An overview of Fractional order PID Controllers and its Industrial applications

A.Sandhya
Department of Mathematics
V.R.Siddhartha Engineering College
Vijayawada, Andhra Pradesh, INDIA

R.Sandhya
Department of Mathematics
V.R.Siddhartha Engineering College
Vijayawada, Andhra Pradesh, INDIA

M.Prameela
Department of Mathematics
P.V.P.Siddhartha institute of technology
Vijayawada, Andhra Pradesh, INDIA

Abstract: In this paper we introduce the different types of the Fractional Order PID controllers and presented some of its application to Industry and robotics

Key-Words: Fractional Order Calculus, Differintegration, Fractional Order Controllers, Control Theory, Control Systems, PID controller, tuning, auto-tuning.

I. INTRODUCTION

"In-between" thinking, e.g., Between integers there are non-integers, Between logic 0 and logic 1, there is the fuzzy logic, Between integer order splines, there are "fractional order splines", Between integer high order moments, there are non-integer order moments, Between "integer dimensions", there are fractal dimensions.

The Fractional Order Calculus (FOC) constitutes the branch of mathematics dealing with differentiation and integration under an arbitrary order of the operation, i.e. the order can be any real or even complex number, not only the integer one [1], [2], [3]. Although the FOC represents more than 300-year-old issue [4], [5], its great consequences in contemporary theoretical research and real world applications have been widely discussed relatively recently. The idea of non-integer derivative was mentioned for the first time probably in a letter from Leibniz to L'Hospital in 1695. Later on, the pioneering works related to FOC have elaborated by personalities such as Euler, Fourier, Abel, Liouville or Riemann. The interested reader can find the more detailed historical background of the FOC e.g. in [1].

According to [4], [6], the reason why FOC remained practically unexplored for engineering applications and why only pure mathematics was "privileged" to deal with it for so long time can be seen in multiple definitions of FOC, missing simple geometrical interpretation, absence of solution methods for fractional order differential equations and seeming adequateness of the Integer Order Calculus (IOC) for majority of problems. However, nowadays the situation is going better and the FOC provides efficient tool for many issues related to fractal dimension, "infinite memory", chaotic behaviour, etc. Thus, the FOC has already came in useful in engineering areas such as bioengineering, viscoelasticity, electronics, robotics, control theory and signal processing [6]. Several control applications are available e.g. in [7], [8], [9].

II. BASIC CONCEPTS OF FRACTIONAL ORDER CALCULUS

Several applications of fractional calculus can be found the area of control systems. Fractional calculus allows the derivatives and integrals to be of any real number. The fractional-order differentiator can be denoted by a general fundamental operator ${}_aD_t^q$ as a generalization of the differential and integral operators, which is defined as follows

$$ {}_aD_t^q = \begin{cases} \frac{d^\alpha}{dt^\alpha} & Re\ \alpha > 0 \\ 1 & Re\ \alpha = 0 \\ \int_a^t (d\tau)^{-\alpha} & Re\ \alpha < 0 \end{cases} $$

Where α is the fractional order which can be a complex number, the constant 'a' is related to the initial conditions. There are two commonly used definitions for the general fractional differentiation and integration, i.e., the Grünwald–Letnikov (GL) and the Riemann Liouville (RL) definitions (Oldham 1974).

The GL definition is given below:

$$ {}_aD_t^\alpha f(t) = \lim_{h\to 0} h^{-\alpha} = \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^j \binom{\alpha}{j} f(t-jh) \text{ Where } \left[\frac{t-a}{h}\right] \text{is an integer} $$

While the RL definition is given by:

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)d\tau}{(t-\tau)^{(\alpha-n+1)}} (n-1 \leq \alpha < n) $$

Where n is an integer and α is real a number. $\Gamma(x)$ is the well-known Euler's Gamma function. Also, there is another definition of fractional differ integral introduced by (Caputo 1967).

Caputo's definition can be written as:

$$ {}_a^cD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)d\tau}{(t-\tau)^{(\alpha-n+1)}} (n-1 \leq \alpha < n) $$

Fractional order differential equations are at least as stable as their integer order counterparts. This is because systems with memory are typically more stable than their memory-less alternatives.

III. APPLICATIONS OF FRACTIONAL CALCULUS IN CONTROL

Classification of dynamic systems according to the order of the plant and the controller can be done as:

i) IO (integer order) plant with IO controller

ii) IO plant with FO (fractional order) controller (*Example1*)

iii) FO plant with IO controller (*Example 2*)

iv) FO plant with FO controller.

From engineering point of view, doing something better is the major concern. This review article will show two examples that the best fractional order controller outperforms the best integer order controller. Then, we try to argue why consider fractional order control even when integer (high) order control works comparatively well

EXAMPLE-1: FO CONTROLLER FOR IO PLANT

In this Example, we focus on using FO-PID controller for an IO plant - “DC-Motor with elastic shaft”, a benchmark system from (Mathworks Inc., 2006). Detailed results can be found in (Xue et al., 2006).

a. Best IO PID vs. Best FO PID

We used constrained optimization routine to search for the best controller parameters. Two optimization criteria are used. One is ITAE (integral of time-weighted absolute error) and another one is ISE (integral of squared error), where the constraint is the maximum torque less than Nm. The reference signal is the unit step function.

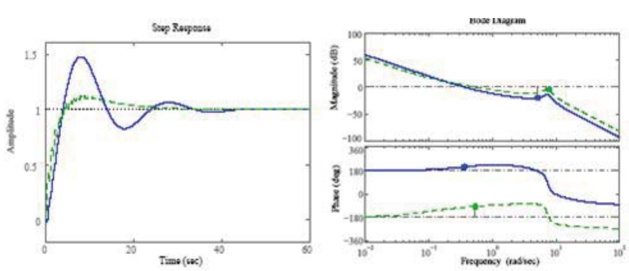
For the optimally searched IO PID using ITAE,

$G_{c1}(s) = 41.94 + \frac{21.13}{s} - 8.26s;$

For optimally searched IO PID using ISE,

$G_{c2}(s) = 110.09 + \frac{10.65}{s} + 30.97s$

Fig. 1 shows the responses to unit step of the angular position controlled by two integer order PID controllers $G_{c1}(s)$ and $G_{c2}(s)$, respectively with the Bode plots of the open-loop controlled system shown in the same figure.



(a) Step responses (b) Bode plots

Fig. 1. Best IO PID Controllers. Solid line: ITAE; Broken line: ISE

Now let us look at the best FO PID controllers. As the first attempt, let us first fix α = 0.5 and μ = 0.6. Doing the numerical search, we get the best ITAE of 2.22 and the corresponding fractional order PID controller is

$G_{c3}(s) = 135.12 + \frac{0.01}{s^{0.7}} - 31.6s^{0.6};$

For optimally searched IO PID using ISE,

$G_{c}(s) = 61.57 + \frac{91.95}{s^{0.5}} + 2.33\, s^{0.6}$

The step responses are compared in Fig. 2 with corresponding Bode plots.

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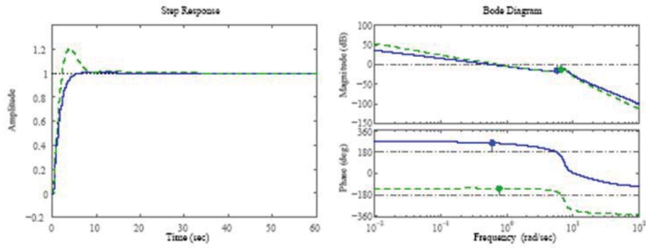

(a) Step responses (b) Bode plots

Fig. 2. Best FO PID Controllers. Solid line: ITAE; Broken line: ISE

The observation is clear. The best FO PID performs better than the best IO PID. This is not surprising but this may not be fair since FO PID has two more extra parameters in optimal search.

EXAMPLE-2: FO CONTROL FOR FO PLANT

In this section, we consider a class of evolution systems described by the one-dimensional time fractional wave equation subject to a fractional order boundary controller. Via hybrid symbolic and numerical simulation and parameter optimization, we confirmed that the fractional order boundary controller not only can stabilize the fractional wave equation, but performs better than an integer order boundary controller as well. Detailed results can be found in (Liang et al., 2004b, 2005).

a.Problem Formulation

We consider a cable made with special smart materials governed by the fractional wave equation, fixed at one end, and stabilized by a boundary controller at the other end. Omitting the mass of the cable, the system can be represented by

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} = \frac{\partial^{2} u}{\partial x^{\alpha}}, \quad 1 < \alpha \leq 2,\ x \in [0,1],\ t \geq 0 \qquad (1)$$

$$u(0,t) = 0, \qquad (2)$$

$$u_x(1,t) = f(t), \qquad (3)$$

$$u(x,0) = u_0(x), \qquad (4)$$

$$u_t(x,0) = v_0(x), \qquad (5)$$

Where u(x, t) is the displacement of the cable at $x \in [0, 1]$ and $t \geq 0$,
f (t) is the boundary control force at the free end of the cable,
$u_0(x)$ and $v_0(x)$ are the initial conditions of displacement and velocity, respectively.

The control objective is to stabilize u(x, t), given the initial conditions (4) and (5).We adopt the following definition for the fractional derivative of order α of function f (t) (see(Mainardi and Paradisi, 1997) and (Mainardi,1996)),

$$\frac{d^{\alpha}}{dt^{\alpha}} f(t) = \begin{cases} f^{(n)}(t) \quad if \ \alpha = n \in N, \\ \frac{t^{n-\alpha-1}}{\Gamma(n-\alpha)} * f^{(n)}(t) \quad if \ n-1 < \alpha < n \end{cases} \qquad (6)$$

where the * denotes the time convolution between two functions.

In this paper, we study the performance and properties of controllers in the following format:

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$$f(t) = -k \frac{d^{\mu}u(1,t)}{dt^{\mu}}, 0 < \mu \leq 1 \quad (7)$$

where k is the controller gain, μ is the order of fractional derivative of the displacement at the free end of the cable. When $\mu = 1$, the controller (7) is called integer order controller and has been widely used in the boundary control of wave equations and beam equations (see (Chen, 1979), (Conrad and Morg¨ul,1998), and (Chen et al., 1987)). The effectiveness has also been verified when applied to the boundary control of fractional wave equation in (Liang et al., 2004a). When $0 < \mu < 1$, can controller (7) stabilize the system? What advantages does a fractional order controller have over integer order controllers?

b. Performance Comparison

To test if fractional order boundary controllers can be used to stabilize the fractional wave equation, the following three different systems were simulated

- Case 1: $\alpha = 1.1, k = 0.1, \mu = 0.5$,
- Case 2: $\alpha = 1.5, k = 0.1, \mu = 0.7$,
- Case 3: $\alpha = 1.9, k = 0.2, \mu = 0.9$.

All cases have the same initial conditions

$u_0(x) = -\sin(0.5\pi x), v_0(x) = 0.$ (8)

For integer order boundary controllers ($\mu = 1$),we seek the best gain k to

$\min_k J(k)$ = max abs (u (1, t)), $t \in [t_f - T, t_f]$ Subject to: $k > 0$.

For fractional order boundary controllers ($0 < \mu \leq 1$), the task to find the best gain k and the fractional order μ to

$\min_{k,\mu} J(k, \mu)$ = max (u (1, t)), $t \in [t_f - T, t_f]$ Subject to: $k > 0$ and $0 < \mu \leq 1$. In the above optimization tasks, u(1, t) is the displacement of the free end of the cable; t_f is the total time of simulation; T is the time period to optimize within the time interval $[t_f - T, t_f]$ which is determined by trial-and-error. The optimization program we chose is SolvOpt (see (Kuntsevich and Kappel, 1997)), a free program for local nonlinear optimization problems.

In fractional order controller given by,

$G_{c(s)} = K_p + T_i s^{-\lambda} + T_d s^{\mu}$

(where K_p, T_i and T_d are proportional, integral and derivative gains respectively) more parameters need to be tuned. It's unfair but, theoretically, always better than integer order controller.

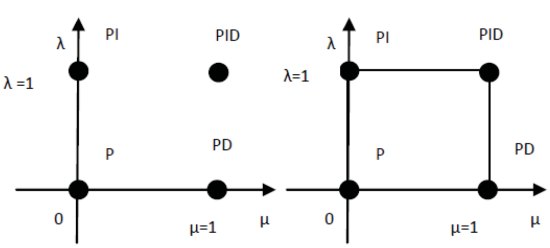

Fig.3 $PI^{\lambda}D^{\mu}$ controller: from points to plane

Many control objects are fractional-order ones, so that the fractional approach for control of the fractional-order systems becomes a meaningful work. This approach (as shown in fig.3) has changed the point based control scheme to plane based scheme. Achieving something better is always the major concern from control engineering point of view. Existing evidences have confirmed that the best fractional order controller outperforms the best integer order controller. It has also been answered in the literature why to consider fractional order control even when integer order control works comparatively quite well [11]. Since integer-order PID control dominates the industry, it can be believed that fractional order-PID control will gain increasing impact and wide acceptance.

IV. STABILITY OF FRACTIONAL ORDER SYSTEMS

Obviously, the stability is the very fundamental and critical requirement during control system design. It is widely known that an integer order continuous-time linear time-invariant system is stable if and only if all roots of its characteristic polynomial have negative real parts. In other words, the roots must lie in the left half of the complex plane. Investigation of stability of the fractional order systems represents the more complicated issue [6], [12].

For example, the stability of commensurate fractional order systems can by analysed via the theorem of Matington [12] or the definition from [6], which describes the way of mapping the poles from s^{α} plane into the *w*-plane. An interesting result is that the poles of the stable fractional order system can be located even in the right half of such complex plane. This is illustrated in Fig. 4 where the stability region for a commensurate fractional order linear time-invariant system with order $0 < \alpha < 1$ is depicted [5], [6].

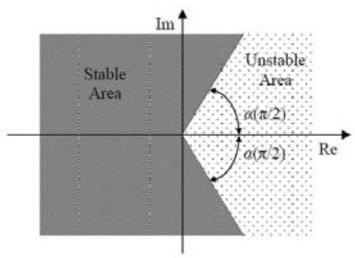

Fig. 4: Region of stability for the commensurate fractional order system with $0 < \alpha < 1$

V. FRACTIONAL ORDER CONTROLLERS (FOC)

The fractional order PID (FOPID) controller is the expansion of the conventional PID controller based on fractional calculus. For many decades, proportional - integral - derivative (PID) controllers have been very popular in industries for process control applications. Their merit consists in simplicity of design and good performance, such as low percentage overshoot and small settling time (which is essential for slow industrial processes). Owing to the paramount importance of PID controllers, continuous efforts are being made to improve their quality and robustness. In the field of automatic control, the fractional order controllers which are the generalization of classical integer order controllers would lead to more precise and robust control performances. Though it is reasonably true, that the fractional order models require the fractional order controllers to achieve the best performance, in most cases the fractional order controllers are applied to regular linear or nonlinear dynamics to enhance the system control performances. Historically there are four major types of fractional order controllers: (Xue and Chen, 2002)

1. CRONE Controller
2. Tilted Proportional and Integral (TID) Controller
3. Fractional Order PI D Controller
4. Fractional Lead-Lag Compensator

5.1 CRONE Controller

CRONE is a French acronym for fractional order robust control. By the use of these controllers it is possible to ensure almost constant closed loop characteristics and ensure small variation of the closed loop system

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stability degree in spite of the plant perturbation and uncertainty in model parameters. It has a frequency domain design methodology employing fractional differentiation. It is possible to control minimum and non-minimum plants, unstable, time varying and non-linear plants with this controller. There are 3 generations of CRONE control successively extending the application fields. Some applications of these controllers have been in the domain of flexible transmission, car suspension control, hydraulic actuator (oustaloup et al. 2006.)

5.2 Tilted Proportional and Integral (TID) Controller

The object of TID is to provide an improved feedback loop compensator having the advantages of the conventional PID compensator, but providing a response which is closer to the theoretically optimal response. In TID scheme the proportional compensating unit is replaced with a compensator having a transfer function characterized by $1/s^{1/n}$ or $s^{-1/n}$. This compensator is herein referred to as a "Tilt" compensator, as it provides a feedback gain as a function of frequency which is tilted or shaped with respect to the gain/frequency of a conventional or positional compensation unit. The entire compensator is herein referred to as a Tilt-Integral-Derivative

(TID) compensator. For the Tilt compensator, n is a nonzero real number, preferably between 2 and 3. Thus, unlike the conventional PID controller, wherein exponent coefficients of the transfer functions of the elements of the compensator are either 0 or -1 or +1, TID scheme exploits an exponent coefficient of -1/n. By replacing the conventional proportional compensator with the tilt compensator of the invention, an overall response is achieved which is closer to the theoretical optimal response determined by Bode.

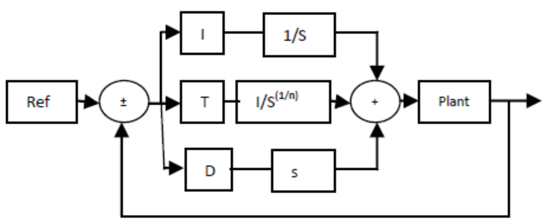

Fig. 5. Tilt-integral-derivative controller

5.3 Fractional Order PI D Controller

$PI^{\lambda}D^{\mu}$ controller, also known as $PI^{\lambda}D^{\delta}$ controller was studied in time domain as well as in frequency domain. In general form, the transfer function of $PI^{\lambda}D^{\delta}$ is given by

$$C(s) = \frac{U(s)}{E(s)} = k_p + k_i s^{-\lambda} + k_d s^{\delta} \quad (9)$$

Where λ and δ are positive real numbers, k_p is the proportional gain, i is the integration constant and d is the differentiation constant. Clearly, taking $\lambda = 1$ and $\delta = 1$, we obtain a classical PIDcontroller. If $\lambda = 0$ ($k_i = 0$) we obtain a PD^{δ} controller, etc. All these types of controllers are particular cases of the $PI^{\lambda}D^{\delta}$ controller. It can be expected that $PI^{\lambda}D^{\delta}$ controller may enhance the systems control performance due to more tuning knobs introduced. Actually, in theory, $PI^{\lambda}D^{\delta}$ itself is an infinite dimensional linear filter due to the fractional order in differentiator or integrator.

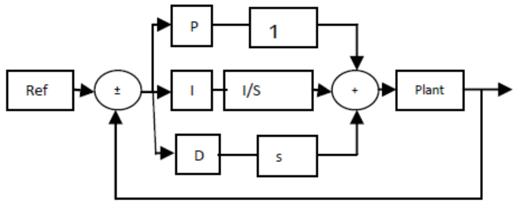

Fig.6. FO-PID ($PI^{\lambda}D^{\mu}$) controller (where, $0 \le \lambda \le 1$ & $0 \le \mu \le 1$)

5.4 Fractional Lead-Lag Compensator

Lead compensators are mostly used to stabilize marginally stable systems. Lag compensators are mostly used to reduce the magnitude of the high frequency loop gain of the system. The use of fractional order elements in this lag-lead compensator gives greater flexibility to designer, to shape the loop frequency responses since the order of the filter can take any real value instead of only integer values the transfer function of a generic FO lead-lag compensator is given by (Monje et al. 2004)

$$C(s) = K_c\left(\frac{s+\frac{1}{\lambda}}{s+\frac{1}{x\lambda}}\right)^{\alpha} = K_c\, x^{\alpha}\left(\frac{\lambda s+1}{x\lambda s+1}\right)^{\alpha}, 0 < x < 1$$

VI. INDUSTRIAL APPLICATIONS OF FRACTIONAL-ORDER $PI^{\lambda}D^{\delta}$ CONTROLLERS:

FRACTIONAL-ORDER $PI^{\lambda}D^{\delta}$ CONTROLLERS

1. INTRODUCTION

Podlubny (1999) proposed a generalization of the PID controller, namely the $PI^{\lambda}D^{\delta}$ controller, involving an integrator of order λ and a differentiator of order δ He demonstrated the better response of this type of controller, in comparison with the classical PID controller, when used for the control of fractional- order systems. A frequency domain approach using fractional PID controllers is also studied in Vinagre et al. (2000a). Further research activities are underway to define new effective tuning techniques for non-integer-order controllers by an extension of the classical *PID* control theory (see Monje et al., 2005 for additional references).

2. TUNING BY ROBUSTNESS DESIGN SPECIFICATIONS

In this section several interesting design specifications are described for the fractional controlled system. The objective is to obtain a controlled system robust to plant uncertainties. Other considerations include noise and disturbance rejection. In this work the specifications considered are:

Steady-state error cancellation. The fractional integrator $s^{(-\lambda)}$ is, for steady-state error cancellation, as efficient as an integer-order integrator. So, this specification is fulfilled with the introduction of the fractional integrator.

Phase margin (ϕ_m) and gain crossover frequency (ω_{cg}) specifications. Gain and phase margins have always served as important measures of robustness. It is known from classical control that the phase margin is related to the damping of the system and therefore can also serve as a performance measure. The equations relating the phase margin and gain crossover frequency are:

$$Arg(C(j\,\omega_{cg})G(j\,\omega_{cg})) = -\pi + \phi_m \quad (10)$$

$$|C(j\,\omega_{cg})G(j\,\omega_{cg})|_{dB} = 0dB \quad (11)$$

Gain margin (g_m) and phase crossover frequency (ω_{cp}) specifications. The gain margin can be taken as one of the main indicators as far as robustness is concerned. The next relation is given for the gain margin and phase crossover frequency:

$$\frac{1}{|C(j\omega_{cp})G(j\omega_{cp})|} = g_m \quad (12)$$

*Robustness to variations in the gain of the plant***.** To this respect, the next constraint can be considered (see Chen et al., 2003):

$$\left(\frac{d(\phi_{F(s)})}{d\omega}\right)_{\omega=\omega cg} \quad (13)$$

where F(s) is the open loop system. With this condition we force the phase to be flat at ω_{cg} and so, to be almost constant within an interval around ω_{cg} .It means that the system is more robust to gain changes and the overshoot of the response is almost constant within the interval.

*High frequency noise rejection***.** To ensure good measurement noise rejection:

$$|T(j\omega) = \frac{(C(j\omega)G(j\omega))}{(1+C(j\omega)G(j\omega))}|_{dB} \leq AdB \text{ for } \omega \geq \omega_t \text{ rad/s}$$

$$\Rightarrow |T(j\omega_t)|_{dB} = AdB \quad (14)$$

where A is the desired noise attenuation for frequencies $\omega \geq \omega_t$ rad/s

*Good output disturbance rejection***.** A constraint on the sensitivity function can be defined:

$$|S(j\omega) = \frac{1}{(1+C(j\omega)G(j\omega))}|_{dB} \leq BdB \text{ for } \omega \geq \omega_s \text{ rad/s}$$

$$\Rightarrow |s(j\omega_s)|_{dB} = BdB \quad (15)$$

with B the desired value of the sensitivity function for frequencies $\omega \leq \omega_s$ rad/s (desired frequency range).

*Limitation of control effort***.** Making use of the sensitivity function, to fulfil this constraint the next condition must be considered:

$$|CS(j\omega) = \frac{(C(j\omega))}{(1+C(j\omega)G(j\omega))}|_{dB} \leq RdB \text{ for } \omega \geq \omega_{cg} \text{ rad/s}$$

$$\Rightarrow |CS(j\omega_{cg})|_{dB} = RdB \quad (16)$$

with R the desired limitation for the control effort.

A tuning method for a fractional-order controller is presented here, so that some of the specifications commented above can be fulfilled. For the fractional $PI^{\lambda}D^{\delta}$ controller up to five design specifications can be met, since there are five parameters to tune: k_p, k_d ,k_i, λ and δ. A set of five nonlinear equations (corresponding to the specifications) with five unknown variables (parameters of the controller) has to be solved in this case. For fractional order controllers such as a PI^{λ} or a PD^{δ} three design specifications could be met (one for each parameter). That is, we can take advantage of the fractional orders λ and δ to fulfil additional specifications or requirements for the controlled system.

3. AUTO-TUNING

.3.1. Initial Considerations for Auto-auning of Fractional Controllers

As commented earlier, one of the purposes of this work is to present the auto-tuning problem for fractional-order controllers. The final aim is to find a method to auto-tune a fractionalorder $PI^{\lambda}D^{\delta}$ controller, formulated as:

$$C(s) = K_C x^{\delta} \left(\frac{\lambda_1 s+1}{s}\right)^{\lambda} \left(\frac{\lambda_2 s+1}{x\lambda_2 s+1}\right)^{\delta} \quad (17)$$

As can be observed, this controller has two different parts given by equations (18) and (19).

$$PI^{\lambda}(s) = \left(\frac{\lambda_1 s+1}{s}\right)^{\lambda} \quad (18)$$

$$PD^{\delta}(s) = K_C x^{\delta}\left(\frac{\lambda_2 s+1}{x\lambda_2 s+1}\right)^{\delta} \quad (19)$$

Equation (18) corresponds to a fractional-order PI^{λ} controller and equation (19) to a fractional-order lead compensator that can be identified as a PD^{δ} controller plus a noise filter.

Some considerations have to be taken into account concerning the auto-tuning method for this fractional-order structure:

a. The simplicity of the auto-tuning method is an important goal to achieve, since it is the aim to implement it for industrial applications by using, for instance, a PLC (Programmable Logic Controller) or a PC with a data acquisition board. That is, the tuning rules for the parameters of the fractional-order controller must be given by simple equations and computable within a sample time appropriate for the PLC or the PC with data acquisition board and for the plant.

b. It would be convenient to apply the relay test to obtain experimentally the information of the plant, due to the reliability of this method.

This considerations are taken into account in the auto-tuning method presented here, which besides aims the robustness to plant gain variations of the controlled system.

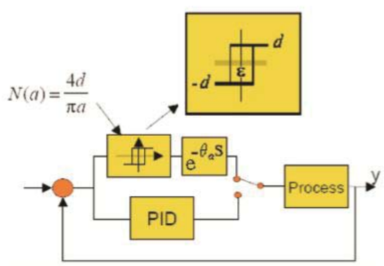

Fig .7. Relay Test for Auto-tuning.

3.2. Auto-tuning of Fractional-Order $PI^{\lambda}D^{\delta}$ Controllers

There are a wide variety of auto-tuning methods for integer controllers. Some of them aim in some way the robustness of the controlled system, for example, forcing the phase of the open loop system to be flat around the crossover frequency so that the system is robust to gain variations (see Chen et al., 2003). However, the complexity of the equations relating the parameters of the controller increases when some kinds of robustness constraints are required for the controlled system. The implementation of this type of auto-tuning method for industrial purposes is very complicate since; in general, industrial devices such as a PLC cannot solve sets of complex nonlinear equations. In this respect, the auto-tuning method

Proposed here allows flexible and direct selection of the parameters of the fractional-order controller, using the relay test and following a flat phase robustness criterion.

The relay test method consists on the use of relay feedback (see Fig 7). This way, processes with the dynamics typically encountered in process control will then exhibit limit cycle oscillations and the auto-tuner will identify one point on the Nyquist curve of the process from the experiment. When the operator decides to tune the controller, he simply presses a button starting a relay experiment. The relay feedback causes the output to oscillate with a controlled amplitude. The frequency of the limit cycle is approximately the ultimate frequency at which the process has a phase lag of 180°. The ratio of the amplitude of the limit cycle and the relay amplitude is approximately the process gain at that frequency. Thus, a point on the Nyquist curve of the open loop dynamics close to the ultimate point is determined.

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As commented earlier, the fractional-order $PI^{\lambda}D^{\delta}$ controller of equation (17) has two different parts given by the fractional-order PI^{λ} controller of equation (18) and the fractional order lead compensator of equation (19) that can be identified as a PD^{δ} controller plus a noise filter. In this method, after having obtained the necessary points in the frequency response of the plant to be controlled using the relay test (see Chen et al., 2003), the fractional order PI^{λ} controller of equation (18) will be used to cancel the slope of the phase of the plant at the gain crossover frequency ω_{cg}. This way we ensure a flat phase around the frequency of interest. Once the slope is cancelled, the PD^{δ} controller of equation (19) will be designed
to fulfil the design specifications of gain crossover frequency, ω_{cg}, and phase margin, ϕ_m, following a robustness criterion based on the flatness of the phase curve of this compensator (see Monje et al., 2004). The resulting phase of the open loop system will be the flattest possible around the crossover frequency ω_{cg}, ensuring the maximum robustness to plant gain variations.

4. IMPLEMENTATION CONSIDERATIONS

To select an auto-tuning method for industrial applications, we have taken into account that, in many cases, industrial control devices cannot solve complex equations, and so, we have used the relay test for this purpose. Having tuned the controllers, to implement them we have to take into account other considerations, such as memory size and computational load required by the algorithm, knowing that, in any case, the fractional orders must be approximated by integer ones. There are many different ways of finding such approximations (Valerio and Sá Da Costa, 2002; Vinagre et al., 2000b), but unfortunately it is not possible to say that any one of them is the best, because even though some of them are better than others
with regard to certain characteristics, the relative merits of each approximation depend on the differentiation order, on whether one is more interested in accurate frequency behaviour or in accurate time responses, on how large admissible transfer functions may be, and other factors like these. For the controller (9), where the fractional-order operators are explicit, the techniques mentioned can be used directly and the coefficients of the difference equations or the values of the circuit elements can be computed beforehand. However, the case of controller (17) and auto-tuning is different. On the one hand, we have to consider only techniques that can be implemented online, allowing a fully-automatic auto-tuning process .On the other hand, we will have to discard techniques requiring a high computational load
or complex operations, for example the identification based techniques. Another possibility for implementing the controllers is the use of specific microelectronic
devices, such as FPGAs, FPAAs and switched capacitors. There is a lot of work to be done on this field.

VII. APPLICATIONS IN ROBOTICS

In industrial environments the robots have to execute their task quickly and precisely, minimizing production time. It requires flexible robots working in large workspaces;

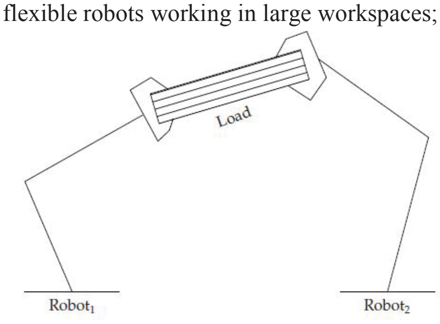

Figure 8: A cooperative cell of robots achieving a desired task.

therefore, they are influenced by nonlinear and fractional order dynamic effects [13]. For instance in [14, 15] the authors analyse the behaviour of two links in a redundant robot (a robot that has more degree of freedom than required to carry out its task) following a circular trajectory in the Cartesian space. By calculating the inverse kinematics, the pseudo inverse matrix does not converge into an optimal solution either for repeatability or manipulability. In fact, the configuration of those links has a chaotic behaviour that can be approximated by fractional order equations, since it is a phenomenon that depends on the long-term history, as introduced in [16]. Another case-fractional order behaviour in robotics was presented in [13], where a robot of three degrees of freedom was analysed by following a circular trajectory, controlled with a predictive control algorithm on each joint. Despite it has an integer order model, the current of all motors at the joints presents clearly a fractional order behaviour. In [17], the authors analyse the effect of a hybrid force and position fractional controller applied to two robotic arms holding the same object, as shown in Figure 13. The load of the object varied and some disturbances are applied as reference of force and position. A $PI^{\alpha}D^{\mu}$ controller was tuned by trial and error. The resulting controller was demonstrated to be robust to variable loads and small disturbances at the reference. Another interesting problem in robotics which can be treated, with FOC is the control of flexible robots, as this kind of light robots use low power actuators, without self-destruction effects when high impacts occurs. Nevertheless significant vibrations over flexible links make a position control difficult to design, because it reveals a complex behaviour difficult to approximate by linear differential equations [18]. However in [19], the authors propose a PD^{α} for a flexible robot of one degree of freedom with variable load, resulting in a system with static phase and constant overshoot, independent of the applied load. Another case was analysed in [20], simulating a robot with two degrees of freedom, and some different physical characteristics, as an ideal robot, a robot with backslash and a robot with flexible joints. In each one of these configurations they applied *PID* and $PI^{\alpha}D^{\mu}$ controllers and their behaviour was compared. These controllers were tuned by trial and error in order to achieve a behaviour close to the ideal and tested 10000 trajectories with different type of accelerations [21]. Over the ideal robot, the *PID* controller had a smaller response time and smaller overshoot peak than the fractional order *PID*. When any kind of nonlinearity is added to the model, the fractional controller has a smaller overshoot and a smaller stationary error, demonstrating that these type of controllers are more robust than classical *PID* to nonlinear effects .A small overshoot in fractional order controllers is an important characteristic when accuracy and speed are desired in small spaces. In [22]the authors used CRONE controllers in order to reduce the overshoot on small displacement over a *XY* robot. The workspace is of 1mm2 and the overshoot obtained was lower than 1%. An application in a robot with legs was presented in [23, 24], designing a set of PD^{α}algorithms in order to control position and force, applied to an hexapod robot with 12 degrees of freedom. The authors defined two performance metrics, one for quantity of energy and the other for position error. The controllers with α =0.5 had the best performance in this robot.

VIII. CONCLUSION

In this paper we first showed two examples that the best fractional order controller outperforms the best integer order controller. Then, we tried to argue why consider fractional order control even when integer (high) order control works comparatively well. Several typical known fractional order controllers are introduced and commented Then, fractional order PID controllers are introduced which may make fractional order controllers ubiquitous in industry.. Two methods for fractional-order PID controller tuning have been presented. The first one uses optimization for tuning the five parameters of the controller, taking into account frequency response and robustness specifications. The second one is an auto-tuning method that uses relay feedback to characterize some points on the frequency response of the plant, and frequency domain specifications for tuning the controller, in this case constructed as a series combination of a fractional orde r PI^{λ} controller and a fractional-order PD^{δ} controller with noise filter. These methods allow the use of some software and hardware strategies for efficient implementations of controllers in industrial applications and robotics.

IX. ACKNOWLEDGEMENT

I would like to thank my Supervisor Prof.B.D.C.N.Prasad and Co-Supervisor Prof.G.V.S.R.Deekshitulu for their encouragement.

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REFERENCES

[1] YangQuan Chen UBIQUITOUS FRACTIONAL ORDER CONTROLS ifac-fda-2006002-01jul-0481chen

[2] K. B. Oldham, J. Spanier, *Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*, Academic Press,New York – London, 1974.

[3] K. S. Miller, B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, John Wiley and Sons, New York, USA,1993.

[4] I. Podlubný, *Fractional Differential Equations*,Academic Press, San Diego, CA, USA, 1999.

[5] Y. Chen, I. Petráš, D. Xue, Fractional Order Control– A Tutorial, In: Proceedings of 2009 American Control Conference, St. Louis, MO, USA, 2009.

[6] I. Petráš, Stability of fractional-order systems with rational orders: A survey, *Fractional Calculus &Applied Analysis*, Vol. 12, No. 3, 2009, pp. 269-298.

[7] R. E. Gutiérrez, J. M. Rosário, J. T. Machado,Fractional Order Calculus: Basic Concepts and Engineering Applications, *Mathematical Problems inEngineering*, Vol. 2010, 2010, 19 p., DOI:10.1155/2010/375858.

[8] M. Axtell, M. E. Bise, Fractional calculus applications in control systems, In: Proceedings of the 1990 National Aerospace and Electronics Conference, Dayton, OH, USA, 1990.

[9] S. E. Hamamci, Stabilization using fractional order PI and PID controllers, *Nonlinear Dynamics*, Vol.51, No. 1-2, 2008, pp. 329-343, DOI:10.1007/s11071-007-9214-5.

[10] S. E. Hamamci, M. Koksal, Calculation of all stabilizing fractional-order PD controllers for integrating time delay systems, *Computers andMathematics with Applications*, Vol. 59, No. 5, 2010, pp. 1621-1629, DOI: 10.1016/j.camwa.2009.08.049.

[11] Chen Yang Quan, ―Applied Fractional Calculus in Control‖, American Control Conference-ACC2009, St. Louis, Missouri, USA, June 10-12, 2009,

[12] D. Matignon, Stability properties for generalized fractional differential systems, *ESAIM: Proc.*, Vol. 5, 1998, pp. 145-158, DOI: 10.1051/proc: 1998004.

[13] J. Rosario, D. Dumur, and J. T. Machado, "Analysis of fractional-order robot axis dynamics," in *Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and Its Applications*, vol. 2, July 2006.

[14] F. B. M. Duarte, M. da Grac¸a Marcos, and J. A. T. Machado, "Fractional-order harmonics in the trajectory control of redundant manipulators," in *Proceedings of the 2nd IFAC Workshop on FractionalDifferentiation and Its Applications*, vol. 2, July 2006.

[15] F. B. M. Duarte and J. A. Machado, "Pseudoinverse trajectory control of redundant manipulators:a fractional calculus perspective," in *Proceedings of the IEEE International Conference on Robotics andAutomation*, vol. 3, pp. 2406–2411, 2002.

[16] F. B. M. Duarte and J. A. T. Machado, "Chaotic phenomena and fractional-order dynamics in the trajectory control of redundant manipulators," *Nonlinear Dynamics*, vol. 29, no. 1–4, pp. 315–342, 2002.

[17] N. M. F. Ferreira and J. A. T. Machado, "Fractional-order position/force control of two cooperatingmanipulators," in *Proceedings of the IEEE International Conference on Computational Cybertnetics*, August2003.

[18] M. F. M. Lima, J. T. Machado, and M. Cris, "Fractional order fourier spectra in robotic manipulatorswith vibrations," in *Proceedings of the 2nd IFACWorkshop on Fractional Differentiation and Its Applications*,vol. 2, July 2006.

[19] C. A. Monje, F. Ramos, V. Feliu, and B. M. Vinagre, "Tip position control of a lightweight flexiblemanipulator using a fractional order controller," *IET Control Theory and Applications*, vol. 1, no. 5, pp.1451–1460, 2007.

[20] BLAS M. VINAGRE CONCEPCIÓN A. MONJE ANTONIO J. CALDERÓN JOSÉ I. SUÁREZ C. "Fractional PID Controllers for Industry Application. A Brief Introduction" in Journal of Vibration and Control, 13(9–10): 1419–1429, 2007

[21] N. M. F. Ferreira and J. A. T. Machado, "Fractional-order hybrid control of robotic manipulators," in*Proceedings of the 11th International Conference on Advanced Robotics*, pp. 393–398, 2003.

[22] N. M. F. Ferreira, J. A. T. Machado, A. M. S. F. Galhano, and J. B. Cunha, "Fractional control of twoarms working in cooperation," in *Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and Its Applications*, vol. 2, July 2006

[23] B. Orsoni, P.Melchior, A. Oustaloup, Th. Badie, and G. Robin, "Fractional motion control: application to an XY cutting table," *Nonlinear Dynamics*, vol. 29, no. 1–4, pp. 297–314, 2002.

[24] M. F. Silva and J. A. T. MacHado, "Fractional order PDα joint control of legged robots," *Journal of Vibration and Control*, vol. 12, no. 12, pp. 1483–1501, 2006.

[25] M. F. Silva, J. A. T. Machado, and R. S. Barbosa, "Comparison of different orders pad fractional order pd0.5 control algorithm implementations," in *Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and Its Applications*, vol. 2, July 2006.