

# No-Reference Image Based On Fuzzy Classification for Quality Assessment

Percy Irine Biju P

*Department of Electrical and Electronics Engineering  
Anna University Regional Campus, Tirunelveli, Tamilnadu, India*

Vinolia Anandan

*Department of Computer Applications  
Anna University Regional Campus, Tirunelveli, Tamilnadu, India*

**Abstract-** Image Quality Assessment (IQA) of distorted or decompressed images without any reference to the original image is challenging from mathematical point of view. Human observers are still considered the best in assessing the quality of an image without any reference to the original image and it is evaluated using subjective measures. In this project using fuzzy logic approach, the informational entropies of visually salient regions of images are considered as features and the quality of the images are assessed using linguistic values. The features are transformed into fuzzy space by designing an algorithm based on Interval Type-2 (IT2) fuzzy sets and its membership function. The uncertainty present in the input-output feature space is measured using this algorithm to predict the image quality accurately as close to human observations. The proposed image quality metric on comparison with other no-reference quality metrics produced more accurate results and adaptable with subjective mean opinion score metric.

**Keywords-** IQA, IT2, No-Reference, Fuzzy, Visual Saliency

## I. INTRODUCTION

A digital image is defined as a 2-D function  $f(x, y)$  where 'x' and 'y' are spatial coordinates and the amplitude of 'f' at any pair of coordinates (x, y) is called the intensity [3] or gray level of the image at that point. Digital image is composed of a finite number of elements, each of which has a particular location and values. These elements are called as pixels. Digital image processing extracts attributes from images and includes the recognition of individual objects. It involves algorithms to detect and isolate various desired features of a digitized image. The types of image features are edges, corners, ridges. The quality evaluation of digital images is critical in many applications of image processing. In the current connected world, many users share and deliver multimedia data. The overall communication process includes manipulation, processing, storing, and transmission over (noisy) channels. Digital images are subjected to loss of information, various kinds of distortions at the time of compression and transmission, which deteriorate visual quality of the images at the receiving end. Quality of an image plays fundamental role to take vital decision and therefore, its assessment [1,2,4] is essential prior to application. Despite rapid advancement in technology, the characteristics of human vision are still considered best performer for quality assessment processes. Modeling physiological and psycho visual features of the Human Visual System (HVS) [1-3] are reported for developing Image Quality Assessment (IQA) methods [6-8]. Due to limited knowledge of HVS, computational HVS modeling used in IQA is far from complete. The most reliable means of assessing image quality is subjective evaluation based on the opinion of the human observers. However, subjective testing is not automatic, lengthy process and expensive too.

IQA techniques [5] can be divided into two groups, namely subjective and objective. The best way for assessing the quality of an image is the subjective quality measurement recommendations given by the ITU (International Telecommunication Union), which consists of Mean Opinion Score (MOS) from a number of expert observers by looking at image. However, for most applications the MOS method is inconvenient because MOS evaluation is slow and costly, since it employs a group of people in the evaluation process. In order to solve this problem i.e. the need for people in the evaluation process, an objective approach is required. Such objective quality assessment system has great potential in a wide range of application environments. Usually the objective image quality approaches can be categorized into three groups depending on the availability of the original image. (1) Full-Reference (FR) methods perform a direct comparison between the image under test and a reference or original image. (2) No-Reference (NR) metrics, are applied when the original image is unavailable. (3) Reduced-Reference (RR) metrics lie between FR

and NR metrics and are designed to predict image quality with only partial information about the reference image. Focusing on NR metrics, the methods can be targeted to estimate the quality of JPEG [4] (Joint Photographic Expert Group) compressed images.

Most objective image quality assessment methods either require access to the original image as reference or only can evaluate images, degraded with predefined distortions and therefore, lacking generalization approach. No-Reference IQA algorithms attempt to perform quality evaluation for specific type of distorted images and designed specifically for JPEG or JPEG2000 compression artifacts. JPEG image quality index and quality metric based on Natural Scene Statistics (NSS) [5] model are the respective metrics proposed for evaluation of image quality. Extreme learning machine classifier based MOS estimator, Discrete Cosine Transform (DCT) domain statistics based metric and blind image quality index are the three no-reference methods reported very recently to assess image quality. But none of them incorporated human centric computation methods that can exploit powerful judgment ability of human observers like subjective methods, best suited for assessing quality of images.

In computer vision global and/or local features are extracted for image analysis and importance of the features depends on the area of application. In subjective image quality evaluation methods, local features are more relevant since human perceive the image based on salient points rather than considering the image as a whole for predicting its quality. Salient features of images create varied degree of impression on human observers in assessing the quality of the images without any reference image. Therefore, designing of a NR-IQA method depends on human interpretation about the interrelation between the input features and output quality of the images and modeled as fuzzy rules using linguistic variables [6, 8].

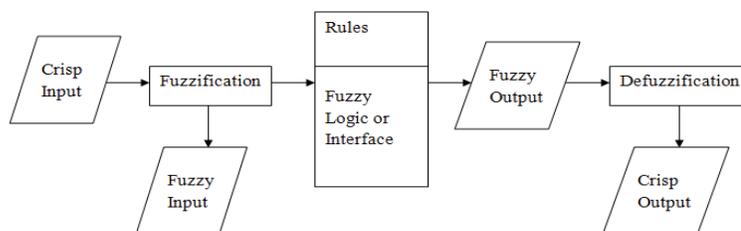


Fig.1. Fuzzy Logic

Fig.1. represents the fuzzy interface to obtain a crisp output.

Fuzzy modeling [10] is used to transform knowledge of human experts into mathematical models (Fig.1). It can be used as a tool to assist human perception about a given task by transforming human observations [9] into mathematical understanding. In type-1 fuzzy systems, the degree of membership of an image belonging to a particular class representing quality of an image is crisp. However, human perception has wide variation both on image features and image quality, therefore it is difficult to interpret and measure the linguistic variables by type-1 fuzzy set. So to evaluate uncertainty associated with human perception in assessing image quality, Type-2 (T2) fuzzy based system is used. The membership functions of input–output variables of T2 fuzzy systems are considered as type-1 fuzzy sets instead of crisp value. This handles uncertainty [7] in human visual perceptual inference generating process by determining image quality using Interval Type-2 (IT2) fuzzy sets. Membership grade of each feature has been represented as an interval of lower and upper bound instead of a crisp value to transform it into fuzzy feature space. IT2 fuzzy set is used within a bounded range for representing degree of membership values, called foot print of uncertainty.

## II. PROPOSED ALGORITHM

### A. Preprocessing

Image preprocessing also called image restoration, involves the correction of noise introduced during image processing. This process produces a corrected image that is as close as possible geometrically to the radiant energy characteristics of the original scene. During No-Reference Image Quality Assessment (NR-IQA) the noise present in the image is not predefined, therefore the filtering technique is used based upon the Peak Signal to Noise

Ratio (PSNR) value. The PSNR value is also used in classifying training images based on their quality into five different classes (Excellent, Good, Average, Bad and Poor).

### B. Feature Selection

Feature selection in image processing is a technique of redefining a large set of redundant data into a set of features of reduced dimension that is transforming the input data into the set of features. Feature selection greatly influences the performance; therefore a correct choice of features is a very crucial step.

Visually salient regions - Visually salient regions (Fig.2) possess a property which maximizes discrimination between the objects in an image. Performance of subjective quality assessment method depends on how quickly Region Of Interest (ROI) in the image is identified by the method. ROI is known as visually salient region and it is observed that in high quality images a large number of such regions exist. Generally, visual saliency refers to the concept that certain parts of the scene are pre-attentively distinctive and create some form of significant visual arousal within the early stages of the HVS.



Fig.2. Visual Saliency

Fig.2. represents the visual saliency between the objects in an image.

Uniform image with peaked histogram indicate low complexity or high predictability while rest with flat intensity distributions demonstrate higher complexity of the image.

Informational entropies of visually salient regions called regional entropy values, are considered as features to develop NR-IQA method. Uncertainty in pixel intensity of local features is evaluated using Shannon's entropy that determines stability of the system.

#### B (a). Kadir and Brady Algorithm

To select visually salient regions and to acquire Shannon's entropy value with respect to the global image descriptor of visually salient regions, Kadir and Brady algorithm is used;

Input: Training Images

Step 1: Measure the gray level pixel intensity within the image size.

Step 2: Estimate the local Probability Distribution Functions (PDF) from the histogram.

Step 3: Calculate global entropy using Eq. (1).

$$H(j, i) = (X(j, i) \cdot \log(X(j, i))) \quad (1)$$

Step 4: Select a range of scale at which the entropy value is highest.

Step 5: Weight (WD) the entropy value, WD is the change of magnitude of PDF as a function of scale at

each peak, known as inter-scale saliency.

Output: A matrix of dimension  $M \times N$  where 'M' is number of parameters of a visually salient region and 'N' represents number of visually salient regions.

### C. Feature Estimation

Feature estimation in image processing is the process of approximation of features, which is a value that is usable even if input data may be incomplete or unstable. The value is not usable because it is derived from the best information available.

Type 2 fuzzy feature space - T2 fuzzy sets and its system theory is gaining popularity for managing uncertainty present in the input-output feature space, difficult to handle by type-1 fuzzy sets. The membership grades of a T2 fuzzy set are in  $[0, 1]$  instead of crisp numbers. Since the boundaries of T2 fuzzy sets are blurred, it is difficult in determining the exact membership grade. For determining the degree of membership value of T2 fuzzy sets either generalized or IT2 method shall be used [7]. IT2 fuzzy sets are used to measure the quality of distorted images when no prior information is available about the original images. IT2FS is described by a primary variable, in the interval  $[0, 1]$  representing the primary Membership Function (MF) and a secondary variable representing the secondary Membership Function (MF). The secondary membership function has constant value for IT2FS, here it is one. Uncertainty is conveyed by the union of all primary memberships, called Footprint Of Uncertainty (FOU).

Considering triangular primary membership function of IT2 fuzzy set, membership functions are themselves fuzzy so the output obtained is transformed into a crisp interval to obtain an exact membership grade.

#### C (a). IT2 Feature Space Algorithm

For evaluating image quality, IT2 fuzzy feature space is built using probability density function.

Input: A matrix of dimension  $M \times N$  where 'M' is number of parameters of a visually salient region and 'N' represents number of visually salient regions.

Step 1: Estimate distribution of entropies using Normal Kernel Distribution.

Step 2: Compute probability densities of the entropy values.

Step 3: Evaluate data distributions at each regional entropy value with respect to their maximum and minimum probability density.

Procedure Compute MAX :

Input:  $E_v^j$  and corresponding  $f(E_v^j)$ , where 'j' is the image index among 'N' input images belong to the same image quality class and 'v' denotes visually salient regions.

Step1: RANGE MAX =  $\text{MAX}_v(\text{MAX}_j(E_v^j))$

Step2: RANGE MIN =  $\text{MIN}_v(\text{MIN}_j(E_v^j))$

Step3:  $i = \text{RANGE MIN}$

Step4: WHILE  $i < \text{RANGE MAX}$

    Begin

    For each regional entropy value  $E_v^j$

    UPD[.] =  $\text{MAX}_j(f(E_v^j))$

    End; where UPD [.] is the set of maximum probability density of entropy values belonging to a particular quality class.

Step5:  $i = i + 1$

Step6: If  $i = \text{RANGE MAX}$  Then END

Else GOTO Step4.

Output: Upper probability distribution UPD [.]

Procedure Compute MIN :

Input:  $E_v^j$  and corresponding  $f(E_v^j)$ , where 'j' is the image index among 'N' input images belong to the same image quality class and 'v' denotes visually salient regions.

Step1:  $\text{RANGE MAX} = \text{MAX}_v(\text{MAX}_j(E_v^j))$

Step2:  $\text{RANGE MIN} = \text{MAX}_v(\text{MIN}_j(E_j^v))$

Step3:  $i = \text{RANGE MIN}$

Step4: WHILE  $i < \text{RANGE MAX}$

Begin

For each regional entropy value  $E_v^j$

$\text{LPD}[.] = \text{MIN}_j(f(E_v^j))$

End; //  $\text{LPD}[.]$  is the set of minimum probability density of entropy values belonging to a particular quality class.

Step5:  $i = i + 1$

Step6: If  $i = \text{RANGE MAX}$  Then END

Else GOTO Step4.

Output: Upper probability distribution LPD [.]

Procedure IT2primary UMF :

Input: UPD [.] of input images belonging to a particular quality class.

Step1:  $\text{RANGE MAX} = \text{MAX}_v(\text{MAX}_j(E_v^j))$

Step2:  $\text{RANGE MIN} = \text{MIN}_v(\text{MIN}_j(E_j^v))$

Step3:  $i = \text{RANGE MIN}$

Step4:  $u(\text{UMF})_{\text{max}} = 1$

Step5: WHILE  $i < \text{RANGE MAX}$

Step6: For each  $E_v^j$

Begin

$u(\text{UMF}) = \text{prob}(\text{UPD}) / \text{max prob}(\text{UPD})$

End;

Step7:  $i = i + 1$

Step8: if  $i = \text{RANGE MAX}$

End.

Else Go to Step4.

Output: Primary UMF of that quality class.

Procedure IT2primary LMF :

Input: LPD [.] of input images belonging to a particular quality class.

Step1:  $\text{RANGE MAX} = \text{MIN}_v(\text{MAX}_j(E_{jv}^i))$

Step2:  $\text{RANGE MIN} = \text{MAX}_v(\text{MIN}_j(E_{jv}^i))$

Step3:  $i = \text{RANGE MIN}$

Step4:  $u(\text{LMF})_{\text{max}} = \text{maxprob}(\text{LPD}) / \text{max prob}(\text{UPD})$

Step5: WHILE  $i < \text{RANGE MAX}$

Step6: For each  $E_{v_i}^j$

    Begin

$u(\text{LMF}) = \text{prob}(\text{LPD}) / \text{max prob}(\text{UPD})$

    End;

Step7:  $i = i + 1$

Step8: if  $i = \text{RANGE MAX}$

End.

Else Go to Step4.

Output: Primary LMF of that quality class.

The set of maximum probability density is called Upper Probability Density (UPD) distribution of the informational entropy values. Similarly, the set of minimum probability density is called Lower Probability Density (LPD) distribution of the informational entropy values. UPD and LPD are used to calculate upper and lower IT2 Primary membership functions respectively. For both the algorithms  $\text{maxprob}(\cdot)$  is the maximum probability density value and  $\text{prob}(\cdot)$  represents probability density value of a distinct entropy value. Therefore, the resulting data distributions provide the Upper Membership Function (UMF) and Lower Membership Function (LMF) for the corresponding image quality class.

#### D. NR-IQA Method

The proposed system is validated using different types of test images taking from TAMPERE and PROFILE (with added effect) image database. Test image is converted to portable gray map format and visually salient regions of the image are obtained using Kadir and Brady algorithm and Shannon's Entropy. Shannon's method is applied to measure local entropy of the regions and mean of the entropies (Mean local entropy) is calculated. Mean local entropy represents features of the test image based on which a distinct quality class is to be predicted. Next the UMF and LMF of the test image corresponding to five different quality classes are calculated. For a test image, maximum T-conorm of the set containing UMF grades of five different classes and maximum T-

conorm of the set containing LMF grades of same five different classes are obtained. Mean conorm is calculated using Eq. (2).

$$\text{Mean conorm} = (\text{MaxTConorm(UMF)} + \text{MAXTConorm(LMF)}) / 2 \quad (2)$$

It has been observed that human eye can record increased value of contrast at the boundary having difference in intensity values than actual. To map the phenomenon, average of maximum T-conorms operator is chosen to evaluate the effect of human vision in predicting quality of an image. Using Eq. (3), the Mean conorm value obtained is matched with the UMF grades of five different classes, obtained during the training phase.

$$\forall i(\text{Absolute difference})_i = \|\text{UMFi} - \text{Mean conorm}\| \quad (3)$$

where ‘i’ indicates five different image quality classes (Excellent, Good, Average, Bad, Poor).

The lowest absolute difference corresponding to a particular class is assigned as quality class label to the test image. It has been assumed that the degree of UMF is dominant in deciding the image quality class compare to LMF grades.

### III. EXPERIMENT AND RESULT

Considering different test images, degree of UMF and LMF of visually salient regions corresponding to each image quality classes are calculated as shown in Table 1. For each test image, maximal UMF and maximal LMF are obtained and Mean conorm is calculated using Eq. (2), as given in Table 2. Absolute difference of the test image with respect to each image quality class is calculated using Eq. (3) and given in the last column of Table 2. We assign the quality class label to a particular test image having lowest Absolute difference among all the given image quality classes. Comparison between our proposed metric of image quality assessment and two other major no-reference image quality metrics (BIQI and JPEG) along with the MOS for the test images are listed in Table 3. The last two columns of Table 3 show absolute and normalized MOS values of the images for corresponding linguistic terms. It is demonstrated in Table 3 that except in case of test images “Aeroplane2” and “Aeroplane3”, the results of all other images are compatible with normalized MOS, which is not true for Blind Image Quality Index (BIQI) and JPEG. We use linguistic variables ‘Average’ and ‘Good’ for visually better images whereas ‘Bad’ and ‘Poor’ for not visually good images as image quality classification labels since perceptually they do not differ much. So accordingly the percentage of correct classification with respect to normalized MOS values for the proposed method is 76.63%, whereas 49.9% and 43.29% for BIQI and JPEG respectively.

**Table 1**  
Mean of Kadir-Brady entropy of visually salient regions of different test images and respective LMF and UMF values for different image quality classes obtained using the proposed method.

Image name	Mean of Kadir-Brady entropy value	LMF of different image quality classes					UMF of different image quality classes				
		Poor	Bad	Avg	Good	Exc	Poor	Bad	Avg	Good	Exc
<i>Bolting</i>	1.6349	0.0032	0.029	0	0.068	0.059	0.342	0.77	0.95	0.67	0.97
<i>Brother1</i>	1.6582	0.0096	0.055	0	0.072	0.092	0.39	0.795	0.96	0.753	0.99
<i>Chiquita</i>	1.9540	0	0.0011	0.009	0	0	0.943	0.935	0.66	0.821	0.25
<i>Gingrich</i>	1.7531	0.093	0.050	0.088	0.0018	0.065	0.568	0.935	0.879	1	0.493
<i>Rowing</i>	1.8192	0.35	0	0.1	0	0.04	0.65	0.4	0.8	0.49	0.5
<i>House_door</i>	1.4772	0	0	0	0	0	0.208	0.59	0.4269	0.297	0.434
<i>Aeroplane1</i>	1.8194	0.062	0.02	0.134	0	0.001	0.541	0.97	1	0.893	0.424
<i>Aeroplane2</i>	1.8197	0.062	0.02	0.134	0	0.001	0.541	0.97	1	0.893	0.424
<i>Aeroplane3</i>	1.8331	0.037	0.017	0.129	0	0	0.579	0.97	0.995	0.8446	0.4211

**Table 2**  
Quality class prediction of test images.

Image name	Mean_conorm	Absolute_difference					Image quality class
		Poor	Bad	Avg	Good	Excellent	
Bolting	0.519	0.0313	0.063	0.185	<b>0.0228</b>	0.203	Good
Brother1	0.541	<b>0.0228</b>	0.064	0.175	0.044	0.201	Poor
Chiquita	0.476	0.218	0.210	<b>0.033</b>	0.119	0.0516	Average
Gingrich	0.546	<b>0.00048</b>	0.151	0.1108	0.206	0.002	Poor
Rowing	0.57	0.0064	0.028	0.078	0.0064	<b>0.0049</b>	Excellent
House_door	0.295	0.0065	0.087	0.0174	<b>0</b>	0.0193	Good
Aeroplane1	0.531	<b>0.0001</b>	0.192	0.219	0.128	0.0112	Poor
Aeroplane2	0.531	<b>0.0001</b>	0.1927	0.219	0.128	0.0112	Poor
Aeroplane3	0.5035	<b>0.0057</b>	0.217	0.241	0.116	0.0067	Poor

**Table 3**  
Comparison between subjective quality metrics and mean opinion score (MOS).

Image name	Image quality assigned by the proposed system (IT2FS)	Blind image quality index (BIQI) by Krishnamoorthy, Bovik	Jpeg quality score index by Wang et al.)	Absolute mean opinion score (linguistic value)	Normalized mean opinion score (linguistic value)
Rowing	<b>Excellent</b>	Average (20.5935)	Good (30.1574)	Infinity (Original)	<b>Infinity (Original)</b>
House_door	<b>Good</b>	Bad (35.3584)	Good (24.9706)	3.9143 (Average)	<b>0.8864 (Good)</b>
Aeroplane1	<b>Poor</b>	Average (15.5362)	Average (15.4587)	2.5429 (Poor)	<b>0.5682 (Bad)</b>
Aeroplane2	Poor	Average (15.1364)	Average (15.3110)	3.3824 (Average)	0.7727 (Average)
Aeroplane3	Poor	Average (14.3383)	Average (15.5740)	4.4286 (Good)	1.0 (Good)
Bolting	<b>Good</b>	Poor (43.127)	Excellent (57.86)	3.5 (Average)	<b>0.7955 (Average)</b>
Brother1	<b>Poor</b>	Poor (Infinity)	Excellent (Infinity)	1.87 (Poor)	<b>0.4091 (Poor)</b>
Chiquita	<b>Average</b>	Excellent (1.27)	Good (28.34)	3.95 (Good)	<b>0.8864 (Good)</b>
Gingrich	<b>Poor</b>	Bad (39.89)	Excellent (50.77)	1.65 (Poor)	<b>0.3864 (Poor)</b>

normalized  $mos_i = absolute\ mos_i / \max(absolute\ mos_i)$   $i =$  number of MOS values, Infinite value of MOS is not taken into consideration for being the maximum value of the set of MOS values. It is apparent from Table 2 that our proposed IT2 fuzzy based NR-IQA method directly infers the quality class of an image whereas other quality metrics provide a numerical value from which the quality class has to be perceived. The perceiving process itself is fuzzy which in the proposed method is the basic paradigm for experimentation. Moreover, it has been observed that the proposed method outperforms subjective image quality metrics in predicting the quality class of the images as evident from the MOS values provided in Table 3. So it is evident that our proposed scheme is compatible with MOS metric.

#### IV. CONCLUSION

The system is developed for the quality assessment of images without any reference to the original image. Informational entropies of salient regions are used as features to measure the uncertainty in the image input-output and it is modeled by IT2 fuzzy sets. This fuzzy system has been used to remove the limitation of type1 fuzzy systems for assessing quality of images. Human observations on visual quality of images are assigned by five different class labels. The variation in features is wide enough for capturing important information from the images. It has been observed that human eye generally considers contradiction at a higher range than actual at the boundary of intensity difference in an image called Mach band effect. So to predict the quality of an image primary UMF of output features is considered to calculate the absolute difference. Linguistic variables such as ‘Average’ and ‘Good’ are used as quality classification labels for visually better images whereas ‘Bad’ and ‘Poor’ for not visually good since perceptually they do not differ much. Accordingly the percentage of correct classification with respect to normalized MOS values for the proposed method is 76.63%, whereas 49.9% and 43.29% for BIQI and JPEG respectively. The proposed method is accurate in assessing quality of the images when the other objective image quality metrics show inconsistent results with subjective MOS.

#### REFERENCES

- [1] Zhou Wang, Guixing Wu, Hamid Rahim Sheikh, Eero P. Simoncelli, En-Hui Yang, and Alan Conrad Bovik, “Quality-Aware Images”, (2006).
- [2] Tomas Brandao, Maria Paula Queluzo, “No-reference image quality assessment based on DCT domain statistics”,(2007).
- [3] T. Oh, J. Park, K. Seshadrinathan, S. Lee, A. C. Bovik, “No-reference sharpness assessment of camera-shaken images by analysis of spectral structure”, (2014).
- [4] L. Y. Zhou, Z. B. Zhang, “No-reference image quality assessment based on noise, blurring and blocking effect”,(2014).
- [5] Q. Sang, X. Wu, C. Li, A. C. Bovik, “Blind image quality assessment using a reciprocal singular value curve”,(2014).

- [6] S. Corchs, F. Gasparini, R. Schettini, "No reference image quality classification for JPEG-distorted images", (2014).
- [7] L. Liu, H. Dong, H. Huang, A.C. Bovik, "No-reference image quality assessment in curvelet domain", (2014).
- [8] Y. Li, X. Lai-Man Po, L. Xu, Feng, "No-reference image quality assessment using statistical characterization in the shearlet domain", (2014).
- [9] L. Liu, B. Liu, H. Huang, A.C. Bovik, "No-reference image quality assessment based on spatial and spectral entropies", (2014).
- [10] Y. Zhanga, A. K. Moorthy, D. M. Chandler, A. C. Bovik, "C-DIIVINE: no-reference image quality assessment based on local magnitude and phase statistics of natural scenes", (2014).
- [11] De, J. Sil, "No Reference Image Quality Assessment Using Fuzzy Relational Classifier", (2011).
- [12] De, J. Sil, "Entropy based fuzzy classification of images on quality assessment", (2012).
- [13] De, J. Sil, "No Reference Image Quality Assessment by Designing Fuzzy Relational Classifier Using MOS Weight Matrix", (2012).
- [14] M. Kumru, "Assessing the visual quality of sanitary ware by fuzzy logic", (2013).
- [15] Wen Lu, Ning Mei, Fei Gao, Lihuo He and Xinbo Gao, "Blind image quality assessment via semi-supervised learning and fuzzy inference", (2015).