The Analysis of Coriolis Effect on a Robot Manipulator

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Abstract: This paper attempts to design a logic controller for controlling the position of the arms of a two-link manipulators taking into account the disturbances caused by inertial loading, coupling reaction forces between joints (Coriolis and centrifugal), and gravity loading effects. The design for the position control incorporates a torque computation block that compensates for the disturbances caused by these effects by a logic controller. The simulation results for the position control of a two-link robotic manipulator using the compensated logic model were then compared with the results obtained using an optimized PID controller (proportional-integral-derivative controller). The results clearly indicated that the performance of the compensated logic controller was much better than the optimized PID.

I. INTRODUCTION

Robotics is a relatively young field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their applications requires knowledge of electrical engineering, mechanical engineering, systems and industrial engineering, computer science, economics, and mathematics. New disciplines of engineering, such as manufacturing engineering, applications engineering, and knowledge engineering have emerged to deal with the complexity of the field of robotics and factory automation. In this paper, a new spatial 3·DOF parallel manipulator with three non-identical chains is developed. The movable platform has three degrees of freedom, which are two degrees of translational freedom and one degree of rotational freedom, with respect to the base plate. The kinematics problems and velocity equation of the new parallel manipulator are given. Three kinds of singularities are presented.

This paper is concerned with the effects of actuation schemes on three measures of kinematic performance which depend upon a manipulators Jacobean Matrix, the manipulability (yoshikawa ,1985) and condition number (Salisbury and craig, 1982; angeles, 1992; kircanski, 1994; angeles,1995). We begin by presenting a simple framework on how to incorporate actuator location in the kinematic model. For each measure we drive properties relating its joints space to its actuator space description. Finally we employ these concepts in the design of a spatial parallel manipulator.

II. LITERATURE SURVEY

David E. Orin and *William W. Schrader* discussed the paper on six different methods for computing the Jacobian matrix for a general ‘n’ degrees of freedom manipulator. In this, they compared several approaches that have been proposed for computing Jacobian matrix, including the new approach introduced. The result is found to be efficient for Jacobian matrix, when it is based on end effector coordinates. B. Roth first investigated the relationship between kinetic geometry and manipulator workspace. *J. Denavit* and *R.S Hartenberg* gave a description of manipulator in terms of link lengths, link twists and joint angles. According to *G.Strang*, the condition number of a matrix is used in numerical analysis to estimate the errors generated in the solution of linear equations by error on the data. The concept was attracted the attention of the several researchers. This concept was utilized by *Salisbury J.K.* and *Craig J.J* [14] for research in kinematics and control issues in contest of an articulated multi-finger mechanical hand.The work is related to analyzing and optimizing the kinematic structure of manipulators with out regard to the form of actuation of the joints. Robot singularities cause many problems in motion synthesis, both for non redundant and for redundant robots. Although redundant robots may be able to avoid singularities inside the workspace boundaries, non-redundant robots have to find the method to pass through the singular configurations, if motion specified in the task coordinates is to be achieved inside the whole robot workspace

Coriolis Effect : It is a deflection occurred in moving objects when they are viewed in a rotating reference frame. In a reference frame with clockwise rotation, the deflection is to the left of the motion of the object; in another with counter-clockwise rotation, the deflection is to the right. Coriolis Effect in case of a robot is the deflection or the error occurred in the target achieved when one arm of a robot is rotating above the another rotating arm The design for the position control incorporates a torque computation block that compensates for the disturbances caused by these effects by a logic controller. The simulation results for the position control of a two-link robotic manipulator using the compensated logic model were then compared with the results obtained using an optimized PID controller (proportional-integral-derivative controller). The results clearly indicated that the performance of the compensated logic controller was much better than the optimized PID.

2.1 Robot Dynamics: Robot dynamics basically deals with mathematical formulations of the equations of robot arm motion. The dynamic equations of motion of a manipulator are a set of mathematical equations describing the dynamic behavior of the manipulator. Such equations of motion are useful for computer simulation of the robot arm motion, design of suitable control equations of a robot arm and evaluation of kinematic design and structure of a robot arm. The actual dynamic model can be obtained from known physical laws such as the laws of Newtonian and Lagrangian mechanics.

2.2 L-E Formulation **:**The general motion equations of a manipulator can conveniently be expressed through the direct application of the Lagrange Euler (L-E) formulation to non - conservative systems. The L-E equation is given as follows

$$d / dt (\partial L / \partial q_i') - \partial L / \partial q_i = \tau_i \quad i = 1,2,3,\ldots\ldots\ldots n$$

where

L = Lagrangian function = kinetic energy K- potential energy P

K = total kinetic energy of the robot arm P = total potential energy of the robot arm

q_i = generalized coordinates of the robot arm

q_i'= first time derivative of the generalized coordinate, q_i

τi = generalized force (or torque) applied to the system at joint i to drive link i

From the above Lagrange-Euler equation, one is required to properly choose a set of generalized coordinates to describe the system. Generalized coordinates are used as a convenient set of coordinates that completely describe the location (position and orientation) of a system with respect to a reference coordinate frame. For a simple manipulator with rotary-prismatic joints, various sets of generalized positions of the joints are readily available because they can be measured by potentiometers or encoders or other sensing devices, they provide a natural correspondence with the generalized coordinates.

Thus, in the case of a rotary joint, $q_i \equiv \theta_i$, the joint angle span of the joint.

2.3 Motion Equations of a manipulator: The derivation of equations of motion of a manipulator requires the computation of the total kinetic energy and the total potential energy of the manipulator.

2.4 Kinetic Energy of the manipulator: If dK_i is the kinetic energy of a particle with differential mass dm in link i; then

$$dK_i = ½(x_i'^2 + y_i'^2 + z_i'^2)\, dm$$

$$= ½ \text{ trace } (vi\, v_i^T)\, dm$$

$$=½ \text{ Tr } (vi\, v_i^T)\, dm$$

where $v_i = (\Sigma_j U_{ij} q_j')\, {}^i r_i$ j varies from 1 to i

$$U_{ij} = {}^0A_{j-1} Q_j {}^{j-1}A_i \quad \text{for } j <= i$$

$$0 \quad \text{for } j > i$$

Q_j is a 4 x 4 matrix, given as

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ for revolute joint} \qquad \textbf{.........(1)}$$

${}^{i}r_i$ = fixed point at rest in a link 'i' w.r.t homogeneous coordinates of the i^{th} link coordinate

${}^{i}A_j$ = homogeneous coordinate transformation matrix

which relates j^{th} coordinate frame to the i^{th} coordinate frame.

The total kinetic energy of all the links can be found out by integrating dK_i over all the links.

The total kinetic energy K of a robot arm after integration will be found as

$K = \int dK_i = ½\, Tr\, [\, \Sigma_p \Sigma_r U_{ip} (J_i) U_{ir}^{T} q_p' q_r')]$ p, r varies from 1 to i **……(2)**

where

J_i is the inertia of all the points on link i given by

$J_i = \int {}^{i}r_i \, {}^{i}r_i^{T} dm$

2.5 Potential Energy of the Manipulator

Let P_i be the i^{th} link's potential energy. The P_i is given by

$P_i = -m_i\, g\, ({}^{0}A_i \, {}^{i}ř_i)$ i = 1, 2, ……….. n

where ${}^{0}A_i$ is the coordinate transformation matrix which relates the i^{th} coordinate frame to the base coordination frame

${}^{i}ř_i$ is the a fixed point in the i^{th} link expressed in homogenous coordinates with respect to the i^{th} link coordinate frame .

The total potential energy P of the robot manipulator is found out by integrating P_i over all the links.

$P = \Sigma_i P_i = \Sigma_i -m_i\, g\, ({}^{0}A_i \, {}^{i}ř_i)$ i varies from 1 to n **……(3)**

Where g = (0, 0, -|g|, 0) and g is the gravitational constant. ($g = 9.8062\ m/sec^2$)

2.6 L-E formulation for the two-link manipulator

From equations (2) and (3), the Lagrangian function L = K-P is given by

$L = ½\, \Sigma_i \Sigma_j \Sigma_k [\, Tr\, (U_{ij} J_i U_{ik}^{T}) q_j' q_k'] + \Sigma_i m_i\, g\, ({}^{0}A_i \, {}^{i}ř_i)$

i varies from 1 to n, j,k varies from 1 to i

Applying the Lagrange-Euler formulation to the Lagrangian function, the generalized torque for joint i actuator to drive the i^{th} link of the manipulator,

$\tau_i = \Sigma_j \Sigma_k Tr\, (U_{jk} J_j U_{ji}^{T}) q_k'' + \Sigma_j \Sigma_k \Sigma_m Tr\, (U_{jkm} J_j U_{ji}^{T}) q_k' q_m' - \Sigma_j m_j\, g\, U_{ji} \, {}^{j}ř_j$

for i = 1, 2, 3…………n

where j varies from i to n; k and m varies from 1 to j

The above equation can be expressed in a much simpler matrix form as

$\tau(t) = D(q(t))\, q(t)'' + h(q(t), q(t)') + c(q(t))$

where $\tau(t)$ = n x 1 generalized torque vector applied at joints i = 1,2……….n, that is,

$$\tau(t) = (\tau_1(t), \tau_2(t), \ldots\ldots, \tau_n(t))^T$$

q(t) = an n x 1 vector of the joint variables of the robot arm and can be expressed as

$$q(t) = (q_1(t), q_2(t), \ldots\ldots, q_n(t))^T$$

q(t)' = an n x 1 vector of the joint velocity of the robot arm and can be expressed as

$$q(t)' = (q_1(t)', q_2(t)', \ldots\ldots, q_n(t)')^T$$

q(t)''= an n x 1 vector of the joint acceleration of the joint variables q(t) and can be expressed as

$$q(t)'' = (q_1(t)'', q_2(t)'', \ldots\ldots, q_n(t)'')^T$$

(q) = an n x n inertial acceleration-related symmetric matrix whose elements are

$$D_{ik} = \Sigma_j \mathrm{Tr}\,(U_{jk} J_j U_{ji}^T) \quad i, k = 1,2 \ldots\ldots n$$

j varies from max(i,k) to n

h(q, q') = an 1x 1 nonlinear Coriolis and centrifugal force vector whose elements are $h(q,q') = (h_1, h_2, \ldots\ldots h_n)^T$

Where $h_i = \Sigma k\, \Sigma_m\, q_k' q_m'$ i =1,2, ………….n. ……(4)

and $h_{ikm} = \Sigma_j \mathrm{Tr}\,(U_{jkm} J_j U_{ji}^T)$ i, k, m = 1,2……..n

j varies from max(i,k,m) to n

c(q) = an n x 1 gravity loading force vector whose elements are

$c(q) = (c_1, c_2, \ldots\ldots c_n)^T$

Where $c_i = = \Sigma_j (-m_j\, g\, U_{ji}\, {}^j\check{r}_j)$ i = 1,2,…….n . …...(5)

j varies from i to n.

The computational complexity of the L-E formulation increases with the 4th power of the number of degrees of freedom of the robot arm. Hence for the simulation purpose a specific simple case of a two-link manipulator with revolute joints is taken. We assume the following in our derivation of the generalized torque equation for the two-link manipulator: joint variables= θ_1, θ_2 ,mass of the links = m_1, m_2 ,

link parameters = $\alpha_1 = \alpha_2 = 0$; $d_1 = d_2 = 0$; $a_1 = a_2 = l$.

The homogenous coordinate transformation matrices ${}^{i-1}A_i$ (i = 1,2) are obtained as

${}^{0}A_1 = [(C_1, -S_1, 0, lC_1), (S_1, C_1, 0, lS_1), (0, 0,1, 0), (0, 0, 0, 1)]$ …..(6)

${}^{1}A_2 = [(C_2, -S_2, 0, lC_2), (S_2, C_2, 0, lS_2), (0, 0,1, 0), (0, 0, 0, 1)]$ …..(7)

${}^{0}A_2 = {}^{0}A_1\ {}^{1}A_2 = [(C_{12}, -S_{12}, 0, l(C_{12} + C_1), (S_{12}, C_{12}, 0, l(S_{12} + S_1), (0, 0,1, 0), (0, 0, 0, 1)]$

…...(8)

where $C_i = \cos\theta_i$; $S_i = \sin\theta_i$; $C_{ij} = \cos(\theta_i + \theta_j)$; $fS_{ij} = \sin(\theta_i + \theta_j)$

From the **eqns 1, 6, 7, 8** we can derive the following results

2.7 Inertia Effects: The elements of the acceleration related symmetric matrix D(θ) can be found as

$D_{11} = 1/3\ m_1 l^2 + 4/3\ m_2 l^2 + m_2 C_2 l^2$. **…….(9)**

$D_{12} = D_{21} = 1/3\ m_2 l^2 + ½\ m_2 C_2 l^2$. **……(10)**

$D_{22} = 1/3\ m_2 l^2$. …..(11)

Where D_{11}, D_{12}, D_{21}, D_{22} are inertia elements.

2.8 Coriolis Effects:

The velocity related coefficients in the Coriolis and Centrifugal terms could be obtained from equations 4 and 5

as $h_1 = -½\ m_2 S_2 l^2 \theta_2'^2 - m_2 S_2 l^2 \theta_1' \theta_2'$ **…….(12)**

$h_2 = -½\ m_2 S_2 l^2 \theta_1'^2$ **……..(13)**

2.9 Gravity Effects:

The gravity loading force vector elements can be obtained from equation 5 as

$c_1 = ½\ m_1 g l C_1 + ½\ m_2 g l C_{12} + m_2 g l C_1$ ……..(14)

$c_2 = ½\ m_2 g l C_{12}$ ...….(15)

Finally, the L-E equations of motion for the two-link manipulator are found to be

$\tau_1 = 1/3\ m_1 l^2 + 4/3\ m_2 l^2 + m_2 C_2 l^2 + 1/3\ m_2 l^2 + ½\ m_2 C_2 l^2 - ½ m_2 S_2 l^2 \theta_2^2 - m_2 S_2 l^2 \theta_1 \theta_2 + ½\ m_1 g l C_1 + ½\ m_2 g l C_{12} + m_2 g l C_1$ ………(16)

$\tau_2 = 1/3\ m_2 l^2 + ½\ m_2 C_2 l^2 + 1/3\ m_2 l^2 - ½ m_2 S_2 l^2 \theta_1^2 + ½\ m_2 g l C_{12}$ …….(17)

The inertia, gravity and Coriolis Effect have been identified as the major contributors for the inaccuracy in the positioning of the robot arms. Hence an accurate controller invariably has to overcome these adverse effects by including some kind of compensation logic in the design. To view robotics as an application of the principles of motions, together with motors to provide motions and sensors to provide location and velocity may miss the inherent complexity of the discipline.

A real robot does face potential errors due to a number of reasons, including: incorrect parameters (for example mass, direction, distance) values, frictional forces ,terrain estimations, play at the link joints, calibration errors in sensors, error in the values read from the sensors. Among such errors centripetal and Coriolis effects are of great importance when robot manipulator is moving at high speeds.

III. CONCLUSION AND FUTURE SCOPE

This superior performance of the controller is complimented by the reduced oscillations of the control variable about the set point .The magnified portion of the simulation graph to demonstrate the amplitude of oscillations experienced by an arm about the set point in the case of optimized PID controllers with accurate position control is required for a sensitive industrial or medical application, this kind of oscillations about the set point is highly undesirable.

Hence the proposed design of controller has the following advantages over the conventional controllers:

1. More accurate tracking of the set point variations.
2. Reduced oscillations about the set point
3. Settling time is comparatively smaller.

REFERENCES

[1] Angeles's, J., 1992, "The Design of Isotropic Manipulator Architectures in the Presence of Redundancies." The International Journal of Robotic Research, Vol.11;No.3, pp 196-201

[2] Angeles's , J.,1995, "Kinematic Isotropy in Humans and machines," processedings of the 9th World Congress of the Theory of Machines and Mechanisms, Vol. I,Milano , Italy

[3] Horn,R..A.,. and Johnson,C.R..1991, "Topics in Matrix Analysis," Cambridge university press

[4] Kircanski, M.V., 1994, "Robotic Isotropic and Optimal Robot Design of Planar Manipulators,".Proc.IEEE International Conference on Robotics And Automation, San Diego, USA, pp.1100-1105.

[5] Klein,C.A.,and Blaho, B,E.,1987, "Dexterity Measures for the Design and Control of Kinematically Redundant Manipulators," The International Journal of Robotic Research, Vol.6, No.2,pp,72-83,

[6] Matone.R, 1997, "The Effects of Actuation Schemes on the Performance of Manipulators," PhD Thesis, Department of Mechanical Engineering, Standard University Stanford, CA.

[7] Liegois, A., 1977, "Automatic Supervisory Control for the Configuration and Behavior of MultibodyMechanisms," IEEE Trans. of System.Man,Cybernatics, SMC-7 (12) ,pp .842-868.

[8] Salisbury.J.K., and Craig,J,J.,1982,"Articulated Hands:Force Control and Kinematic Issues," The International Journal of Robotic Research, Vol.,I.,No.1, pp..4-17.