A Fuzzy Entropy-based Method for Texel identification

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Abstract - The entropy definition primarily assumes the exponential behavior for information gain. In this paper, the fuzzy entropy function is critically analyzed to exhibit the flexibility under certain constraints on parameters. The optimization of entropy function is done for identification of a texel in any regular texture pattern. To analyze the texture, the neighborhood of each pixel in an image is aggregated using a modified Gaussian membership function. The Anirban and Hanmandlu's fuzzy entropy function is used to optimize all parameters through a simple gradient decent technique. The entropy values for different window sizes are statistically computed and the point where the standard deviation of the entropy values reaches the first local minimum would indicate the texel size.

Index Terms: Entropy, Fuzzy feature, Fuzzy Entropy Optimization, Texel

I. INTRODUCTION

Most of the entropy functions are not suitable for representing the information in a fuzzy set because of the fact that these are not parametric. These are Shannon's, Reny's and Pal's [5, 10, 1] entropy functions. In this paper we have used Anirban and Hanmandlu's Fuzzy entropy function [13].

The function involved in the entropy need not be a membership function; it could be any feature. It may be noted when we use a membership function, the unknown parameters of this membership function will parameterize the entropy function indirectly. But, the problem is the choice of a suitable membership function is not that easy all the times. Hence, the main motivation behind the use of the entropy is our ability to represent the information/uncertainty contained in a fuzzy set. There are many applications such as image analysis, texture analysis, pattern recognition we come across the utility of the fuzzy entropy function. In order to illustrate the usefulness of the entropy, we determine the size of the window enclosing a texel that repeats itself in a Brodatz image.

The paper is organized as follows: Section 2 presents the fuzzy entropy function. Normalized entropy and conditional entropy functions are presented. Section 3 briefly describes the fuzzy based texture features from which are derived fuzzy descriptors. Taking these descriptors as features, an application of the entropy function for the identification of a texel in window is given in Section 4. Finally conclusions are drawn in Section 5.

II. FUZZY ENTROPY FUNCTION

The information gain corresponding to the occurrence of the ith event is defined as

$$I(p_i) = e^{-(ap_i^3 + bp_i^2 + cp_i + d)} \quad (1)$$

for all probabilities $p_i \in [0,1]$, $\sum p_i = 1$ and a, b, c and d are real-valued parameters. We define the entropy as

$$H = E(I(p_i)) = \sum_{i=1}^{n} p_i * I(p_i) \quad (2)$$

2.1 *Normalized Entropy*

The normalized entropy H_N can be defined as $H_N = (H - e^{-(a+b+c+d)}) / \lambda$ (3)

where the constant $\lambda = e^{-(\frac{a}{n^3}+\frac{b}{n^2}+\frac{c}{n}+d)} - e^{-(a+b+c+d)}$ and a, b, c and d are real valued parameters and n is the number of events in the probabilistic experiment (or the number of states in the system). The normalized entropy satisfies all the properties of an entropy function.

2.2 Conditional Entropy

Consider two partitions A=[A_1, A_2, … A_n] and B=[B_1, B_2, … B_m] and let us define that the product of two partitions, A=[A_i] and B=[B_j] is a partition whose elements are all intersections A_iB_j and the product partition is denoted by A.B = [A_iB_j]

Let p_{ij} be the probability of the event A_iB_j , i.e. p_{ij} = Pr [A_iB_j] and the marginal probabilities p_i = Pr[A_i] is defined as $p_i = \sum_{j=1}^{m} p_{ij}$. Similarly q_j = Pr[B_j] and $q_j = \sum_{i=1}^{n} p_{ij}$.

The conditional entropy of A_i given that B_j has occurred is denoted by

$$\Pr[A_i \mid B_j] = \Pr[A_iB_j]/\Pr[B_j] = p_{ij}/q_j = p_{i|j} \text{ (Say)} \quad (4)$$

Similarly $\Pr[B_j \mid A_i] = \Pr[A_iB_j]/\Pr[A_i] = p_{ij}/p_i = q_{j|i}$ (Say) (5)

Therefore the entropy of a partition A, given that B_j has occurred is given by H[A|B_j] where $H[A \mid B_j] = \sum_{i=1}^{n} p_{i|j} e^{-[ap_{i|j}^3 + bp_{i|j}^2 + cp_{i|j} + d]}$. Thus the conditional entropy of A assuming B is by definition .

$$H[A \mid B] = \sum_{j=1}^{m}\sum_{i=1}^{n} q_j p_{i|j} e^{-[ap_{i|j}^3 + bp_{i|j}^2 + cp_{i|j} + d]} \quad (6)$$

Similarly, $H[B \mid A] = \sum_{i=1}^{n}\sum_{j=1}^{m} p_i q_{j|i} e^{-[aq_{j|i}^3 + bq_{j|i}^2 + cq_{j|i} + d]}$. (7)

And the entropy of the product partition A.B is defined as

$$H[A.B] = \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij} e^{-[ap_{ij}^3 + bp_{ij}^2 + cp_{ij} + d]} \quad (8)$$

III. AN APPLICATION: TEXEL IDENTIFICATION

Texture is one of the important characteristic of any image and easy to recognize visually but difficult to define it. There is no proper definition; the texture can be described as a tiling of a smallest visual pattern. The repetitive pattern in regular texture is often referred to as a texel.

Structural texture analysis aims at identifying the periodicity in the texture or texel itself. Several attempts had been made earlier using Fourier analysis [6], Autocorrelation function [8], Cooccurrence matrices [3]. Apart from the conventional methods, second order Renyi's entropy was used to identify the texel size [4].

In this paper, we propose an information theoretic approach for texel identification after characterizing the texture in the fuzzy domain [11]. The ambiguities in texture that arise due to fuzzy nature of image function give an opportunity to devise fuzzy texture features. Since texture is region based, we consider arrangement of image functions (i.e., intensities) of pixels in a local region, say, a window, in order to characterize the texture using modified Gaussian type membership function. Next, these features are utilized to measure the amount of uncertainty contained in the image. The fuzzy entropy function gives the optimized value of uncertainty for different sizes of window. It has been observed that the entropy values remain closely the same for the texel size in comparison to the other window sizes.

3.1 Extraction of Fuzzy feature

To convert spatial domain image into fuzzy domain, we consider the spatial arrangement of gray levels of pixels over a window. The fuzzy property can be expressed in terms of a membership function. A membership function to this effect is defined by Gaussian type function.

$$\mu_{(k,l)}(i,j) = \exp\left[-\{(x(i,j) - x(k,l))/\tau\}^2\right] \quad (9)$$

where x(i, j) is the gray level of the pixel at the (i,j)th position and τ is the fuzzifier which is taken as window size. In our experiment, we take the value of τ as 3. We note that

$$\mu_{(i,j)}(k,l) = 1 \text{ if } x(i,j) = x(k,l) \quad (10)$$

To consider the response from the neighboring pixels, we obtain the cumulative response of (i,j)th pixel as follows

$$y(i,j) = \sum_{k,l} \mu_{(k,l)}(i,j) * x(k,l) / \sum_{k,l} \mu_{(k,l)}(i,j) \quad (11)$$

This is the defuzzified response of the (i,j)th pixel over the window of size 3. This process is repeated for all pixels in the image and that yields a textured image consisting of all defuzzified values. For our convenience of notation, we designate the matrix formed by y(i,j) as response matrix, Y.

3.2 Texture Descriptor

We will derive the descriptor that captures the essence of texture of a subimage of size w×w of the original image of size M×N (w«M and w«N). We can use the entropy as a descriptor to represent the information content in the response matrix.

Let Z = {z_1, z_2, … z_n} be the set of events and φ: Z→[0,1] be a fuzzy set. Let us interpret φ(z_i) as the probability that z_i possesses a property P. In this context, we define φ(z_i) as a relative measure of individual response with respect to the total response of the sub matrix w×w. Let D = $[d_{ij}]_{w\times w}$ be a sub matrix of the response matrix Y. Then eventually the event z_{ij} corresponds to the pixel at (i,j)th position and φ(z_{ij}) is defined as

$$\varphi(z_{ij}) = d_{ij} / \sum_{k\in w}\sum_{l\in w} d_{kl} \quad (12)$$

The descriptor is the normalized entropy function, H_N defined by

$$H = \sum_{i\in w}\sum_{j\in w} \varphi(z_{ij}) * \exp[-\{a\varphi^3(z_{ij}) + b\varphi^2(z_{ij}) + c\varphi(z_{ij}) + d\}] \quad (13)$$

And $$H_N = (H - e^{-(a+b+c+d)})/\lambda \quad (14)$$

where $\lambda = [e^{-(\frac{a}{n^3}+\frac{b}{n^2}+\frac{c}{n}+d)} - e^{-(a+b+c+d)}]$ and $n = w*w$

In the entropy function (6), a, b and c are three tunable parameters with $0 \le H_N \le 1$.

3.3 Fuzzy Entropy Optimization

We employ the fuzzy entropy optimization to estimate three tunable parameters: a, b, and c. The derivatives of H_N with respect to a, b, c and d are obtained as follows-

$$\frac{\partial H_N}{\partial a} = \frac{[-\sum_{i\in w}\sum_{j\in w}\varphi^4(z_{ij}) * \exp[-\{a\varphi^3(z_{ij}) + b\varphi^2(z_{ij}) + c\varphi(z_{ij}) + d\}] + H_1]}{H_2 - H_1}$$
$$+ \frac{[\sum_{i\in w}\sum_{j\in w}\varphi(z_{ij}) * \exp[-\{a\varphi^3(z_{ij}) + b\varphi^2(z_{ij}) + c\varphi(z_{ij}) + d\}] - H_1]}{(H_2 - H_1)^2} * (\frac{H_2}{n^3} - H_1) \quad (15)$$

$$\frac{\partial H_N}{\partial b} = \frac{[-\sum_{i\in w}\sum_{j\in w}\varphi^3(z_{ij}) * \exp[-\{a\varphi^3(z_{ij}) + b\varphi^2(z_{ij}) + c\varphi(z_{ij}) + d\}] + H_1]}{H_2 - H_1}$$

$$+\frac{[\sum_{i\in w}\sum_{j\in w}\varphi(z_{ij})*\exp[-\{a\varphi^3(z_{ij})+b\varphi^2(z_{ij})+c\varphi(z_{ij})+d\}]-H_1]}{(H_2-H_1)^2}*(\frac{H_2}{n^3}-H_1) \qquad (16)$$

$$\frac{\partial H_N}{\partial c}=\frac{[-\sum_{i\in w}\sum_{j\in w}\varphi^2(z_{ij})*\exp[-\{a\varphi^3(z_{ij})+b\varphi^2(z_{ij})+c\varphi(z_{ij})+d\}]+H_1]}{H_2-H_1}$$
$$+\frac{[\sum_{i\in w}\sum_{j\in w}\varphi(z_{ij})*\exp[-\{a\varphi^3(z_{ij})+b\varphi^2(z_{ij})+c\varphi(z_{ij})+d\}]-H_1]}{(H_2-H_1)^2}*(\frac{H_2}{n^3}-H_1) \qquad (17)$$

$$\frac{\partial H_N}{\partial d}=\frac{[-\sum_{i\in w}\sum_{j\in w}\varphi(z_{ij})*\exp[-\{a\varphi^3(z_{ij})+b\varphi^2(z_{ij})+c\varphi(z_{ij})+d\}]+H_1]}{H_2-H_1}$$
$$+\frac{[\sum_{i\in w}\sum_{j\in w}\varphi(z_{ij})*\exp[-\{a\varphi^3(z_{ij})+b\varphi^2(z_{ij})+c\varphi(z_{ij})+d\}]-H_1]}{(H_2-H_1)^2}*(\frac{H_2}{n^3}-H_1) \qquad (18)$$

These derivatives are used in the recursive learning of parameters a, b, c and d by the gradient descent technique:

$$a^{New}=a^{Old}-\varepsilon_1\frac{\partial H_N}{\partial a} \quad ..... \ (19) \qquad b^{New}=b^{Old}-\varepsilon_2\frac{\partial H_N}{\partial b} \quad ..... \ (20)$$

$$c^{New}=c^{Old}-\varepsilon_3\frac{\partial H_N}{\partial c} \quad ...... \ (21) \qquad d^{New}=d^{Old}-\varepsilon_4\frac{\partial H_N}{\partial d} \quad .... \ (22)$$

where ε_1, ε_2, ε_3, ε_4 are learning rates for all three parameters. The nearest optimization points of both positive and negative search directions are taken. The values of ε_1, ε_2, ε_3, ε_4 are taken as 0.01, 0.01, 0.01 and 0.01respectively.

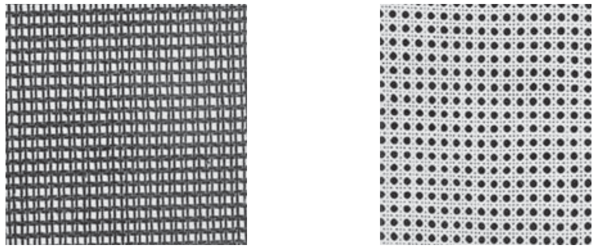
3.4 An Algorithm for the Texel Identification

The steps of the algorithm to determine the texel size are:

1. Calculate the value μ(i,j) for each pixel (i,j) to fuzzify the original image of size M×N
2. Determine the Response matrix Y of size M×N
3. For different window size w do
4. Divide the matrix Y into boxes of size w×w
5. For each box D do
6. Determine the matrix $[\varphi(z_{ij})]_{w\times w}$
7. Calculate the Entropy function, H with random initialization of a, b, c and d
8. Optimize the parameter a, b, and c
9. Calculate the optimal normalized entropy value H_N
10. Calculate the standard deviation of entropy values of all boxes
11. The window size w_T which gives the local minimum standard deviation would be the desired texel size.

IV. Results and Discussions

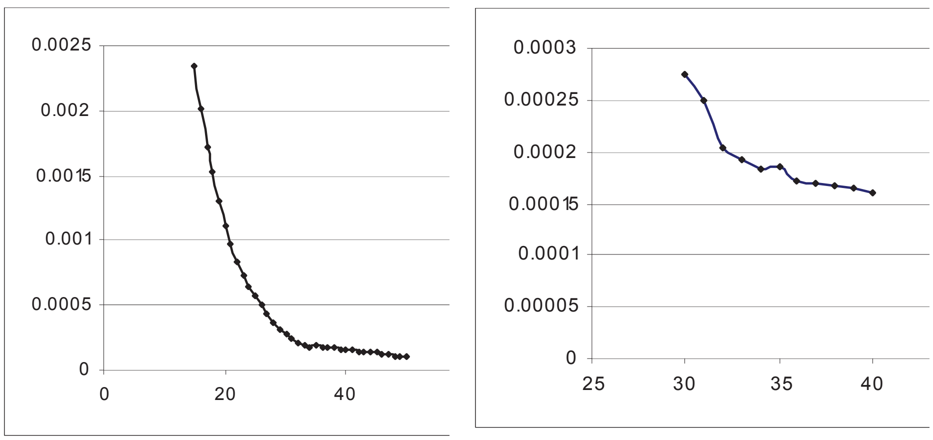
In our experiment, we use the Brodatz textures [12] shown in Figure 1 for the determination of texel size. The images D20 and D101 are of size 640×640, and no preprocessing has been done.



(a) D20 (b) D101

Figure 1: Brodatz regular textures having squared shape texel

The results obtained for D20 are shown in Figure 2. In Section 3.4, we have noted that the standard deviations of optimal normalized entropy values for different window sizes need to be considered. We therefore plot the standard deviations against window sizes.



(a) (b) $w_T = 34$

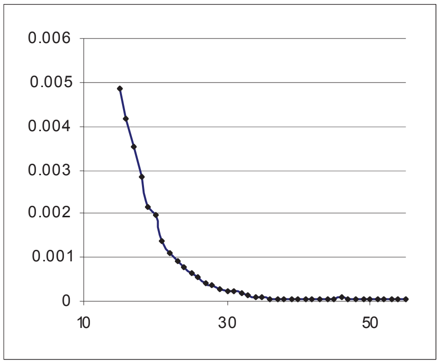
Window Size	s.d	a	b	c	d
30	0.000274	0.013764	6.19944	2.12637	8.90613
31	0.00025	0.013764	6.19944	2.12637	8.90613
32	0.000205	0.013764	6.19944	2.12637	8.90613
33	0.000192	0.013764	6.19944	2.12637	8.90613
34	0.000182	0.013764	6.19944	2.12637	8.90614
35	0.000186	0.013764	6.19944	2.12637	8.06914
36	0.000172	0.013764	6.19944	2.12637	8.06914
37	0.000169	0.013764	6.19944	2.12637	8.06914
38	0.000167	0.013764	6.19944	2.12637	8.06914
39	0.000164	0.013764	6.19944	2.12637	8.06914
40	0.000161	0.013764	6.19944	2.12637	8.06914

(c)

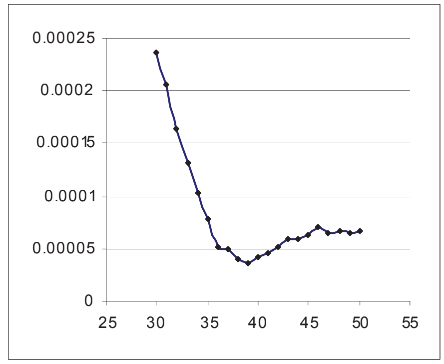
Figure 2: (a) Standard deviations of fuzzy entropy values for texture D20 at different window sizes; (b) Close up view of the graph given in Figure 2 (a); (c) Table shows the s.d for optimal normalized entropy values at different window sizes and the optimized values of parameters a, b, c and d

It is obvious that the entropy values of smaller window than the texel size, would have more deviations that results higher standard deviation, and as window size increases, the standard deviation decreases till the window size reaches the actual size of texel. Definitely for the window size, next to texel size, standard deviation increases. For higher window sizes than the texel size, it is observed that standard deviation decreases because higher window

sizes contained the texel itself. Therefore we can conclude that the window size for first local minimum would represent the texel size of the texture.
From Figure 2, we can observe that at window size 34, standard deviation attains at the first local minimum. This confirms that the texel size of texture D20 is either exactly 34 or closest to it. The reason behind this assertion is that the standard deviation increases beyond this size because of overlapping of partial texture pattern or decreases whenever the window encloses a full texture pattern. That way we get an oscillating curve for the window sizes that cross the texel size.



(a)



(b) $w_T = 39$

Window Size	s.d	a	b	C	d
35	7.71E-05	9.05124	8.21265	1.9152	9.45837
36	5.16E-05	9.05124	8.21265	1.9152	9.45837
37	4.98E-05	9.05124	8.21265	1.9152	9.45838
38	4.04E-05	9.05124	8.21265	1.9152	9.45838
39	3.72E-05	9.05124	8.21265	1.9152	9.45838
40	4.19E-05	9.05124	8.21265	1.9152	9.45838
41	4.63E-05	9.05124	8.21265	1.9152	9.45838
42	5.1E-05	9.05124	8.21265	1.9152	9.45838
43	5.89E-05	9.05124	8.21265	1.9152	9.45838
44	5.93E-05	9.05124	8.21265	1.9152	9.45838
45	6.22E-05	9.05124	8.21265	1.9152	9.45838

(c)

Figure 3: (a) Standard deviations of fuzzy entropy values for texture D101 at different window sizes; (b) Close up view of the graph given in figure 3 (a); (c) Table shows the s.d for optimal normalized entropy values at different window sizes and the optimized values of parameters a, b, c and d

In Figure 3 for the window size 39, the curve attains the first local minimum which indicates that the texel size of the regular texture D101 is 39. Thus the results demonstrate the usefulness of our entropy function in identifying a texel in a regular texture.

V. Conclusions

This paper presents the fuzzy entropy function that is aimed at representing the information in a fuzzy set. As an application of the entropy function, a texel that repeats itself in a texture pattern is identified from the fuzzy based descriptors. The descriptor is based on the response of a pixel about its chosen neighborhood. The fuzzy set formed from the values descriptor is used in the entropy function for optimization with respect to the parameters of the entropy function itself. The size of the window superimposed on the texture pattern, for which the entropy drops down to the first local minimum is said to contain a texel as verified by the visual assessment. The reason is beyond this size of the window the standard deviation oscillates.

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