then $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$ =0 Consequently equation (3-4) reduce to the ordinary linear differential equation:

$$\frac{d}{dt} P_A(t) = -\lambda P_A(t) + H_0(t) \quad (9)$$

$$[\frac{d}{dt} + \beta] P_{\bar{A}}(t) = \lambda P_A(t) \quad (10)$$

The differential equation (9) – (10) can be solved analytically by using Laplace transformation method, matrix calculus or numerically following the approach of Gupta (2003) by using the initial conditions:

$P_A(t)$ = 1and $P_{\bar{A}}(t) = 0$

Probability considerations give the following system of differential difference equation associated with the state transition of these systems at time (t+Δt).We first devolep the equation in transient zeroth state by using mnemonic ruleas under

$$P_0(t+\Delta t) = [1-\alpha_1(y)\Delta t-\alpha_2(y)\Delta t-\alpha_3(y)\Delta t-\alpha_4(y)\Delta t]P_0(t) + \int \mu(x)P_1(x,y,t)\,dx\,dy\Delta t + \int \sigma(x)P_2(x,y,t)\,dxdy\Delta t + \int \phi(x)P_4(x,y,t)\,dxdy\,\Delta t + \int \psi(x)P_5(x,y,t)\,dxdy\,\Delta t$$

$$P_0(t+\Delta t)-P_0(t) = -[\alpha_1(y)\Delta t+\alpha_2(y)\Delta t+\alpha_3(y)\Delta t+\alpha_4(y)\Delta t]P_0(t) + \int \mu(x)P_1(x,y,t)\,dxdy\,\Delta t + \int \sigma(x)P_2(x,y,t)\,dxdy\Delta t + \int \phi(x)P_4(x,y,t)\,dx\,\Delta t + \int \psi(x)P_5(x,y,t)\,dxdy\,\Delta t$$

dividing both sides by Δt, we get

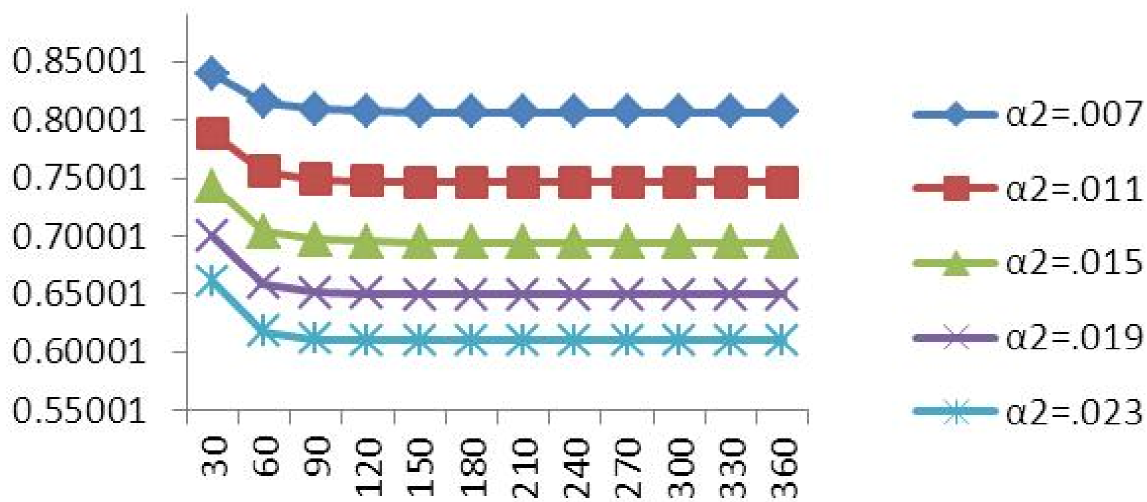
$$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} = -[\alpha_1(y)+\alpha_2(y)+\alpha_3(y)+\alpha_4(y)]P_0(t) + \int \mu(x)P_1(x,y,t)dxdy + \int \sigma(x)P_2(x,y,t)\,dxdy + \int \phi(x)P_4(x,y,t)dx\,dy + \int \psi(x)P_5(x,y,t)dxdy$$

On solving the differential equations together with initial and boundary conditions we can find all the probabilities in terms of $P_0(t)$ and then time dependent availability A(t) can be computed shown in table below fror any one variable say α_2

α_2 Time	.007	.011	.015	.019	.023
30	0.8394	0.7889	0.7427	0.7004	0.6616
60	0.8148	0.7559	0.7041	0.6585	0.6180
90	0.8087	0.7486	0.6965	0.6511	0.6112
120	0.8072	0.7469	0.6950	0.6498	0.6101
150	0.8068	0.7466	0.6947	0.6496	0.6099
180	0.8067	0.7465	0.6946	0.6495	0.6099
210	0.8067	0.7465	0.6946	0.6495	0.6099
240	0.8067	0.7465	0.6946	0.6495	0.6099
270	0.8067	0.7465	0.6946	0.6495	0.6099
300	0.8067	0.7465	0.6946	0.6495	0.60993
330	0.8067	0.7465	0.6946	0.6495	0.6099
360	0.8067	0.7465	0.6946	0.6495	0.6099
MTBF	285.69	263.50	245.35	232.01	220.46

As a special case we shall now discuss how to develop Chapman Kolmogorov differential equation in transient as well as steady states when both failure and repair rates are constant.

Effect of failure rate (α_2) of the subsystem Extruder B on availability



V. CONCLUSION

We have calculated the availability by assuming the subsystem cutter and die fail simultaneously .In this assumptions the subsystem extruder comes out as the sensitive subsystem,in fact ,these assumptions are provided by industry itself. Therefore , a more criticl analysis of this industry can further be carried out by assuming other subsystem being in failed state simultaneously and independently .

The reliability management of complex industrials system is highly sensitive issues for modeling and evaluating the performance of the system, especially during strategic maintenance planning. Rigorous efforts have been made by researchers to evolve methods to study the effect of subsystem conditions and maintenance policies on system performance. These methods involve complex computations and grow tremendously with further growth in number of subsystems.

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