Inventory Optimization with Stock-Dependent Demand and Time Varying Holding Cost

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Abstract- In this article demand elasticity of product has been considered in developing an optimal order quantity and inventory cost based on the assumption that greater product availability tends to stimulate sales. Previous models incorporating stock-level dependent demand rate assume that the holding cost is constant for the entire inventory cycle. Unlike previous models this model considers holding cost per unit per unit time as an increasing function of quantity in storage along with non-zero lead time and no shortage. Lead time is considered a function of quantity ordered and lead time demand is a function of time. This model can be further extended for deteriorating items as well.

Keywords – Elastic demand, lead time, stock-dependent demand, time varying holding cost.

I. INTRODUCTION

As we know that in all conventional inventory models, the demand rate is assumed to be a given constant. There are various inventory models that have been constructed for dealing with deterministic and stochastic demand. All these models implicitly assume that the demand rate is independent, i.e. an outside parameter is not influenced by the internal inventory policy. In real life, however, it is recurrently observed that demand for a particular product can undeniably be influenced by internal factors such as price and availability. The change in the demand in response to inventory or marketing decisions is commonly referred to as demand elasticity.

Harris [1915] developed the first inventory model, Economic Order Quantity [1], which was generalized by Wilson [1934] who gave a formula to attain economic order quantity [2]. Whit [1957] considered the decline of the fashion goods at the end of the agreed shortage period [3]. A model was developed by Ghare and Schrader [1963] for an exponentially reducing inventory[4]. Dave and Patel [1981] were the first to study a deteriorating inventory with linearly increasing demand when shortages are not allowed [5]. In this field some of the recent work has been done by Chung and Ting [1993] [6]. Also Wee [1995] developed an inventory model with deteriorating items [7]. Chang and Dye [1999] studied an inventory model with time-varying demand and partial backlogging [8]. Goyal and Giri [2001] developed the recent trends of modeling in deteriorating item inventory [9]. They classified inventory models based on demand variations and various other conditions or constraints. Ouyang and Cheng [2007] gave an inventory model for deteriorating items with exponential declining demand and partial backlogging [10]. The effects of learning and forgetting on the optimal production lot size for deteriorating items with time-varying demand and deterioration rates were studied by Alamri and Balkhi [2007] [11]. Dye et al. [2007] formulated an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging [12]. They assume that a portion of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases. In 2008, Roy developed a deterministic inventory model when the deterioration rate is time proportional. Holding cost is time dependent and demand rate is a function of selling price. Liao gave an economic order quantity (EOQ) model with non-instantaneous receipt and exponential deteriorating item under two level trade credits [13]. Lee and Wu [2004] developed a note on EOQ model for items with mixtures of exponentially distributed deterioration, shortages and time varying demand [14]. Huang and Hsu [2008] presented a simple algebraic approach to find the exact optimal lead time and the optimal cycle time in steady market demand situations [15]. Pareek et al. [2009] developed a deterministic inventory model for deteriorating items with resale value and shortages [16]. Skouri et al.[2009] developed an inventory model with ramp-type demand rate, partial backlogging, and Weibull's deterioration rate [17]. Mishra and Singh [2011] developed a deteriorating inventory model for waiting time partial backlogging when demand and deterioration rate is constant [18]. They made the work of Abad[34&35] more realistic and applicable in practice. Mandal developed an EOQ inventory model for Weibull-distributed deteriorating items under ramp-type demand and shortages [19]. Mishra and Singh [2013] gave an inventory model for ramp-type demand, time-dependent deteriorating items with salvage value and shortages and deteriorating inventory model for time-dependent demand and holding cost with partial backlogging. Hung studied an inventory model with generalized-type demand, deterioration, and backorder rates [20]. Mishra et al. [2013] developed an inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging[21]. Inventory model for deteriorating items with predetermined life under quadratic demand and nonlinear holding cost has been developed by Niketa D. Trivedi, Nehal J. Shukla and Nita H. Shah[2014] [22]

The pioneer researcher who formulated inventory models taking initial stock - level dependent demand is Gupta and Vrat [1986] [23]. Mandal and Phaujdar [1989] [24] corrected the flaw in Gupta and Vrat [1986] [25] model using profit maximization rather than cost minimization as the objective. Baker and Urban [1988] developed an inventory model taking demand rate in polynomial functional form dependent on inventory level [26]. The same functional form was used by Datta and Pal [1990]. Datta and Pal [1990] proposed an inventory model for deteriorating items, inventory-level dependent demand and shortages [27]. Mandal and Phaujdar [1989] proposed an inventory model in which shortages are allowed and demand is dependent on stock-level. In this model the rate of deterioration is assumed to be variable. Sarker, Mukherjee and Balan [1997] took demand to be dependent on inventory level incorporating an entirely new concept of decrease in demand (due to ageing of inventory or products reaching closer to their expiry date) [28].

Montgomery, Bazarra and Keswani [1973] developed both deterministic and stochastic models considering the situation in which a fraction of demand during the stock out period is backordered and remaining is lost forever [29]. Rosenberg [1979] developed a lot-size inventory model with partial backlogging taking “fictitious demand rate” that simplifies the analysis [30]. Padmanabhan and Vrat [1995] proposed an inventory model for perishable items taking constant rate of deterioration, incorporating the three cases of complete, partial and no backlogging [31]. Zeng [2001] developed an inventory model using partial backordering approach and minimizing the total cost function [32]. This model identifies the conditions for partial backordering policy to be feasible.

In classical inventory models, the demand rate and holding cost are assumed to be constant. In reality, the demand and holding cost for physical goods may be time or stock dependent. Time also plays an important role in the inventory system; therefore, in this article, we consider that demand is stock dependent up to reorder point and holding cost is an increasing function of time. In this paper, we considered demand rate as linear function of inventory before reorder point and developed an inventory model with non zero lead time and lead time demand as a function of time. This makes the work of Rathod and Bhatawala [2013] more realistic with non-zero lead time and avoiding shortage cost [33].

II. PROPOSED METHODOLOY, ASSUMPTIONS AND MODEL FORMULATION

A. Proposed methodology –

The main objective of this paper is to determine the optimum (i.e. minimum cost) inventory policy for an inventory system with stock dependent demand rate and variable holding cost with no shortages. Assuming the demand rate to be inventory-level dependent means the demand is higher for greater inventory levels. Assuming the holding cost per unit of the item per unit time to be time-dependent means the unit holding cost is higher for longer storage periods. The model will be developed for the inventory system where the lead time demand is time dependent and lead time is order quantity proportional. Variable unit holding costs are considered in the model in determining the optimal inventory policy. In other words, the holding cost per unit per unit time is an increasing step function of the storage time. The following steps are followed to find the optimal order quantity:

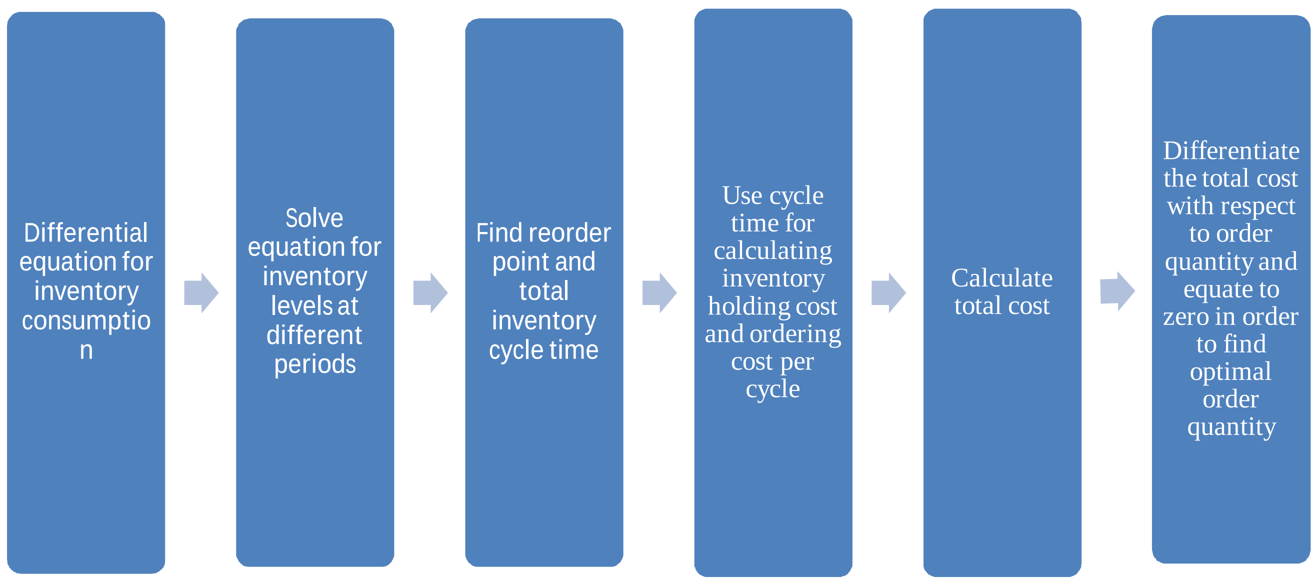

Figure 1:Steps in calculating optimal order quantity for time varying demand

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B. *NOTATIONS*-

I(t) = The inventory on-hand at time t
R = Constant demand rate
N = Number of distinct time periods with different holding cost rates
t = Time
A = Ordering cost per order
$h_k(t)$ = Holding cost of an item at time t
T = Cycle time
β = Demand parameter indicating elasticity in relation to the inventory level
Q = Quantity ordered
S = Reorder point
HC = Holding cost of a full inventory cycle
TIC = Total inventory cost

C. *ASSUMPTIONS*-

1. Demand rate is stock dependent up to reorder point and time dependent thereafter.
2. Replenishment is not instantaneous that is Lead time is not zero and proportional to quantity ordered.
3. Shortage does not occur.
4. Order quantity is fixed which in turn makes lead time and lead time demand both constant .
5. Every time we place the order of Q quantity after the stock reaches to order level S at time $t = t_k$
6. The holding cost is varying as an increasing step function of the quantity in storage or a decreasing function of time.
7. The demand rate R is linear function of the inventory level q up to reorder point which is expressed as, $R(I(t)) = R + \beta I(t)$; $R > 0$, $0 < \beta < 1$, $I(t) \geq 0$
8. The inventory cycle starts at I(t)=Q and at the end of the cycle the inventory level drops to zero and the second cycle starts with the receipt of order at the end of previous period.

D. MODEL FORMULATION -

$$\frac{dI(t)}{dt} = -\{R + \beta I(t)\} \ ; 0 \le t \le t_k$$
$$= -R \quad ; \quad t_k < t \le T$$

In order to solve for I(t), we get from the above equation by integrating both sides

$$\int_0^t \frac{dI(t)}{R+\beta I(t)} = \int_0^t dt$$

$$\frac{1}{\beta} ln \frac{R+\beta I(t)}{R+\beta I(0)} = -t \quad ;$$

Using the boundary condition I(0) = Q , we get

$$R + \beta I(t) = (R + \beta Q)e^{-\beta t}$$

$$I(t) = \frac{1}{\beta}\{(R + \beta Q)e^{-\beta t} - R\} \quad \text{when} \quad (0 \le t \le t_k) \tag{1}$$

Now for the 2nd period that is during lead time,

$$\int_{t_k}^{t} dI(t) = -R \int_{t_k}^{t} dt$$

Using the boundary condition $I(t_k) = s$, we get

$$I(t) - s = -R(t - t_k)$$

$$I(t) = s - R(t - t_k) \quad \text{when} \quad (t_k < t \le T) \tag{2}$$

At $t = t_k$, I(t)= s

Therefore from equation (1),

$$s = \frac{1}{\beta}\{(R + \beta Q)e^{-\beta t_k} - R\}$$

$$e^{-\beta t_k} = ln \frac{R+\beta S}{R+\beta Q}$$

$$t_k = \frac{1}{\beta} ln \frac{R+\beta Q}{R+\beta S} \tag{3}$$

At t = T, I(t) = 0
Therefore from equation (2) ,

$$0 - s = -R(T - t_k)$$

$$T = t_k + \frac{S}{R} = \frac{1}{\beta} ln \frac{R+\beta Q}{R+\beta S} + \frac{S}{R} \tag{4}$$

The holding cost is a decreasing function of inventory with time. The holding cost can be divided into two segments. The holding cost up to reorder point from start period and the holding cost from the start of lead time to the end of a cycle or the start of a new cycle.

Therefore, holding cost $HC = h_k \int_0^{t_k} I(t)dt + h_k \int_{t_k}^{T} I(t)dt$

$$HC = h_k \int_0^{t_k} \frac{1}{\beta}\{(R+\beta Q)e^{-\beta t} - R\}dt + h_k \int_{t_k}^{T} \{S - R(t-t_k)\}\mathrm{dt}$$

$$HC = \frac{h_k}{\beta}(R\int_0^{t_k} e^{-\beta t}dt + \beta Q \int_0^{t_k} e^{-\beta t}dt - R\int_0^{t_k} dt) + h_k(S\int_{t_k}^{T} dt - R\int_{t_k}^{T} tdt + R\int_{t_k}^{T} t_k dt)$$

$$HC = \frac{h_k}{\beta}\left\{-\frac{1}{\beta}(R+\beta Q)(e^{-\beta t_k} - 1) - Rt_k\right\} + h_k\{(S + Rt_k)(T - t_k) - \frac{R}{2}(T^2 - {t_k}^2)\}$$

$$HC = \frac{h_k}{\beta^2}(R+\beta Q)(1 - e^{-\beta t_k}) - \frac{Rh_k}{\beta}t_k + h_k(T-t_k)(S + Rt_k - \frac{RT}{2} - \frac{Rt_k}{2})$$

$$HC = \frac{h_k}{\beta^2}(R+\beta Q)(1 - e^{-\beta t_k}) - \frac{Rh_k}{\beta}t_k + h_k S(T-t_k) - \frac{h_k R}{2}(T-t_k)^2 \qquad (5)$$

Now, Total inventory cost, TIC = Ordering cost + Holding cost

Ordering cost per order = A

There ordering cost of an inventory cycle = $\frac{A}{T}$

$$TIC = \frac{A}{T} + \frac{h_k}{\beta^2}(R+\beta Q)(1 - e^{-\beta t_k}) - \frac{Rh_k}{\beta}t_k + h_k S(T-t_k) - \frac{h_k R}{2}(T-t_k)^2$$

Putting the values of T and t_k from equation (3) and (4) we get,

$$TIC = \frac{A}{\frac{1}{\beta}ln\frac{R+\beta Q}{R+\beta S} + \frac{S}{R}} + \frac{h_k}{\beta^2}(R+\beta Q)\left(1 - e^{-\beta * \frac{1}{\beta}ln\frac{R+\beta Q}{R+\beta S}}\right) - \frac{Rh_k}{\beta} * \frac{1}{\beta}ln\frac{R+\beta Q}{R+\beta S} + h_k S\left(\frac{S}{R}\right) - \frac{h_k R}{2}\left(\frac{S}{R}\right)^2$$

$$TIC = \frac{A}{\frac{1}{\beta}ln\frac{R+\beta Q}{R+\beta S} + \frac{S}{R}} + \frac{h_k}{\beta^2}(R+\beta Q)\left(1 + \frac{R+\beta Q}{R+\beta S}\right) - \frac{Rh_k}{\beta^2}ln\frac{R+\beta Q}{R+\beta S} + \frac{h_k S^2}{2R}$$

$$\frac{dTIC}{dQ} = -A\left(\frac{1}{\beta}ln\frac{R+\beta Q}{R+\beta S} + \frac{S}{R}\right)^{-2}\left(\frac{1}{R+\beta Q}\right) + \frac{h_k}{\beta} + \frac{2h_k(R+\beta Q)}{\beta(R+\beta S)} - \frac{Rh_k}{\beta(R+\beta Q)}$$

$$\frac{dTIC}{dQ} = \frac{-A}{(R+\beta Q)\left(\frac{1}{\beta}ln\frac{R+\beta Q}{R+\beta S} + \frac{S}{R}\right)^2} + \frac{h_k}{\beta} + \frac{2h_k(R+\beta Q)}{\beta(R+\beta S)} - \frac{Rh_k}{\beta(R+\beta Q)}$$

For minimum inventory cost we equate $\frac{dTIC}{dQ} = 0$

Thus, $$\frac{-A}{(R+\beta Q)\left(\frac{1}{\beta}ln\frac{R+\beta Q}{R+\beta S} + \frac{S}{R}\right)^2} + \frac{2h_k(R+\beta Q)}{\beta(R+\beta S)} - \frac{Rh_k}{\beta(R+\beta Q)} + \frac{h_k}{\beta} = 0 \qquad (6)$$

By solving the above equation we get the optimum inventory level that minimizes the total inventory cost. The following numerical problem illustrates the validity of this equation.

III. Numerical example

The following numerical values of the parameter in proper unit were considered as input for numerical and graphic0al analysis of the model,

A = \$100/order, R= 150/day, S = 250, $\beta = 0.4$, h_k = \$.005/unit/day

The output of the model has been found by solving Equation (6) by Matlab 2013. The optimum order quantity obtained Q = 333units

The cycle time T = 3days

And total inventory cost for the optimum quantity = \$76.2

IV.CONCLUSION

The model presented here obtained an optimal order quantity for stock dependent demand and non-instantaneous receipt of orders. The case of the decreasing holding cost considered in this paper applies rented storage facilities, where lower rent rates are normally obtained for longer-term leases. In this model a linearly decreasing holding cost has been used to find the optimal result. And conclusion can be drawn that inventory cost increases with increase in reordering quantity. This model provides the basis for further extensions like the incorporation of deteriorating items, shortage cost and stochastic lead time in order to develop more complex and realistic inventory models. Although this paper does not include any statistical inventory models but it still puts a platform for developing statistical optimal inventory policy if demand or lead time uncertainties are incorporated.

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