

# Intuitionistic Fuzzy Completely Weakly $\pi$ Generalized Continuous Mapping

K.Kadambavanam

*Department of Mathematics, Sri Vasavi College, Erode-638 316, Tamilnadu, India*

K.Vaithiyalingam

*Department of Mathematics, Kongu Engineering College, Perundurai, Erode-638 052, Tamilnadu, India.*

**Abstract** - In this paper the concepts of intuitionistic fuzzy completely weakly  $\pi$  generalized continuous mappings on intuitionistic fuzzy topological space is introduced with numerical examples. Some of its properties are investigated.

**Keywords** - Intuitionistic fuzzy topology, intuitionistic fuzzy weakly  $\pi$  generalized closed set and open set, intuitionistic fuzzy  $w\pi$  space and intuitionistic fuzzy  $w\pi$  space.

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## I. INTRODUCTION

Fuzzy set (FS) proposed by Zadeh [19] in 1965, as a framework to model a system with uncertainty, vagueness and partial truth, by means of degree of membership for each member of the universe of discourse to a subset of it. In 1986, it is generalized into intuitionistic fuzzy (IF) by Atanassov [1] in. After that many research articles have been published in the study of examining and exploring, how far the basic concepts and theorems, defined in crisp sets and in fuzzy sets remain true in IF sets. In 1968, the concept of fuzzy topology was introduced by Chang [3] and in 1997, Coker [4] initiated the concept of generalization of fuzzy topology into IF topology. The apprehension of semi closed,  $\alpha$  closed, semi pre-closed, weakly closed were introduced in his paper. Further its properties are derived.

In this paper, the concept of IF completely weakly  $\pi$  generalized continuous mappings is introduced and studied, with suitable examples. This paper provides some characterizations of IF completely weakly  $\pi$  generalized continuous mappings and establishing the relationships among the other classes of, early defined forms of intuitionistic fuzzy mappings. The derivations of some of its properties are presented. Numerical illustrations are also given to clarify the derived results.

This paper is organized into four sections. Historical developments of the concepts are briefed in the first section. In the second section, the basic definitions and results, needed for this work are listed. Section three discusses some properties of the IF completely weakly  $\pi$  generalized continuous mappings, with suitable examples. Conclusion remarks are presented in section four.

## II. PRELIMINARIES

**Definition 2.1:** Let  $X$  be a non-empty crisp set. An intuitionistic fuzzy (IF) set  $A$ , in  $X$  is defined as an object of the form [1]

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \},$$

where the functions  $\mu_A(x) : X \rightarrow [0, 1]$  and  $\nu_A(x) : X \rightarrow [0, 1]$  are denote the degree of membership (briefly  $\mu_A$ ) and the degree of non-membership (briefly  $\nu_A$ ) of each element  $x \in X$  to the set  $A$ , respectively, and

$0 \leq \mu_A(x) + \nu_A(x) \leq 1$ , for each  $x \in X$ . The collection of all intuitionistic fuzzy sub-sets in  $X$ , is denoted by IFS(X).

Definition 2.2: Let  $A$  and  $B$  be two different IFSs of the form [1],  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$ ,  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X \}$ . The operations  $\wedge$  and  $\vee$  are defined on  $\mu_A(x), \mu_B(x), \nu_A(x)$ , and  $\nu_B(x)$  as follows:

- i)  $\mu_A(x) \vee \mu_B(x) = \max \{ \mu_A(x), \mu_B(x) \}$ ,
- ii)  $\mu_A(x) \wedge \mu_B(x) = \min \{ \mu_A(x), \mu_B(x) \}$ ,
- iii)  $\nu_A(x) \vee \nu_B(x) = \max \{ \nu_A(x), \nu_B(x) \}$ , and
- iv)  $\nu_A(x) \wedge \nu_B(x) = \min \{ \nu_A(x), \nu_B(x) \}$ .

Then,

- i)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$ , similarly  $A \supseteq B$  can also be defined,
- ii)  $A = B$  if and only if both  $A \subseteq B$  and  $B \subseteq A$  are valid,
- iii)  $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \}$ , here the membership grade of  $x$  in  $A$  is the non- membership grade of  $x$  in  $A^c$  and vice versa,
- iv)  $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle \mid x \in X \}$ , and
- v)  $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in X \}$ .

For the sake of simplicity, the notations  $A = \langle x, \mu_A, \nu_A \rangle$  is used instead of  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$ ,

and  $B = \langle x, \mu_B, \nu_B \rangle$  is used instead of  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X \}$ .

The intuitionistic fuzzy sets  $0_X$  and  $1_X$  are defined respectively as,  $0_X = \{ \langle x, 0, 1 \rangle \mid x \in X \}$  and  $1_X = \{ \langle x, 1, 0 \rangle \mid x \in X \}$ . The sets  $0_X$  and  $1_X$  are known as the empty IF set and the whole IF set of  $X$  respectively.

Definition 2.3: An intuitionistic fuzzy topology (IFT) is a family  $\tau$  of IFS defined on  $X$ , satisfying the following axioms [4]:

- i)  $0_X, 1_X \in \tau$ ,
- ii)  $G_1 \cap G_2 \in \tau$ , wherever  $G_1, G_2 \in \tau$ ,
- iii)  $\cup G_i \in \tau$  for any arbitrary family  $\{G_i \mid i \in I\} \subseteq \tau$ .

Then the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS) and any IFS in  $\tau$  are known as an intuitionistic fuzzy open set (IFOS) in  $X$ .

The complement  $A^c$  of  $A$ , is an IFOS in an IFTS  $(X, \tau)$  then  $A^c$  is called an intuitionistic fuzzy closed set (IFCS) in  $X$ .

Definition 2.4: Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in  $X$ . Then the intuitionistic fuzzy closure and an intuitionistic fuzzy interior are defined by [4],

$$cl(A) = \cap \{ G \mid G \text{ is an IFCS in } X \text{ and } A \subseteq G \}, \text{ and}$$

$$int(A) = \cup \{ K \mid K \text{ is an IFOS in } X \text{ and } K \subseteq A \}.$$

Note that for any IFS,  $A$  in  $X$ ,  $cl(A^c) = (int(A))^c$  and  $int(A^c) = (cl(A))^c$ .

Definition 2.5: An IFS,  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS  $(X, \tau)$  is said to be an

- i) intuitionistic fuzzy closed set [4] (IFCS) in  $X \Rightarrow cl(A) = A$ , and
- ii) intuitionistic fuzzy open set [4] (IFOS) in  $X \Rightarrow int(A) = A$ .

Definition 2.6: A subset  $A$  of a space  $(X, \tau)$  is called

- i) regular open [13] if  $A = int(cl(A))$ , and
- ii)  $\pi$  open [13] if  $A$  is the union of regular open sets, symbolically  $A$  is an IF $\pi$ OS in  $X$ .

Definition 2.7: An IFS  $A = \langle X, \mu_A, \nu_A \rangle$  in an IFTS  $(X, \tau)$  is said to be an

- i) intuitionistic fuzzy semi- closed set [5] (IFSCS) if  $int(cl(A)) \subseteq A$ , and
- ii) intuitionistic fuzzy semi- open set [5] (IFSOS) if  $A \subseteq cl(int(A))$ .

Definition 2.8: Let  $A$  be an IFS of an IFTS  $(X, \tau)$ . Then the semi closure of  $A$  (denoted by  $sci(A)$ ) and semi interior of  $A$  (denoted by  $sint(A)$ ) are defined as [17],

$$(i) sci(A) = \bigcap \{G \mid G \text{ is an IFSCS in } X \text{ and } A \subseteq G\}, \text{ and}$$

$$(ii) sint(A) = \bigcup \{K \mid K \text{ is an IFSOS in } X \text{ and } K \subseteq A\}.$$

Result 2.1: Let  $A$  be an IFS in  $(X, \tau)$ , then [15]

- i)  $sci(A) = A \cup int(cl(A))$ , and
- ii)  $sint(A) = A \cap cl(int(A))$ .

Definition 2.9: An IFS  $A = \langle X, \mu_A, \nu_A \rangle$  in an IFTS  $(X, \tau)$  is said to be an

- i) intuitionistic fuzzy  $\alpha$  closed set [5] (IF $\alpha$ CS) if  $cl(int(cl(A))) \subseteq A$ , and
- ii) intuitionistic fuzzy  $\alpha$  open set [5] (IF $\alpha$ OS) if  $A \subseteq int(cl(int(A)))$ .

Definition 2.10: Let  $A = \langle X, \mu_A, \nu_A \rangle$  be an IFS of an IFTS  $(X, \tau)$ . Then the  $\alpha$  closure of  $A$  ( $\alpha cl(A)$ ) and  $\alpha$  interior of  $A$  ( $\alpha int(A)$ ) are defined as [10],

$$\alpha cl(A) = \bigcap \{G \mid G \text{ is an IF}\alpha\text{CS in } X \text{ and } A \subseteq G\}, \text{ and}$$

$$\alpha int(A) = \bigcup \{K \mid K \text{ is an IF}\alpha\text{OS in } X \text{ and } K \subseteq A\}.$$

Result 2.2: Let  $A$  be an IFS in  $(X, \tau)$ , then [11]

- i)  $\alpha cl(A) = A \cup cl(int(cl(A)))$ , and
- ii)  $\alpha int(A) = A \cap int(cl(int(A)))$ .

Definition 2.11: An IFS,  $A = \langle X, \mu_A, \nu_A \rangle$  in an IFTS  $(X, \tau)$  is said to be an

- i) intuitionistic fuzzy pre-closed set [5] (IFPCS) if,  $cl(int(A)) \subseteq A$ ,
- ii) intuitionistic fuzzy regular closed set [5] (IFRCS) if,  $cl(int(A)) = A$ ,

iii) intuitionistic fuzzy generalized closed set [16] (IFGCS) if,  $cl(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is an IFOS in  $X$ ,

iv) intuitionistic fuzzy generalized semi closed set [12] (IFGSCS) if,  $sccl(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is an IFOS in  $X$ ,

v) intuitionistic fuzzy  $\alpha$  generalized closed set [11] (IF $\alpha$ GCS) if,  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$ ,  $U$  is an IFOS in  $X$ .

Definition 2.12: An IFS,  $A$  is said to be an intuitionistic fuzzy weakly  $\pi$  generalized closed set [7] (IFW $\pi$ GCS) in  $(X, \tau)$ , if  $cl(int(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IF $\pi$ OS in  $X$ .

The family of all IFW $\pi$ GCS of an IFTS  $(X, \tau)$  is denoted by IFW $\pi$ GCS( $X$ ).

Result 2.3: Every IFCS, IF $\alpha$ CS, IFGCS, IFRCS, IFPCS, IF $\alpha$ GCS are IFW $\pi$ GCS [7] but the converse need not be true.

Definition 2.13: An IFS,  $A$  is said to be an intuitionistic fuzzy weakly  $\pi$  generalized open set [7] (IFW $\pi$ GOS) in  $(X, \tau)$  if, the complement  $A^c$  is an IFW $\pi$ GCS in  $X$ .

The family of all IFW $\pi$ GOS of an IFTS  $(X, \tau)$  is denoted by IFW $\pi$ GOS ( $X$ ).

Definition 2.14: Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into IFTS  $(Y, \sigma)$ . Then  $f$  is said to be intuitionistic fuzzy continuous [5] (IF cts) if,  $f^{-1}(B) \in \text{IFOS}(X)$  for every  $B \in \sigma$ .

Definition 2.15: Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into IFTS  $(Y, \sigma)$ . Then  $f$  is said to be

- i) intuitionistic fuzzy weakly  $\pi$  generalized continuous mapping [8] (IFW $\pi$ G cts) if,  $f^{-1}(B)$  is an IFW $\pi$ GCS in  $(X, \tau)$  for every IFCS,  $B$  of  $(Y, \sigma)$ ,
- ii) intuitionistic fuzzy semi continuous mapping [18] (IFS cts) if,  $f^{-1}(B) \in \text{IFSO}(X)$  for every  $B \in \sigma$ ,
- iii) intuitionistic fuzzy  $\alpha$  continuous mapping [18] (If $\alpha$  cts) if,  $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$  for every  $B \in \sigma$ ,
- iv) intuitionistic fuzzy pre-continuous mapping [18] (IFP cts) if,  $f^{-1}(B) \in \text{IFPO}(X)$  for every  $B \in \sigma$ ,
- v) intuitionistic fuzzy completely continuous mapping [6] if,  $f^{-1}(B) \in \text{IFRO}(X)$  for every  $B \in \sigma$ ,
- vi) intuitionistic fuzzy generalized continuous mapping [16] (IFG cts) if,  $f^{-1}(B) \in \text{IFGO}(X)$  for every  $B \in \sigma$ ,
- vii) intuitionistic fuzzy generalized semi continuous mapping [12] (IFGS cts) if,  $f^{-1}(B) \in \text{IFGSO}(X)$  for every  $B \in \sigma$ ,
- viii) intuitionistic fuzzy  $\alpha$  generalized continuous mapping [11] (If $\alpha$ G cts) if,  $f^{-1}(B) \in \text{IF}\alpha\text{GO}(X)$  for every  $B \in \sigma$ .

Definition 2.16: A mapping  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy weakly  $\pi$  generalized closed mapping (IFW $\pi$ GCM) [ 9 ], if  $f(A)$  is an IFW $\pi$ GCS in  $Y$ , for every IFCS,  $A$  in  $X$ . In other words, every IFCS in  $X$  are mapped into IFW $\pi$ GCS in  $Y$ .

Definition 2.17: An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $w\pi T_{1/q}$  space (IF  $w\pi T_{1/q}$ ) [7], if every IFW $\pi$ GCS in  $X$  is an IFCS in  $X$ .

Definition 2.18: An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $w\pi T_q$  space (IF  $w\pi T_q$ ) [7], where  $0 < q < 1$ , if every IFW $\pi$ GCS in  $X$  is an IFPCS in  $X$ .

### III. INTUITIONISTIC FUZZY COMPLETELY WEAKLY $\pi$ GENERALIZED CONTINUOUS MAPPINGS

In this section, intuitionistic fuzzy completely weakly  $\pi$  generalized continuous mappings is defined. Some of its properties are studied.

Definition 3.1: A mapping  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy completely weakly  $\pi$  generalized continuous mapping (IF completely  $W\pi$ G cts) if,  $f^{-1}(B)$  is an IFRCS in  $X$  for every IFW $\pi$ GCS  $B$  of  $Y$ .

Proposition 3.1: Every IF completely  $W\pi$ G cts mapping is an IF continuous mapping.

Proof: Let  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi$ G cts mapping. Let  $B$  be an IFCS in  $Y$ . Since every IFCS is an IFW $\pi$ GCS,  $B$  is an IFW $\pi$ GCS in  $Y$ . Then  $f^{-1}(B)$  is an IFRCS in  $X$ , by definition. Since every IFRCS is an IFCS,  $f^{-1}(B)$  is an IFCS in  $X$ . Therefore  $f$  is an IF cts mapping.

Proposition 3.2: Every IF completely  $W\pi$ G cts mapping is an IF $\alpha$  continuous mapping.

Proof: Let  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi$ G cts mapping. Let  $B$  be an IFCS in  $Y$ . Since every IFCS is an IFW $\pi$ GCS,  $B$  is an IFW $\pi$ GCS in  $Y$ . Then  $f^{-1}(B)$  is an IFRCS in  $X$ , by definition. Since every IFRCS is an IF $\alpha$ CS,  $f^{-1}(B)$  is an IF $\alpha$ CS in  $X$ . Therefore  $f$  is an IF $\alpha$  cts mapping.

Proposition 3.3: Every IF completely  $W\pi$ G cts mapping is an IFP continuous mapping.

Proof: Let  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi$ G cts mapping. Let  $B$  be an IFCS in  $Y$ . Since every IFCS is an IFW $\pi$ GCS,  $B$  is an IFW $\pi$ GCS in  $Y$ . Then  $f^{-1}(B)$  is an IFRCS in  $X$ , by definition. Since every IFRCS is an IFPCS,  $f^{-1}(B)$  is an IFPCS in  $X$ . So  $f$  is an IFP cts mapping.

Remark 3.1: The proof of proposition 3.3 can also be seen as follows

Proof: Let  $f$  be an IF completely  $W\pi$ G cts mapping from  $(X, \tau)$  to  $(Y, \sigma)$ . If  $B$  be an IFCS in  $Y$ , then  $B$  is an IFW $\pi$ GCS in  $Y$ , because every IFCS is an IFW $\pi$ GCS. Therefore, by definition  $f^{-1}(B)$  is an IFRCS in  $X$ . Since every IFRCS is an IFPCS,  $f^{-1}(B)$  is an IFPCS in  $X$ . So  $f$  is an IFP cts mapping.

Proposition 3.4: Every IF completely  $W\pi$ G cts mapping is an IFG continuous mapping.

Proof: Let  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi$ G cts mapping. Let  $B$  be an IFCS in  $Y$ . Since every IFCS is an IFW $\pi$ GCS,  $B$  is an IFW $\pi$ GCS in  $Y$ . Then  $f^{-1}(B)$  is an IFRCS in  $X$ , by definition. Since every IFRCS is an IFGCS,  $f^{-1}(B)$  is an IFGCS in  $X$ . Therefore  $f$  is an IFG cts mapping.

Proposition 3.5: Every IF completely  $W\pi$ G cts mapping is an IF $\alpha$ G continuous mapping.

Proof: Let  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  cts mapping. Let  $B$  be an IFCS in  $Y$ . Since every IFCS is an IFW $\pi$ GCS,  $B$  is an IFW $\pi$ GCS in  $Y$ . Then  $f^{-1}(B)$  is an IFRCS in  $X$ , by definition. Since every IFRCS is an IF $\alpha$ GCS,  $f^{-1}(B)$  is an IF $\alpha$ GCS in  $X$ . Therefore  $f$  is an IF $\alpha$ G cts mapping.

Proposition 3.6: Every IF completely  $W\pi G$  cts mapping is an IFS continuous mapping.

Proof: Let  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  cts mapping. Let  $B$  be an IFCS in  $Y$ . Since every IFCS is an IFW $\pi$ GCS,  $B$  is an IFW $\pi$ GCS in  $Y$ . Then  $f^{-1}(B)$  is an IFRCS in  $X$ , by definition. Since every IFRCS is an IFS,  $f^{-1}(B)$  is an IFS in  $X$ . Therefore  $f$  is an IFS cts mapping.

Proposition 3.7: Every IF completely  $W\pi G$  cts mapping is an IFGS continuous mapping.

Proof: Let  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  cts mapping. Let  $B$  be an IFCS in  $Y$ . Since every IFCS is an IFW $\pi$ GCS,  $B$  is an IFW $\pi$ GCS in  $Y$ . Then  $f^{-1}(B)$  is an IFRCS in  $X$ , by definition. Since every IFRCS is an IFGSCS,  $f^{-1}(B)$  is an IFGSCS in  $X$ . Therefore  $f$  is an IFGS cts mapping.

Proposition 3.8: Every IF completely  $W\pi G$  cts mapping is an IFW $\pi$ G cts mapping.

Proof: Let  $f_1 (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  cts mapping. Let  $B$  be an IFCS in  $Y$ . Since every IFCS is an IFW $\pi$ GCS,  $B$  is an IFW $\pi$ GCS in  $Y$ . Then  $f^{-1}(B)$  is an IFRCS in  $X$ , by definition. Since every IFRCS is an IFW $\pi$ GCS,  $f^{-1}(B)$  is an IFW $\pi$ GCS in  $X$ . Therefore  $f$  is an IFW $\pi$ G cts mapping.

Remark 3.2: The non-validity of the converse of the propositions (3.1) to (3.8) are shown by means of the examples (3.1) to (3.8) as follows:

Example 3.1: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$  and  $T_1 = \langle x, (0.2, 0.4, 0.5), (0.8, 0.6, 0.5) \rangle$ ,  $T_2 = \langle y, (0.2, 0.4, 0.5), (0.8, 0.6, 0.5) \rangle$ . Then  $\tau = \{0_{\tau}, T_1, 1_{\tau}\}$  and  $\sigma = \{0_{\sigma}, T_2, 1_{\sigma}\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$ ,  $f(b) = v$  and  $f(c) = w$ . The IFS,  $B = \langle y, (0.6, 0.7, 0.8), (0.4, 0.3, 0.2) \rangle$  is IFW $\pi$ GCS in  $Y$ , but  $f^{-1}(B)$  is not an IFRCS in  $X$ , since  $(\text{cl}(\text{int}(f^{-1}(B)))) = T_1^c \neq f^{-1}(B)$ . Therefore  $f$  is an IF continuous mapping but not an IF completely  $W\pi G$  cts mapping.

Example 3.2: Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$ , where  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$ , by  $f(a) = u$ ,  $f(b) = v$  and  $f(c) = w$ . Let  $T_1 = \langle x, (0.1, 0.2, 0.3), (0.8, 0.8, 0.6) \rangle$ ,  $T_2 = \langle y, (0.1, 0.2, 0.3), (0.8, 0.8, 0.6) \rangle$ . Then  $\tau = \{0_{\tau}, T_1, 1_{\tau}\}$  and  $\sigma = \{0_{\sigma}, T_2, 1_{\sigma}\}$  are IFTs on  $X$  and  $Y$  respectively. Let  $B = \langle y, (0.9, 0.9, 0.8), (0.1, 0.1, 0.2) \rangle$  is an IFW $\pi$ GCS in  $Y$ , but  $f^{-1}(B) = \langle x, (0.9, 0.9, 0.8), (0.1, 0.1, 0.2) \rangle$  is not an IFRCS in  $X$ , since  $(\text{cl}(\text{int}(f^{-1}(B)))) = T_1^c \neq f^{-1}(B)$ . Therefore  $f$  is an IF $\alpha$  continuous mapping but not an IF completely  $W\pi G$  continuous mapping.

Example 3.3: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$  and  $T_1 = \langle x, (0.5, 0.4, 0.5), (0.5, 0.6, 0.5) \rangle$ ,  $T_2 = \langle x, (0.2, 0.3, 0.3), (0.8, 0.7, 0.7) \rangle$  and  $T_3 = \langle y, (0.2, 0.3, 0.3), (0.8, 0.7, 0.7) \rangle$ . Then  $\tau = \{0_{\tau}, T_1, T_2, 1_{\tau}\}$  and  $\sigma = \{0_{\sigma}, T_3, 1_{\sigma}\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow$



$(Y, \sigma)$  by  $f(a) = u$ ,  $f(b) = v$  and  $f(c) = w$ . The IFS,  $B = \langle \gamma, (0.8, 0.5, 0.2), (0.1, 0.6, 0.7) \rangle$  is IFW $\pi$ GCS in  $Y$ , but  $f^{-1}(B)$  is not an IFRCS in  $X$ . Therefore  $f$  is an IFP continuous mapping but not an IF completely W $\pi$ G cts mapping.

Example 3.4: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$  and  $T_1 = \langle \alpha, (0.5, 0.4, 0.5), (0.5, 0.6, 0.5) \rangle$ ,  $T_2 = \langle \alpha, (0.5, 0.3, 0.3), (0.5, 0.7, 0.7) \rangle$  and  $T_3 = \langle \gamma, (0.5, 0.3, 0.3), (0.5, 0.7, 0.7) \rangle$ . Then  $\tau = \{0_\alpha, T_1, T_2, 1_\alpha\}$  and  $\sigma = \{0_\gamma, T_3, 1_\gamma\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$ ,  $f(b) = v$  and  $f(c) = w$ . Then  $f$  is an IFG continuous mapping but not an IF completely W $\pi$ G continuous mapping, since IFS,  $B = \langle \gamma, (0.6, 0.8, 0.7), (0.3, 0.2, 0.3) \rangle$  is IFW $\pi$ GCS in  $Y$ , but  $f^{-1}(B) = \langle \alpha, (0.6, 0.8, 0.7), (0.3, 0.2, 0.3) \rangle$  is not an IFRCS in  $X$ , since  $cl(int(f^{-1}(B))) = T_1^\alpha \neq f^{-1}(B)$ .

Example 3.5: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$  and  $T_1 = \langle \alpha, (0.4, 0.2, 0.1), (0.6, 0.7, 0.8) \rangle$ ,  $T_2 = \langle \alpha, (0.3, 0.1, 0.1), (0.6, 0.8, 0.8) \rangle$  and  $T_3 = \langle \gamma, (0.3, 0.1, 0.1), (0.6, 0.8, 0.8) \rangle$ . Then  $\tau = \{0_\alpha, T_1, T_2, 1_\alpha\}$  and  $\sigma = \{0_\gamma, T_3, 1_\gamma\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$ ,  $f(b) = v$  and  $f(c) = w$ . The IFS,  $B = \langle \gamma, (0.6, 0.7, 0.8), (0.2, 0.3, 0.2) \rangle$  is IFW $\pi$ GCS in  $Y$ , but  $f^{-1}(B)$  is not an IFRCS in  $X$ . Therefore  $f$  is an IF $\alpha$ G continuous mapping but not an IF completely W $\pi$ G cts mapping.

Example 3.6: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$  and  $T_1 = \langle \alpha, (0.4, 0.3, 0.2), (0.5, 0.5, 0.7) \rangle$ ,  $T_2 = \langle \gamma, (0.4, 0.3, 0.2), (0.5, 0.5, 0.7) \rangle$ . Then  $\tau = \{0_\alpha, T_1, 1_\alpha\}$  and  $\sigma = \{0_\gamma, T_2, 1_\gamma\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$ ,  $f(b) = v$  and  $f(c) = w$ . Then  $f$  is an IFS continuous mapping but not an IF completely W $\pi$ G continuous mapping, since IFS,  $B = \langle \gamma, (0.7, 0.6, 0.5), (0.2, 0.4, 0.5) \rangle$  is IFW $\pi$ GCS in  $Y$ , but  $f^{-1}(B) = \langle \alpha, (0.7, 0.6, 0.5), (0.2, 0.4, 0.5) \rangle$  is not an IFRCS in  $X$ , since  $cl(int(f^{-1}(B))) = T_1^\alpha \neq f^{-1}(B)$ .

Example 3.7: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$  and  $T_1 = \langle \alpha, (0.5, 0.3, 0.2), (0.5, 0.6, 0.7) \rangle$ ,  $T_2 = \langle \alpha, (0.2, 0.1, 0.2), (0.6, 0.8, 0.6) \rangle$  and  $T_3 = \langle \gamma, (0.2, 0.1, 0.2), (0.6, 0.8, 0.6) \rangle$ . Then  $\tau = \{0_\alpha, T_1, T_2, 1_\alpha\}$  and  $\sigma = \{0_\gamma, T_3, 1_\gamma\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$ ,  $f(b) = v$  and  $f(c) = w$ . The IFS,  $B = \langle \gamma, (0.6, 0.8, 0.6), (0.2, 0.1, 0.2) \rangle$  is IFW $\pi$ GCS in  $Y$ , but  $f^{-1}(B)$  is not an IFRCS in  $X$ . Therefore  $f$  is an IFGS continuous mapping but not an IF completely W $\pi$ G cts mapping.

Example 3.8: Let  $X = \{a, b, c\}$ ,  $Y = \{u, v, w\}$  and  $T_1 = \langle \alpha, (0.2, 0.3, 0.3), (0.8, 0.7, 0.7) \rangle$ ,  $T_2 = \langle \alpha, (0.8, 0.7, 0.7), (0.2, 0.3, 0.3) \rangle$  and  $T_3 = \langle \gamma, (0.8, 0.7, 0.7), (0.2, 0.3, 0.3) \rangle$ . Then  $\tau = \{0_\alpha, T_1, T_2, 1_\alpha\}$  and  $\sigma = \{0_\gamma, T_3, 1_\gamma\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$ ,  $f(b) = v$  and  $f(c) = w$ . Then  $f$  is an IFW $\pi$ G continuous mapping but not an IF completely W $\pi$ G continuous mapping, since IFS,  $B = \langle \gamma, (0.8, 0.9, 0.8), (0.2, 0.1, 0.2) \rangle$  is IFW $\pi$ GCS in  $Y$ , but  $f^{-1}(B) = \langle \alpha, (0.8, 0.9, 0.8), (0.2, 0.1, 0.2) \rangle$  is not an IFRCS in  $X$ , since  $cl(int(f^{-1}(B))) = T_1^\alpha \neq f^{-1}(B)$ .

Remark 3.3: The derived relationship among the terms, defined above, have the following schematic representation.

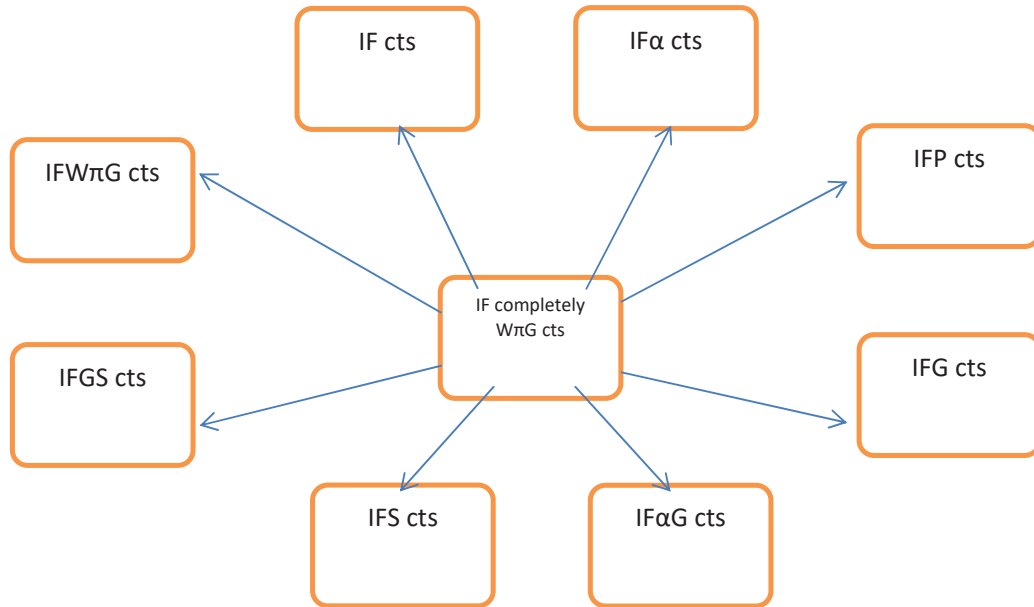


Fig.3.1 Relationships between the intuitionistic fuzzy weakly  $\pi$  generalized closed mapping, and the other existing closed mapping on intuitionistic fuzzy closed sets.

Theorem 3.1: Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \theta)$  be any two mappings where  $(Z, \theta)$  is an IF  $w\pi T_{1/2}$  space. Then the following statements are hold good:

- (1) If  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  continuous mapping and  $g: (Y, \sigma) \rightarrow (Z, \theta)$  is an IF continuous mapping, then their composition  $g \circ f: (X, \tau) \rightarrow (Z, \theta)$  is an IF completely  $W\pi G$  continuous mapping.
- (2) If  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  continuous mapping and  $g: (Y, \sigma) \rightarrow (Z, \theta)$  is an IF $\alpha$  continuous mapping, then their composition  $g \circ f: (X, \tau) \rightarrow (Z, \theta)$  is an IF completely  $W\pi G$  continuous mapping.
- (3) If  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  continuous mapping and  $g: (Y, \sigma) \rightarrow (Z, \theta)$  is an IF $\alpha G$  continuous mapping, then their composition  $g \circ f: (X, \tau) \rightarrow (Z, \theta)$  is an IF completely  $W\pi G$  continuous mapping.
- (4) If  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  continuous mapping and  $g: (Y, \sigma) \rightarrow (Z, \theta)$  is an IFP continuous mapping, then their composition  $g \circ f: (X, \tau) \rightarrow (Z, \theta)$  is an IF completely  $W\pi G$  continuous mapping.



- (5) If  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  continuous mapping and  $g: (Y, \sigma) \rightarrow (Z, \theta)$  is an IFG continuous mapping, then their composition  $g \circ f: (X, \tau) \rightarrow (Z, \theta)$  is an IF completely  $W\pi G$  continuous mapping.
- (6) If  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  continuous mapping and  $g: (Y, \sigma) \rightarrow (Z, \theta)$  is an IFWG continuous mapping, then their composition  $g \circ f: (X, \tau) \rightarrow (Z, \theta)$  is an IF completely  $W\pi G$  continuous mapping.
- (7) If  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF completely  $W\pi G$  continuous mapping and  $g: (Y, \sigma) \rightarrow (Z, \theta)$  is an IF completely continuous mapping, then their composition  $g \circ f: (X, \tau) \rightarrow (Z, \theta)$  is an IF completely  $W\pi G$  continuous mapping.

*Proof:*

- (1) Let  $A$  be an IFW $\pi$ GCS in  $Z$ . Since  $Z$  is a  $IF_{w\pi T_{1/2}}$  space,  $A$  is an IFCS in  $Z$ . Then  $g^{-1}(A)$  is a IFCS in  $Y$ , by definition. Since every IFCS is an IFW $\pi$ GCS,  $g^{-1}(A)$  is an IFW $\pi$ GCS in  $Y$ . Therefore  $f^{-1}(g^{-1}(A))$  is an IFRCS in  $X$ , by definition. Thus  $g \circ f$  is an IF completely  $W\pi G$  continuous mapping.
- (2) Let  $A$  be an IFW $\pi$ GCS in  $Z$ . Since  $Z$  is a  $IF_{w\pi T_{1/2}}$  space,  $A$  is an IFCS in  $Z$ . Then  $g^{-1}(A)$  is a IF $\alpha$ CS in  $Y$ , by definition. Since every IF $\alpha$ CS is an IFW $\pi$ GCS,  $g^{-1}(A)$  is an IFW $\pi$ GCS in  $Y$ . Therefore  $f^{-1}(g^{-1}(A))$  is an IFRCS in  $X$ , by definition. Thus  $g \circ f$  is an IF completely  $W\pi G$  continuous mapping.
- (3) Let  $A$  be an IFW $\pi$ GCS in  $Z$ . Since  $Z$  is a  $IF_{w\pi T_{1/2}}$  space,  $A$  is an IFCS in  $Z$ . Then  $g^{-1}(A)$  is a IF $\alpha$ GCS in  $Y$ , by definition. Since every IF $\alpha$ GCS is an IFW $\pi$ GCS,  $g^{-1}(A)$  is an IFW $\pi$ GCS in  $Y$ . Therefore  $f^{-1}(g^{-1}(A))$  is an IFRCS in  $X$ , by definition. Thus  $g \circ f$  is an IF completely  $W\pi G$  continuous mapping.
- (4) Let  $A$  be an IFW $\pi$ GCS in  $Z$ . Since  $Z$  is a  $IF_{w\pi T_{1/2}}$  space,  $A$  is an IFCS in  $Z$ . Then  $g^{-1}(A)$  is a IFPCS in  $Y$ , by definition. Since every IFPCS is an IFW $\pi$ GCS,  $g^{-1}(A)$  is an IFW $\pi$ GCS in  $Y$ . Therefore  $f^{-1}(g^{-1}(A))$  is an IFRCS in  $X$ , by definition. Thus  $g \circ f$  is an IF completely  $W\pi G$  continuous mapping.
- (5) Let  $A$  be an IFW $\pi$ GCS in  $Z$ . Since  $Z$  is a  $IF_{w\pi T_{1/2}}$  space,  $A$  is an IFCS in  $Z$ . Then  $g^{-1}(A)$  is a IFGCS in  $Y$ , by definition. Since every IFGCS is an IFW $\pi$ GCS,  $g^{-1}(A)$  is an IFW $\pi$ GCS in  $Y$ . Therefore  $f^{-1}(g^{-1}(A))$  is an IFRCS in  $X$ , by definition. Thus  $g \circ f$  is an IF completely  $W\pi G$  continuous mapping.
- (6) Let  $A$  be an IFW $\pi$ GCS in  $Z$ . Since  $Z$  is a  $IF_{w\pi T_{1/2}}$  space,  $A$  is an IFCS in  $Z$ . Then  $g^{-1}(A)$  is a IFW $\pi$ GCS in  $Y$ , by definition. Therefore  $f^{-1}(g^{-1}(A))$  is an IFRCS in  $X$ , by definition. Thus  $g \circ f$  is an IF completely  $W\pi G$  continuous mapping.
- (7) Let  $A$  be an IFW $\pi$ GCS in  $Z$ . Since  $Z$  is a  $IF_{w\pi T_{1/2}}$  space,  $A$  is an IFCS in  $Z$ . Then  $g^{-1}(A)$  is a IFRCS in  $Y$ , by

definition. Since every IFRCs is an IFW $\pi$ GCS,  $g^{-1}(A)$  is an IFW $\pi$ GCS in  $Y$ . Therefore  $f^{-1}(g^{-1}(A))$  is an IFRCs in  $X$ , by definition. Thus  $g \circ f$  is an IF completely W $\pi$ G continuous mapping.

#### IV. CONCLUSION

In this paper, intuitionistic fuzzy completely weakly  $\pi$  generalized continuous mapping is defined, with example. The relationship among the intuitionistic fuzzy completely weakly  $\pi$  generalized continuous mapping and other existing types of intuitionistic fuzzy continuous mappings are obtained.

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