Reliability measures of two Unit Parallel System with Different Repair Policies of the Servers

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Abstract—This paper analyzes a reliability model of parallel system of two identical units with different repair policies of the servers subject to maximum repair time. Initially, ordinary server who takes some time to arrive at the system repairs the failed unit. If ordinary server is unable to repair that unit in a pre-specified time (called maximum repair time), than inspection of the unit is conducted by him to see the feasibility of its repair by an expert server who visits the system immediately. Furthermore, if repair of the unit is not possible by the expert server, it is replaced by the ordinary server giving some replacement time. The ordinary server remains with the system during repair of the unit by an expert server. All random variables are statistically independent. The switch over is instantaneous and perfect. The failure rate and the rate by which unit undergoes for inspection to see the feasibility of repair are taken as constant while the distribution of arrival time of server, inspection time, repair time and replacement time of the unit are assumed as arbitrary with different probability density functions. The expressions for various measures of the system effectiveness have been derived in steady state using semi-Markov process and regenerative point technique. The graphical results for MTSF, availability and profit are obtained giving particular values to various parameters and costs.

Keywords—Arrival time of server, Cost-Benefit Analysis ,Maximum Repair Time, Parallel-Unit System and Reliability.

MATHEMATICS SUBJECT CLASSIFICATION (2000): 90B25 AND 60K10

I. INTRODUCTION

In the field of reliability, redundant system has been widely studied as these are frequently used in modern technology. Various researchers including Shriniwasan and Gopalan [1973], Singh [1995] have analyzed the system with cold standby redundancy. But sometimes, to attain better reliability and system performance, the introduction of standby redundancy is not suggestive. So it is desirable to introduce parallel redundancy. Gaver [1963] considered a two-unit parallel repairable system with a repairman. Nakagawa and Osaki [1975] analyzed stochastic behavior of parallel system with repair maintenance while Gupta and Kumar [1994] analyzed a parallel system with periods of working and rest. In most of these papers it is assumed that every server is capable in repairing the unit and its repair facility becomes available immediately as and when required. But in practice there are many situations when service facility may take some time to arrive at the system and also sometimes single server is not able to repair the failed unit in fixed period of time. Sridharan and Mohanavadivu (1998) studied the stochastic behavior of a two-unit cold standby redundant system, with two types of repairmen. Furthermore, system can be made more reliable by making replacement of the failed unit by new ones in case server is not able to repair the unit in pre-specific period of time. Malik and Gitanjali [2012] discussed a parallel system with arrival time of the server and replacement of the unit after maximum repair time of the server. It can be pointed out from above that no research paper has been written so far in reliability theory on parallel system considering two types of server, arrival and maximum repair time to the server and replacement of unit by new ones all together in a paper.

Keeping all the above circumstances in mind, a parallel system of two identical units with different repair policies of the servers subject to maximum repair time. Initially, ordinary server who takes some time to arrive at the system repairs the failed unit. If ordinary server is unable to repair that unit in a pre-specified time (called maximum repair time), than inspection of the unit is conducted by him to see the feasibility of its repair by an expert server who visits the system immediately. Furthermore, if repair of the unit is not possible by the expert server, it is replaced by the ordinary server giving some replacement time. The ordinary server remains with the system during repair of the unit by an expert server. All random variables are statistically independent. The switch over is instantaneous and perfect. The failure rate and the rate by which unit undergoes for inspection to see the feasibility of repair are taken as constant while the distribution of arrival time of server, inspection time, repair time and replacement time of the unit are assumed as arbitrary with different probability density functions. The expressions for various measures of the system effectiveness have been derived in steady state using semi-Markov process and regenerative point technique. The graphical results for MTSF, availability and profit are obtained giving particular values to various parameters and costs.

II. NOTATIONS

E	:	Set of regenerative states
O	:	Unit is operative
λ	:	Constant failure rate of the unit
α_0	:	Maximum constant rate of repair time taken by the server
α/β	:	Probability that failed unit is not repairable / repairable by an expert server
$f(t)/F(t)$	:	pdf / cdf of the replacement time of the unit
$g(t)/G(t)$	:	pdf / cdf of the repair time of t he unit
$g_1(t)\,/\,G_1(t)$	:	pdf /cdf of the repair time of the unit taken by expert server
$w(t)\,/W(t)$	:	pdf/cdf of the arrival time of the ordinary server
$h(t)/H(t)$	:	pdf/cdf of the inspection time of the unit taken by ordinary server
$FU_r\,/\,FU_R$	:	Unit is failed and under repair / under repair continuously from previous state
$FW_r\,/\,FW_R$	:	Unit is failed and waiting for repair/waiting for repair continuously from previous state
$FU_i\,/\,FU_I$	:	Unit is failed and under inspection by ordinary server / waiting for inspection by ordinary server continuously from previous state.
FU_{re}/FU_{Re}	:	Unit is failed and under repair by expert server / under repair continuously from previous state by expert server
$FU_{Rp}\,/\,FU_{RP}$	:	Unit is failed and under replacement with ordinary server / under replacement continuously from previous state with ordinary server
~/*	:	Symbol for Laplace Stieltjes transform / Laplace transform
$\boxed{S}$/©	:	Symbols for Stieltjes convolution / Laplace convolution.
‘ (desh)	:	Symbol for derivative of the function

The possible transitions between states along with transitions rates for the system model are shown in Figure 1.The states S_0,S_1 ,S_2,S_3,S_4,$S_5$and S_6 are regenerative while the other states are non-regenerative.

III. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements $p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\, dt$ as:

$$p_{01} = 1, \quad p_{12} = w^*(\lambda), \quad p_{1,11} = \left(1 - w^*(\lambda)\right),$$
$$p_{20} = g^*(\lambda + \alpha_0), \quad p_{23} = \frac{\alpha_0}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)],$$
$$p_{2,14} = \frac{\lambda}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)], \quad p_{34} = a h^*(\lambda),$$
$$p_{35} = b h^*(\lambda), \quad p_{37} = \left(1 - h^*(\lambda)\right), \quad p_{40} = f^*(\lambda),$$
$$p_{46} = \left(1 - f^*(\lambda)\right), \quad p_{50} = g_1{}^*(\lambda),$$
$$p_{5,10} = \left(1 - g_1{}^*(\lambda)\right) \quad p_{62} = f^*(0), \quad p_{78} = a, p_{79} = b,$$
$$p_{82} = f^*(0), \quad p_{92} = g_1{}^*(0), \quad p_{10,2} = g_1{}^*(0),$$

$$p_{11,12} = w^*(0), \quad p_{12,2} = g^*(\alpha_0), \quad p_{13,8} = a,$$
$$p_{13,9} = b, \quad p_{14,2} = g^*(\alpha_0), \quad p_{14,13} = \left(1 - g^*(\alpha_0)\right),$$
$$p_{12.11,12} = \left(1 - w^*(\lambda)\right) g^*(\alpha_0),$$
$$p_{12.11,12,13,8} = a\left(1 - w^*(\lambda)\right)\left(1 - g^*(\alpha_0)\right),$$
$$p_{12.11,12,13,9} = b\left(1 - w^*(\lambda)\right)\left(1 - g^*(\alpha_0)\right),$$
$$p_{22.14} = \frac{\lambda}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)] g^*(\alpha_0),$$
$$p_{22.14,13,8} = \frac{a\lambda}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)][1 - g^*(\alpha_0)],$$
$$p_{22.14,13,9} = \frac{b\lambda}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)][1 - g^*(\alpha_0)],$$
$$p_{32.78} = a\left(1 - h^*(\lambda)\right), \quad p_{32.79} = b\left(1 - h^*(\lambda)\right),$$
$$p_{42.6} = \left(1 - f^*(\lambda)\right), \quad p_{52.10} = \left(1 - g_1{}^*(\lambda)\right) \quad \ldots (1)$$

It can easily be verified that

$$p_{01} = p_{12} + p_{1,11} = p_{20} + p_{23} + p_{2,14} = p_{20} + p_{23} + p_{22.14} + p_{22.14,13,8} + p_{22.14,13,9} = p_{34} + p_{35} + p_{37} = p_{34} + p_{35} + p_{32.78} + p_{32.79} = p_{40} + p_{46} = p_{40} + p_{42.6} = p_{50} + p_{5,10} = p_{50} + p_{52.10} = p_{62} = p_{78} + p_{79} = p_{82} = p_{92} = p_{10,2} = p_{11,12} = p_{12,2} + p_{12,13} = p_{13,8} + p_{13,9} = p_{14,2} + p_{14,13} = 1 \quad \ldots (2)$$

The mean sojourn times μ_i in state S_i is given by

$$\mu_0 = \int_0^\infty P(T > t)\,dt = m_{01} = \frac{1}{2\lambda},$$
$$\mu_1 = m_{12} + m_{1,11} = \frac{1}{\lambda}\left(1 - w^*(\lambda)\right),$$
$$\mu_2 = m_{20} + m_{23} + m_{2,14} = \frac{1}{\alpha_0+\lambda}[1 - g^*(\lambda + \alpha_0)],$$
$$\mu_3 = m_{34} + m_{35} + m_{37} = \frac{1}{\lambda}\left(1 - h^*(\lambda)\right),$$
$$\mu_4 = m_{40} + m_{46} = \frac{1}{\lambda}\left(1 - f^*(\lambda)\right),$$
$$\mu_5 = m_{50} + m_{5,10} = \frac{1}{\lambda}\left(1 - g_1{}^*(\lambda)\right),$$
$$\mu_1' = m_{12} + m_{12.11,12} + m_{12.11,12,13,8} + m_{12.11,12,13,9}$$
$$- \left(1 - w^*(\lambda)\right)\left(\left(\frac{1}{\lambda} - w^{*\prime}(0)\right) + \left(1 - g^*(\alpha_0)\right)\right.$$
$$\left.\left(\frac{1}{\alpha_0} - b g_1{}^{*\prime}(0) - a f^{*\prime}(0) - h^{*\prime}(0)\right)\right)$$
$$\mu_2' = m_{20} + m_{23} + m_{22.14} + m_{22.14,13,8} + m_{22.14,13,9} =$$

$$\left[\left(\frac{1}{\alpha_0} - \quad b g_1^{*\prime}(0) - a f^{*\prime}(0) - h^{*\prime}(0)\right) 1 + \lambda\left(1 - g^{*}(\alpha_0)\right)\right] \frac{[1 - g^{*}(\alpha_0+\lambda)]}{\alpha_0+\lambda}$$

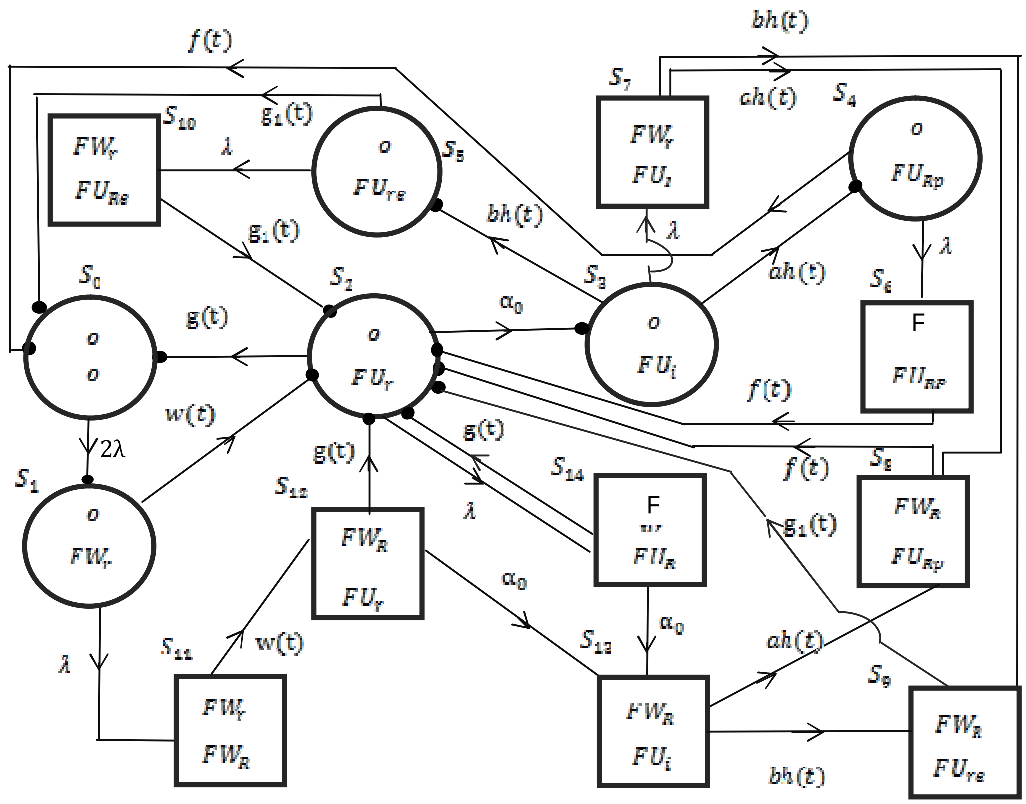
,

$$\mu_3' = m_{34} + m_{35} + m_{32.78} + m_{32.79} = \left(1 - h^{*}(\lambda)\right)\left[\left(1 - \lambda\left(b g_1^{*\prime}(0) + a f^{*\prime}(0) + h^{*\prime}(0)\right)\right)\right],$$

$$\mu_4' = m_{40} + m_{42.6} = \left(1 - f^{*}(\lambda)\right)\left(\frac{1}{\lambda} - f^{*\prime}(0)\right),$$

$$\mu_5' = m_{50} + m_{52.10} = \left(1 - g_1^{*}(\lambda)\right)\left(\frac{1}{\lambda} - g_1^{*\prime}(0)\right) \quad .... \; (3)$$

Figure1: State Transition Diagram



● : Regenerative Point O : Upstate ▭ : Failed State

IV. MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for $\phi_i(t)$:

$$\varphi_i(t) = \sum_j{}' Q_{i,jj}(t) \boxed{S} \varphi_j(t) + \sum_k{}' Q_{i,jk}(t) \qquad ... \; (4)$$

whereSj is an un-failed regenerative state to which the given regenerative state Si can transit and k is a failed state to which the state $i$can transit directly. Taking $L.S.T$ of relation (4) and solving for $\varphi_0^{**}(s)$, we get

$$MSTF(T_0) = \lim_{s \to 0} \frac{1 - \emptyset_0^{**}(s)}{s} = \frac{N_1}{D_1} \qquad \dots(5)$$

Where

$N_1 = \mu_0 + \mu_1 + p_{12}\mu_2 + p_{12}p_{23}\mu_3 + p_{12}p_{23}p_{34}\mu_4 + p_{12}p_{23}p_{35}\mu_5$

and $D_1 = 1 - p_{12}p_{20} - p_{12}p_{23}(p_{34}p_{40} + p_{35}p_{50})$.

V. AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is in upstate at instant t given that the system entered regenerative state i at $t = 0$.The recursive relation for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}{}^{(n)}(t) © A_j(t) \qquad \dots (6)$$

Where j is any successive regenerative state to which the regenerative state i can transit through n≥1 (natural number) transitions and Mi(t) is the probability that the system is up initially in regenerative state Si∈E at time $'t`$ without visiting to any other regenerative state. We have

$M_0(t) = e^{-2\lambda t}$, $M_1(t) = e^{-\lambda t}\overline{W}(t)$, $M_2(t) = e^{-(\lambda+\alpha_0)t}G(t)$, $M_3(t) = e^{-\lambda t}(t)H(t)$ $M_4(t) = e^{-\lambda t}(t)\bar{F}(t)$ and $M_6(t) = e^{-\lambda t}(t)\overline{G_1}(t)$

Taking $L.T.$ of relations (6) and solving for $A_0^*(s)$, we get steady-state availability as

$$A_0 = \lim_{s \to 0} s\, A_0^*(s) = \frac{N_2}{D_2} \qquad \dots (7)$$

Where $N_2 = (\mu_0 + \mu_1)(p_{20} + p_{23}(p_{34}p_{40} + p_{35}p_{50})) + \mu_2 + p_{23}(\mu_3 + p_{34}\mu_4 + p_{35}\mu_5)$

and $D_2 = (\mu_0 + \mu_1')(p_{20} + p_{23}(p_{34}p_{40} + p_{35}p_{50})) + \mu_2' + p_{23}(\mu_3' + p_{34}\mu_4' + p_{35}\mu_5')$.

VI. BUSY PERIOD ANALYSIS OF ORDINARY SERVER

Let $B_i^r(t)$ be the probability that the ordinary server is busy at an instant 't'given that the systementered regenerative state S_i at $t = 0$. The following are the recursive relations for $B_i^r(t)$ are given as

$$B_i^r(t) = W_i(t) + \sum_j q_{i,j}{}^{(n)}(t) © B_j^r(t) \qquad \dots(8)$$

where j is a subsequent regenerative state to which state i transits through n≥1(natural number) transitions and

$W_2(t) = e^{-(\lambda+\alpha_0)t}\bar{G}(t) + (\lambda e^{-\lambda t} © 1 © e^{-\alpha_0 t})\bar{G}(t)$,
$W_3(t) = e^{-\lambda t}\bar{H}(t) + (\lambda e^{-\lambda t} © 1)\bar{H}(t)$ and
$W_4(t) = e^{-\lambda t}\bar{F}(t) + (\lambda e^{-\lambda t} © 1 ©)\bar{F}(t) \qquad \dots (9)$

Taking $L.T.$ of relation (8) and solving for $B_0^{r^*}(s)$, we get in the long run the time for which the ordinary server is busy in steady state given by

$$B_0^r = \lim_{s \to 0} s\, B_0^{r^*}(s) = \frac{N_3}{D_2} \qquad \dots(10)$$

Where $N_3 = w_2^*(0) + p_{23}w_3^*(0) + p_{23}p_{34}w_4^*(0)$ and D_2 is already specified.

VII. BUSY PERIOD ANALYSIS OF EXPERT SERVER

Let $B_i^e(t)$ be the probability that the expert sever is busy in repairing the unit at an instant 't'given that the system entered regenerative state S_i at $t = 0$. The recursive relation for $B_i^e(t)$ are given by:

$$B_i^e(t) = W_i(t) + \sum_j q_{i,j}{}^{(n)}(t) © B_j^e(t) \qquad \dots (11)$$

Where j is a subsequent regenerative state to which state i transits through n≥1(natural number) transitions and

$W_5(t) = e^{-\lambda t}\overline{G_1}(t) + \left(\lambda e^{-\lambda t} © 1\right)\overline{G_1}(t)$ ...(12)

Taking $L.T.$ of relation (11) and solving for $B_0^{e*}(s)$, we get the time for which the system is under repair done by expert server is given by

$$B_0^e = \lim_{s\to 0} s\, B_0^{e*}(s) = \frac{N_4}{D_2} \qquad \ldots(13)$$

where $N_4 = p_{23}\, p_{35}\, w_5^*(0)$ and D_2 is already specified.

VIII. EXPECTED NUMBER OF VISITS BY THE ORDINARY SERVER

Let $N_i(t)$ be the expected number of visits by the ordinary server in $(0,t]$ given that thesystem entered the regenerative state S_i at $t = 0$.The recursive relation for $N_i(t)$ are given by

$$N_i(t) = \sum_j Q_{i,j}(t) \boxed{S} [\delta_j + N_j(t)] \qquad \ldots(14)$$

Where j is any regenerative state to which the given regenerative state i transits and δ_i=1, if j is the regenerative state where the ordinary server does job afresh, otherwise δ_i= 0. Taking $L.S.T.$ of relation (14) and solving for $N_0^{**}(s)$, we get the expected number of visits by ordinary server per unit time as

$$N_0 = \lim_{s\to 0} s\; N_0^{**}(s) = \frac{N_5}{D_2} \qquad \ldots(15)$$

Where $N_5 = \left(p_{20} + p_{23}\left(p_{34}p_{40} + p_{35}p_{50}\right)\right)$and $D_2$is already specified.

IX. EXPECTED NUMBER OF VISITS BY THE EXPERTSERVER

Let $N_i^e(t)$ be the expected number of visits by expert server $(0,t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relation for $N_i^e(t)$ are given by:

$$N_i^e(t) = \sum_j Q_{i,j}(t) \boxed{S} [\delta_j + N_j^e(t)] \qquad \ldots(16)$$

Where j is any regenerative state to which the given regenerative state i transits and δ_i=1, if j is the regenerative state where the expert server does job afresh, otherwise δ_i= 0. Taking $L.S.T.$of relation(16) and solving for $N_0^{e**}(s)$, we get the expected number of visits by expert server per unit time as

$$N_0^e = \lim_{s\to 0} s\, N_0^{e**}(s) = \frac{N_6}{D_2} \qquad \ldots(17)$$

Where

$N_6 = p_{12.11,11,13,9}\left(p_{20} + p_{23}\left(p_{34}p_{40} + p_{35}p_{50}\right)\right) +$
$p_{22.14,13,9} + p_{23}\left(p_{32.79} + p_{35}\right)$

and $D_2$is already specified.

X. EXPECTED NUMBER OF REPLACEMENTS OF THE UNIT

Let $R_i(t)$ be the expected number of replacements by the unit in $(0,t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relation for $R_i(t)$ are given by:

$$R_i(t) = \sum_j Q_{i,j}(t) \boxed{S} [+R_j(t)] \qquad \ldots(18)$$

Where j is any regenerative state to which the given regenerative state i transits and δ_i=1, if j is the regenerative state where the failed unit replaced by new ones, otherwise δ_i= 0. Taking$L.S.T.$ of relation(18) and solving for $R_0^{**}(s)$, we get the expected number of replacements per unit time as

$$R_0 = \lim_{s\to 0} s\, R_0^{**}(s) = \frac{N_7}{D_2} \qquad \ldots(19)$$

Where $N_9 = p_{23}\left(p_{34} + p_{32.78}\right)$ and $D_2$is already specified.

XI. COST-BENEFIT ANALYSIS

Profit incurred to the system model in steady state is given by:

$P = K_1 A_0 - K_2 B_0^R - K_3 B_0^e - K_4 R_0 - K_5 N_0 - K_6 N_0^e$

where

K_1 = Revenue per unit uptime of the system
K_2 = Cost per unit time for which ordinary server is busy due to repair
K_3 = Cost per unit time for which ordinary server is busy due to Replacement
K_4 = Cost per unit time replacement of the unit by ordinary server
K_5 = Cost per unit time for which ordinary server is busy due to inspection
K_6 = Cost per unit time for which expert server is busy due to repair
K_7 = Cost per unit visits by the ordinary server
K_6 = Cost per unit visits by the expert server

XII. PARTICULAR CASE

Let us consider $g(t) = \theta e^{-\theta t}$, $f(t) = \gamma e^{-\beta t}$.By using the non-zero element p_{ij}, we obtain the following results:

$MTSF(T_0) = \frac{N_1}{D_1}$,

Availability $(A_0) = \frac{N_2}{D_2}$,

Busy Period for repair $(B_0^r) = \frac{N_3}{D_2}$,

Busy period for expert sever due to repair $(B_0^e) = \frac{N_4}{D_2}$,

Expected number of visits by ordinary server $(N_0) = \frac{N_5}{D_2}$

Expected number of visits by expert server $(N_0^e) = \frac{N_6}{D_2}$,

Expected number of replacement of the unit $(R_0) = \frac{N_7}{D_2}$,

Where

$$N_1 = \frac{1}{2\lambda} + \frac{1}{\gamma+\lambda} + \left(\frac{\gamma}{\gamma+\lambda}\right)\frac{1}{(\theta+\lambda+\alpha_0)}\left[\left(1 + \frac{\alpha_0}{\eta+\lambda}\left(1 + \eta\left(\frac{a}{\beta+\lambda} + \frac{b}{\gamma_0+\lambda}\left(1 + \frac{\gamma_0}{\theta_0+\lambda}\right)\right)\right)\right)\right]$$

,

$$D_1 = \left[1 - \frac{1}{\theta+\lambda+\alpha_0}\left(\frac{\gamma}{\gamma+\lambda}\right)\left[\theta + \frac{\eta}{\eta+\lambda}\left(\frac{a\beta}{\beta+\lambda} + \frac{b\gamma_0}{\gamma_0+\lambda}\left(\frac{\theta_0}{\theta_0+\lambda}\right)\right)\right]\right],$$

$$N_2 = \frac{1}{\theta+\lambda+\alpha_0}\left[1 + \left(\frac{1}{\gamma+\lambda} + \frac{1}{2\lambda}\right)\left(\theta + \alpha_0\left(\frac{\eta}{\eta+\lambda}\right)\left(\frac{a\beta}{\beta+\lambda} + \frac{b\theta_0}{\theta_0+\lambda}\frac{\gamma_0}{\gamma_0+\lambda}\right)\right) + \alpha_0\left(\frac{1}{\eta+\lambda} + \left(\frac{\eta}{\eta+\lambda}\right)\left(\frac{a}{\beta+\lambda} + \frac{b}{\theta_0+\lambda}\frac{\gamma_0}{\gamma_0+\lambda} + \frac{b}{\gamma_0+\lambda}\right)\right)\right],$$

$$N_3 = \frac{1}{(\theta+\lambda+\alpha_0)}\left[1 + \lambda + \left(\frac{\alpha_0}{\eta+\lambda}\right)\left[\frac{a\eta(1+\lambda)}{\beta+\lambda} + \lambda\left(1 + \frac{1}{\lambda} + \left(\frac{\eta}{\eta+\lambda}\right)\left(\frac{a}{\beta+\lambda} + \frac{b}{\theta_0+\lambda}\right)\right)\right]\right]$$

,

$$N_4 = \frac{1}{\theta+\lambda+\alpha_0}\left[\frac{b\alpha_0(1+\lambda)}{\theta_0+\lambda}\left(\frac{\eta}{\eta+\lambda}\right)\right],$$

$$N_5 = \frac{1}{(\theta+\lambda+\alpha_0)}\left[\theta + \alpha_0\left(\frac{\eta}{\eta+\lambda}\right)\left(\frac{a\beta}{\beta+\lambda} + \frac{b\theta_0}{\theta_0+\lambda}\right)\right],$$

$$N_6 = \frac{b\lambda}{\theta+\lambda+\alpha_0}\left[\left(\frac{\theta}{\gamma+\lambda} + \frac{\alpha_0}{\gamma+\lambda}\left(\left(\frac{\eta}{\eta+\lambda}\right)\left(\frac{a\beta}{\beta+\lambda} + \frac{b\,\theta_0}{\theta_0+\lambda}\right)\right)\right) + 1\right] + (b\alpha_0)$$

, $N_7 = \frac{a\alpha_0}{\theta+\lambda+\alpha_0}$ and

$$D_2 = \frac{1}{\theta+\lambda+\alpha_0}\left[1 + \left[\frac{1}{2\lambda} + \frac{\lambda}{\gamma+\lambda}\left(\frac{1}{\lambda} + \frac{1}{\gamma} + \frac{\alpha_0}{\theta_0+\lambda}\left(\frac{1}{\eta} + \frac{1}{\alpha_0} + \frac{a}{\beta} + \frac{b}{\theta_0}\right)\right)\left(\theta + \alpha_0\left(\frac{\eta}{\eta+\lambda}\right)\left[\left(\frac{a\beta}{\beta+\lambda} + \frac{b\theta_0}{\theta_0-\lambda}\right)\right]\right)\right] + \frac{\lambda\alpha_0}{\theta_0+\alpha_0}\left(\frac{1}{\alpha_0} + \frac{1}{\eta} + \frac{a}{\beta} + \frac{b}{\theta_0}\right) + \alpha_0\left[\frac{1}{\eta+\lambda}\left[\left(1 + \lambda\left(\frac{1}{\eta} + \frac{a}{\beta} + \frac{b}{\theta_0}\right)\right)\right] + \left(\frac{\eta\lambda}{\eta+\lambda}\right)\left[\frac{a}{\beta+\lambda}\left(\frac{1}{\lambda} + \frac{1}{\beta}\right) + \frac{b}{\theta_0+\lambda}\left(\frac{1}{\lambda} + \frac{1}{\theta_0}\right)\right]\right]\right]$$

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XIII. CONCLUSION

The graphical behavior of mean time to system failure (MTSF), availability and profit with respect to replacement rate (β) of the unit for fixed values of other parameters including K1=5000, K2=1150, K3=50, K4=900, K5=150, K6=200 with a=0.6 and b=0.4 can be observed respectively from fig. 2 to 4. It is analyzed that MTSF, availability and profit of the system model go on increasing with the increase of replacement rate (β), repair rates of ordinary and expert servers (θ and θ0), inspection rate (η) of the unit and arrival rate (γ) of the ordinary server while they decline with the increase of failure rate (λ). However, the effect of arrival and repair rates of the ordinary server on these measures is more as compared to the other parameters. It is also revealed that MTSF and Availability of the system increase with the increase of rate ($\alpha 0$) but profit declines. Thus a parallel system of two identical units with different repair policies of the servers can be made profitable by increasing the arrival and repair rates of ordinary server and calling an expert server immediately when ordinary server fails to repair the system in a pre-specified repair time.

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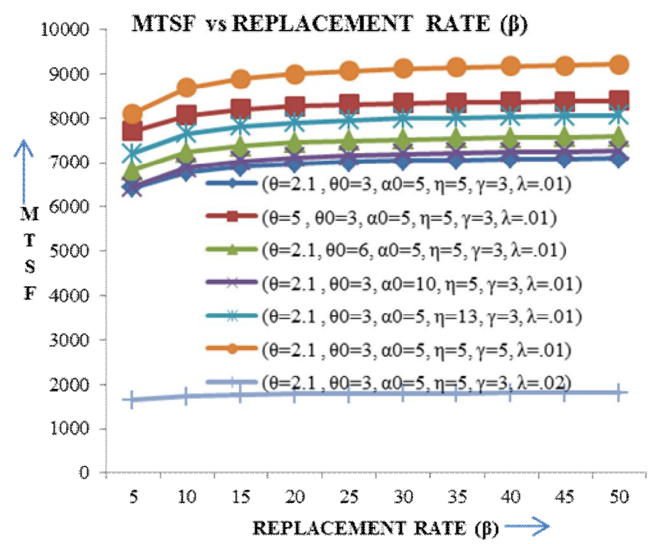

Figure.2: MTSF Vs. REPLACEMENT RATE

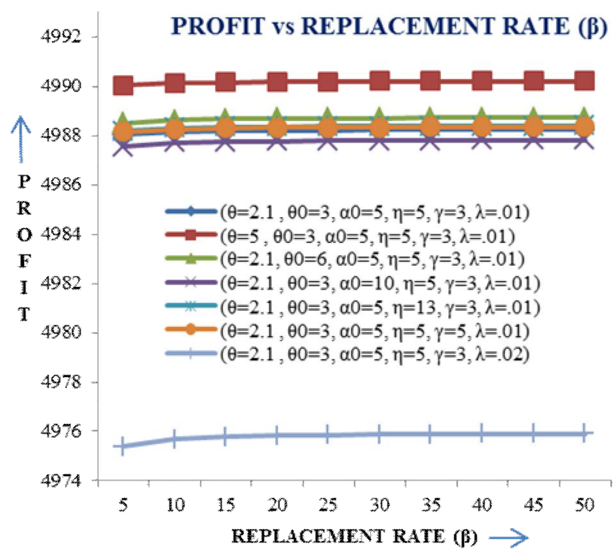

Figure.4: PROFIT Vs. REPLACEMENT RATE

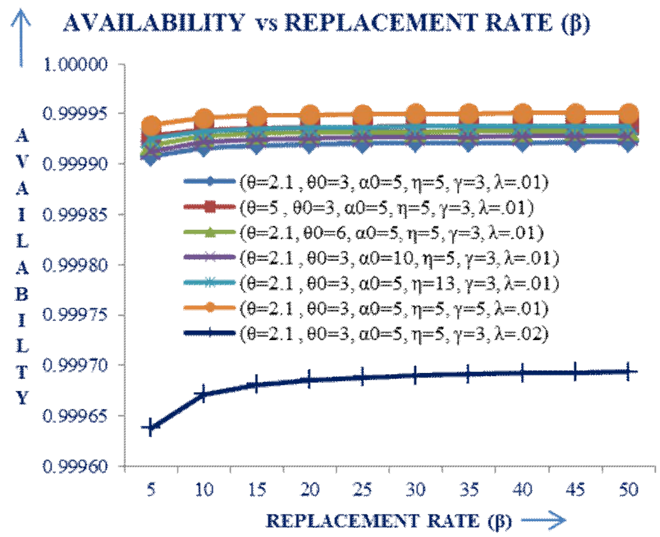

Figure.3: AVAILABILTY Vs. REPLACEMENT RATE