Outcomes of Thermal Stresses Concerning by Internal Moving Point Heat Source in Rectangular Plate

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Abstract-In this paper we deals with the study of thermal stresses in thin rectangular plate in presence of instantaneous moving point heat source which change its position along x and y axes with non- homogenous boundary condition. The solution of governing heat conduction equation has been found by using integral transform technique. Results are obtained in the form of infinite series. Also special case has been derived numerically and graphically.

Key words: rectangular plate, internal moving point heat source, integral transform, thermal stresses.

I. INTRODUCTION

Material properties are an important characteristic of every material. These properties are dependent on temperature. Thus an important problem in thermo elasticity is that the temperature field. In many engineering application such as civil engineering, aerodynamics heating it is important to know the effect of temperature on thermal properties of materials. W.Nowaki [10] has determined the steady state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper surface with zero temperature on the lower surface and with the circular edge thermally insulated. Roy chaudhary [9] considered a quasi static thermal stresses in a thin circular plate due to transient temperature taken along the circumference of a circle on the upper face with the lower face at zero temperature and fixed circular edge.N.W.Khobragade and P.C. Wankhede[7] studied an inverse unsteady state thermoelastic problem of a thin rectangular plate. D.T.Solanke and M.H.Durge[1] have determine the temperature distribution and thermal stresses in thin rectangular plate with moving line heat source taking second kind boundary condition by using integral transform technique and Green's theorem. K.R.Gaikwad et al.[3] have been calculated quasi static thermoelastic problem of an infinitely long circular cylinder. M.S.Thakare et al.[4] have considered the problem on rectangular plate and find thermal stresses by integral transform with internal moving point heat source. Y.M.Ahire et al.[11]studied the thermal stresses in thin rectangular plate subjected to point heat source using integral transform technique and also illustrated graphically. Recently, Y.M.Ahire and K.P.Ghadle[12] studied three dimensional unsteady state temperature distribution of thin rectangular plate with moving point heat source placed at the point $(x',0,0)$ change its position along x-axis and considering homogeneous boundary condition along x and y direction and non homogeneous boundary condition along z direction.

In this paper an attempt to determine, thermal stresses in thin rectangular plate with internal moving point heat source that change its position along the x and y axes with constant velocity. Here an attempt is made to solve thermoelastic problem to determinethe temperature and stress function of thin rectangular plate occupying the space D: $0 \le x \le a, 0 \le y \le b, -h \le z \le h$ with boundary condition that nonhomogeneous boundary condition of first kind is maintained on the edge $x = 0, a$ second kind of nonhomogeneous boundary condition is maintained on edge $y = 0, b$ and third kind homogeneous boundary condition is maintained on the edge $z = -h, h$. The solution of the governing heat conduction equation has been solved by using integral transform technique[5]. Results are obtained in form of infinite series. A special case is considered and computed numerically and graphically.

II.FORMULATION OF THE PROBLEM

Consider three dimensional thin rectangular plates occupying the space D: $0 \le x \le a, 0 \le y \le b, -h \le z \le h$ where $h < b < a, h$ is thickness of plate which is very small. The plate is subjected to the motion of moving point heat source at the point $(x', y', 0)$ which moves its position along x, y, z axes with constant velocity. The heat conduction equation of the rectangular plate is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x,y,z)}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (1)$$

Where K is thermal conductivity and $\propto$ is thermal diffusivity of the material of the plate.

Consider an instantaneous moving point heat source at point $(x', y', 0)$ which releasing its heat spontaneously at time t' such volumetric moving heat source in rectangular coordinates is given by

$$g(x, y, z) = g_p^i \delta(x - x')\delta(y - y')\delta(z)\delta(t - t') \quad (2)$$

Where g_p^i is instantaneous point heat source.

Hence equation (1) becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{K} g_p^i \delta(x - x')\delta(y - y')\delta(z)\delta(t - t') = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (3)$$

Where $x' = v_1 t$ and $y' = v_2 t$

With initial and boundary conditions are given by

$$[T]_{t=0} = 0 \quad (4)$$

$$[T]_{x=0} = f_1(y, z, t) \quad (5)$$

$$[T]_{x=a} = f_2(y, z, t) \quad (6)$$

$$\left[\frac{\partial T}{\partial y}\right]_{y=0} = f_3(x, z, t) \quad (7)$$

$$\left[\frac{\partial T}{\partial y}\right]_{y=b} = f_4(x, z, t) \quad (8)$$

$$\left[T + k_1 \frac{\partial T}{\partial z}\right]_{z=-h} = 0 \quad (9)$$

$$\left[T + k_2 \frac{\partial T}{\partial z}\right]_{z=h} = 0 \quad (10)$$

Let us consider thermal stress function χ is $\chi = \chi_c + \chi_p$ where χ_c is complementary function and χ_p is particular integral. They are governed by equations,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi_c = 0 \quad \text{And} \quad (11)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \chi_p = -\alpha E \Gamma \quad (12)$$

Since plate is thin z is negligible and where $\Gamma = T - T_0$, where T_0 is initial temperature. Also component of stress function are given by

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2}$$
(13)

$$\sigma_{yy} - \frac{\partial^2 \chi}{\partial x^2}$$
(14)

$$\sigma_{xy} = \frac{\partial^2 \chi}{\partial x \partial y}$$
(15)

With boundary conditions

$\sigma_{yy} = 0,\ \sigma_{xy} = 0$ at y=b.

III.SOLUTION OF THE PROBLEM

Applying Finite Fourier sine transform, Finite Fourier cosine transform and Finite Marchi-Fasulo transform to equation (2), also using boundary conditions we get,

$$\frac{d\bar{T}^{*\wedge}}{dt} + \alpha A \bar{T}^{*\wedge} = \frac{m\pi\alpha}{a}\left[(-1)^{m+1}\overline{f_1}^{\wedge} + \overline{f_2}^{\wedge}\right] + \alpha\left[(-1)^n f_3^{*\wedge} - f_4^{*\wedge}\right] + \frac{\alpha g_p^i}{K}\delta(t-t')\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)p_l(0)$$
(16)

Where A $=\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2} + a_l^2\right)$

Solving above equation we get

$$\bar{T}^{*\wedge} = e^{-\alpha A t}\left[\int_0^t e^{\alpha A t}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}\overline{f_1}^{\wedge} + \overline{f_2}^{\wedge}\right] + \alpha\left[(-1)^1 f_3^{*\wedge} - f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{K}e^{\alpha A t'}p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]$$
(17)

Taking inverse Marchi –fasulo transform ,Finite Fourier cosine transform and Finite Fourier sine transform on equation (17) we get,

$$T = \frac{4}{ab}\sum_{m=n=l=1}^{\infty} e^{-\alpha A t}\left[\int_0^t e^{\alpha A t}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}\overline{f_1}^{\wedge} + \overline{f_2}^{\wedge}\right] + \alpha\left[(-1)^n f_3^{*\wedge} - f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{K} e^{\alpha A t'}p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\frac{p_l(z)}{\lambda_l}$$
(18)

And $\Gamma = T \quad T_0$

$$\Gamma = \frac{4}{ab}\sum_{m=n=l=1}^{\infty} e^{-\alpha A t}\left[\int_0^t e^{\alpha A t}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}\overline{f_1}^{\wedge} + \overline{f_2}^{\wedge}\right] + \alpha\left[(-1)^n f_3^{*\wedge} - f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{K}e^{\alpha A t'}p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\mathrm{sir}\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\frac{p_l(z)}{\lambda_l}$$
(19)

IV.DETERMINATION OF STRESS FUNCTIONS

Let χ_c and $\chi_p$satisfying equation (11) and (12) respectively are given by

$$\chi_c = \sum_{m=1}^{\infty} y\left[c_1 e^{\frac{m\pi y}{a}} + c_2 e^{\frac{-m\pi y}{a}}\right]\cos\left(\frac{m\pi x}{a}\right) + y\left[\left[c_3 e^{\frac{m\pi y}{a}} + c_4 e^{\frac{-m\pi y}{a}}\right]\right]\sin\left(\frac{m\pi x}{a}\right)$$
(20)

$$\chi_p = \sum_{m=n=l=1}^{\infty} \frac{4E\alpha b\alpha}{\pi^2(b^2m^2+a^2n^2)} e^{-\alpha At}\left[\int_0^t e^{\alpha At}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}\overline{f_1}^{\wedge}+\overline{f_2}^{\wedge}\right]+\alpha\left[(-1)^n f_3^{*\wedge}-f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{\kappa} e^{\alpha At'} p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\frac{p_l(z)}{\lambda_l} \tag{21}$$

Therefore,

$$\chi = \sum_{m=1}^{\infty} y\left[c_1 e^{\frac{m\pi y}{a}} + c_2 e^{\frac{-m\pi y}{a}}\right]\cos\left(\frac{m\pi x}{a}\right) + y\left[\left[c_3 e^{\frac{m\pi y}{a}} + c_4 e^{\frac{-m\pi y}{a}}\right]\right]\sin\left(\frac{m\pi x}{a}\right) + \sum_{m=n=l=1}^{\infty} \frac{4E\alpha b\alpha}{\pi^2(b^2m^2+a^2n^2)} e^{-\alpha At}\left[\int_0^t e^{\alpha At}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}\overline{f_1}^{\wedge}+\overline{f_2}^{\wedge}\right]+\alpha\left[(-1)^n f_3^{*\wedge}-f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{\kappa} e^{\alpha At'} p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\frac{p_l(z)}{\lambda_l} \tag{22}$$

Using (22) in (13),(14) and (15) we get,

$$\sigma_{xx} = \sum_{m=1}^{\infty}\left\{\left[\left(\frac{ym^2\pi^2}{a^2}+\frac{2m\pi}{a}\right)c_1 e^{\frac{m\pi y}{a}} + \left(\frac{ym^2\pi^2}{a^2}-\frac{2m\pi}{a}\right)c_2 e^{\frac{-m\pi y}{a}}\right]\cos\left(\frac{m\pi x}{a}\right) + \left[\left(\frac{ym^2\pi^2}{a^2}+\frac{2m\pi}{a}\right)c_3 e^{\frac{m\pi y}{a}} + \left(\frac{ym^2\pi^2}{a^2}-\frac{2m\pi}{a}\right)c_4 e^{\frac{-m\pi y}{a}}\right]\sin\left(\frac{m\pi x}{a}\right)\right\} - \sum_{m=n=l=1}^{\infty}\frac{4E\alpha b\alpha}{\pi^2(b^2m^2+a^2n^2)} e^{-\alpha At}\left[\int_0^t e^{\alpha At}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}f_1^{\wedge}+f_2^{\wedge}\right] + \alpha\left[(-1)^n f_3^{*\wedge}-f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{\kappa} e^{\alpha At'} p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\frac{p_l(z)}{\lambda_l}\frac{n^2\pi^2}{b^2} \tag{23}$$

$$\sigma_{yy} = \sum_{m=1}^{\infty}\left\{-y\left[c_1 e^{\frac{m\pi y}{a}} + c_2 e^{\frac{-m\pi y}{a}}\right]\cos\left(\frac{m\pi x}{a}\right)\left(\frac{m^2\pi^2}{a^2}\right) - y\left[\left[c_3 e^{\frac{m\pi y}{a}} + c_4 e^{\frac{-m\pi y}{a}}\right]\right]\sin\left(\frac{m\pi x}{a}\right)\left(\frac{m^2\pi^2}{a^2}\right)\right\} - \sum_{m=n=l=1}^{\infty}\frac{4E\alpha b\alpha}{\pi^2(b^2m^2+a^2n^2)} e^{-\alpha At}\left[\int_0^t e^{\alpha At}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}\overline{f_1}^{\wedge}+\overline{f_2}^{\wedge}\right]+\alpha\left[(-1)^n f_3^{*\wedge}-f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{\kappa} e^{\alpha At'} p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\frac{p_l(z)}{\lambda_l}\left(\frac{m^2\pi^2}{a^2}\right) \tag{24}$$

$$\sigma_{xy} = \sum_{m=1}^{\infty}\left\{-\left[\left(1+\frac{ym\pi}{a}\right)c_1 e^{\frac{m\pi y}{a}} + \left(1-\frac{ym\pi}{a}\right)c_2 e^{\frac{-m\pi y}{a}}\right]\sin\left(\frac{m\pi x}{a}\right)\left(\frac{m\pi}{a}\right) + \left[\left(1+\frac{ym\pi}{a}\right)c_3 e^{\frac{m\pi y}{a}} + \left(1-\frac{ym\pi}{a}\right)c_4 e^{\frac{-m\pi y}{a}}\right]\cos\left(\frac{m\pi x}{a}\right)\left(\frac{m\pi}{a}\right)\right\} - \sum_{m=n=l=1}^{\infty}\frac{4E\alpha b\alpha}{\pi^2(b^2m^2+a^2n^2)} e^{-\alpha At}\left[\int_0^t e^{\alpha At}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}f_1^{\wedge}+f_2^{\wedge}\right] + \alpha\left[(-1)^n f_3^{*\wedge}-f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{\kappa} e^{\alpha At'} p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\frac{p_l(z)}{\lambda_l}\frac{nm\pi^2}{ab} \tag{25}$$

Using boundary conditions $\sigma_{yy} = 0$, $\sigma_{xy} = 0$ at $y = b$ we get,

$$c_1 = c_2 = 0 \; ,$$

$$c_3 = \frac{(a-m\pi b)}{2m\pi b} e^{\frac{-m\pi b}{a}} \frac{4E\alpha a}{\pi^2(b^2m^2+a^2n^2)} e^{-\alpha At}\left[\int_0^t e^{\alpha At}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}\overline{f_1}^{\wedge}+\overline{f_2}^{\wedge}\right]+\alpha\left[(-1)^n f_3^{*\wedge}-f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{\kappa} e^{\alpha At'} p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\frac{p_l(z)}{\lambda_l}(-1)^{n+2},$$

$$c_4 = \frac{a+m\pi b}{2m\pi b} e^{\frac{m\pi b}{a}} \frac{4E\alpha a}{\pi^2(b^2m^2+a^2n^2)} e^{-\alpha At}\left[\int_0^t e^{\alpha At}\left\{\frac{m\pi\alpha}{a}\left[(-1)^{m+1}\overline{f_1}^{\wedge}+\overline{f_2}^{\wedge}\right]+\alpha\left[(-1)^n f_3^{*\wedge}-f_4^{*\wedge}\right]\right\}dt + \frac{\alpha g_p^i}{\kappa} e^{\alpha At'} p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\frac{p_l(z)}{\lambda_l}(-1)^{n+1}$$

Substituting these values in equation (23),(24) and(25) we get,

$$\sigma_{xx} = \sum_{m=n=l=1}^{\infty}\left\{\left[\left(\frac{ym^2\pi^2}{a^2}+\frac{2m\pi}{a}\right)\left(\frac{a-m\pi b}{2m\pi b}\frac{4E\alpha a(-1)^{n+2}\emptyset}{\pi^2(b^2m^2+a^2n^2)}e^{\frac{-m\pi b}{a}}e^{\frac{m\pi y}{a}}\right)+\left(\frac{ym^2\pi^2}{a^2}-\frac{2m\pi}{a}\right)\left(\frac{a+m\pi b}{2m\pi b}\frac{4E\alpha ab(-1)^{n+1}\emptyset}{\pi^3(b^2m^2+a^2n^2)}e^{\frac{m\pi b}{a}}e^{\frac{-m\pi y}{a}}\right)\right]\sin\left(\frac{m\pi x}{a}\right)\right\}-\sum_{m=n=l=1}^{\infty}\frac{4E\alpha ab}{(b^2m^2+a^2n^2)}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\frac{n^2}{b^2}\emptyset \tag{26}$$

$$\sigma_{yy} = \sum_{m=n=l=1}^{\infty}\left\{-y\left[\left[\left(\frac{a-m\pi b}{2m\pi b}\frac{4E\alpha a}{\pi^2(b^2m^2+a^2n^2)}(-1)^{n+2}\emptyset e^{\frac{-m\pi b}{a}}\right)e^{\frac{m\pi y}{a}}+\left(\frac{a+m\pi b}{2m\pi b}\frac{4E\alpha a}{\pi^2(b^2m^2+a^2n^2)}(-1)^{n+1}\emptyset e^{\frac{m\pi b}{a}}\right)e^{\frac{-m\pi y}{a}}\right]\right]\sin\left(\frac{m\pi x}{a}\right)\left(\frac{m^2\pi^2}{a^2}\right)\right\}-\sum_{m=n=l=1}^{\infty}\frac{4E\alpha ab}{(b^2m^2+a^2n^2)}\emptyset\sin\left(\frac{m\pi x}{a}\right)\left(\frac{m^2}{a^2}\right)\cos\left(\frac{n\pi y}{b}\right) \tag{27}$$

$$\sigma_{xy} = \sum_{l=m=n=1}^{\infty}\left\{\left(1+\frac{m\pi y}{a}\right)\frac{4E\alpha a(a-m\pi b)}{2m b\pi^3(b^2m^2+a^2n^2)}(-1)^{n+2}\emptyset e^{\frac{-m\pi b}{a}}e^{\frac{m\pi y}{a}}+\left(1-\frac{m\pi y}{a}\right)\frac{4E\alpha a(a+m\pi b)}{2mb\pi^3(b^2m^2+a^2n^2)}\times(-1)^{n+1}\emptyset e^{\frac{m\pi b}{a}}e^{\frac{-m\pi y}{a}}\right\}\cos\left(\frac{m\pi x}{a}\right)\frac{m\pi}{a}-\sum_{m=n=l=1}^{\infty}\frac{4E\alpha ab}{(b^2m^2+a^2n^2)}\emptyset\sin\left(\frac{n\pi y}{b}\right)\left(\frac{mn}{ab}\right)\cos\left(\frac{m\pi x}{a}\right) \tag{28}$$

Where

$$\emptyset = e^{-\alpha A t}\left[\int_0^t e^{\alpha A t}\left\{\frac{m\pi a}{a}\left[(-1)^{m+1}\overline{f_1}^{*\wedge}+\overline{f_2}^{*\wedge}\right]+\alpha\left[(-1)^n f_3^{*\wedge}-f_4^{*\wedge}\right]\right\}dt+\frac{\alpha g_p^i}{K}e^{\alpha A t'}p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\frac{p_l(z)}{\lambda_l}$$

V.SPECIAL CASE AND NUMERICAL RESULTS

Let $f_1 = 2a(y^2+by)t(z^2-h^2)^2, f_2 = a(y^2+by)t(z^2-h^2)^2, f_3 = 3b(x+a)t(z^2-h^2)^2,$
$f_4 = b(x+a)t(z^2-h^2)^2$

Therefore,

$$\sigma_{xx} = \sum_{m=n=l=1}^{\infty}\left\{\beta\sin\left(\frac{m\pi x}{a}\right)-\frac{4E\alpha ab}{(b^2m^2+a^2n^2)}\sin\left(\frac{m\pi x}{a}\right)\times\cos\left(\frac{n\pi y}{b}\right)\frac{n^2}{b^2}\right\}e^{-\alpha A t}\times\left[\frac{ab}{\pi}S(1+2(-1)^{m+1})(3(-1)^n-1)\left(\frac{mb^2}{n^2}+\frac{a^2}{m}\right)\times\left\{\frac{te^{\alpha At}}{\alpha A}-\frac{e^{\alpha At}}{(\alpha A)^2}+\frac{1}{(\alpha A)^2}\right\}+\frac{\alpha g_p^i}{K}e^{\alpha A t'}p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\frac{p_l(z)}{\lambda_l}$$

Where

$$\beta = \left(\frac{ym^2\pi^2}{a^2}+\frac{2m\pi}{a}\right)\left(\frac{a-m\pi b}{2m\pi b^3}\frac{4E\alpha a(-1)^{n+2}}{\pi^2(b^2m^2+a^2n^2)}\right)e^{\frac{-m\pi b}{a}}e^{\frac{m\pi y}{a}}+\left(\frac{ym^2\pi^2}{a^2}-\frac{2m\pi}{a}\right)\left(\frac{a+m\pi b}{2m\pi b}\frac{4E\alpha ab(-1)^{n+1}}{\pi^3(b^2m^2+a^2n^2)}\right)e^{\frac{m\pi b}{a}}e^{\frac{-m\pi y}{a}}$$

$$\sigma_{yy} = \sum_{m=n=l=1}^{\infty}\left\{-y\left[\left[\left(\frac{a-m\pi b}{2m\pi b}\frac{4E\alpha a}{\pi^2(b^2m^2+a^2n^2)}(-1)^{n+2}e^{\frac{-m\pi b}{a}}\right)e^{\frac{m\pi y}{a}}+\left(\frac{a+m\pi b}{2m\pi b}\frac{4E\alpha a}{\pi^2(b^2m^2+a^2n^2)}(-1)^{n+1}e^{\frac{m\pi b}{a}}\right)e^{\frac{-m\pi y}{a}}\right]\right]\sin\left(\frac{m\pi x}{a}\right)\left(\frac{m^2\pi^2}{a^2}\right)-\frac{4E\alpha ab}{(b^2m^2+a^2n^2)}\times\sin\left(\frac{m\pi x}{a}\right)\times\left(\frac{m^2}{a^2}\right)\cos\left(\frac{n\pi y}{b}\right)\right\}e^{-\alpha A t}\left[\frac{ab}{\pi}S(1+2(-1)^{m+1})(3(-1)^n-1)\times\left(\frac{mb^2}{n^2}+\frac{a^2}{m}\right)\times\left\{\frac{te^{\alpha At}}{\alpha A}-\frac{e^{\alpha At}}{(\alpha A)^2}+\frac{1}{(\alpha A)^2}\right\}+\frac{\alpha g_p^i}{K}e^{\alpha A t'}p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\frac{p_l(z)}{\lambda_l}$$

$$\sigma_{xy} = \sum_{l=m=n=1}^{\infty}\left\{\left(\left(1+\frac{m\pi y}{a}\right)\frac{4E\alpha a(a-m\pi b)}{2mb\pi^4(b^4m^4+a^4n^4)}(-1)^{n+2}e^{\frac{-m\pi b}{a}}e^{\frac{m\pi y}{a}}+\left(1-\frac{m\pi y}{a}\right)\frac{4E\alpha a(a+m\pi b)}{2mb\pi^3(b^2m^2+a^2n^2)}(-1)^{n+1}e^{\frac{m\pi b}{a}}e^{\frac{-m\pi y}{a}}\right)\cos\left(\frac{m\pi x}{a}\right)\frac{m\pi}{a}-\frac{4E\alpha ab}{(b^2m^2+a^2n^2)}\sin\left(\frac{n\pi y}{b}\right)\left(\frac{mn}{ab}\right)\cos\left(\frac{m\pi x}{a}\right)\right\}e^{-\alpha At}\left[\frac{ab}{\pi}S(1+2(-1)^{m+1})(3(-1)^n-1)\times\left(\frac{mb^2}{n^2}+\frac{a^2}{m}\right)\times\left\{\frac{te^{\alpha At}}{\alpha A}-\frac{e^{\alpha At}}{(\alpha A)^2}+\frac{1}{(\alpha A)^2}\right\}+\frac{\alpha g_b^i}{K}e^{\alpha At'}p_l(0)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right)\right]\frac{p_l(z)}{\lambda_l}$$

Where $S=\left[-16h^2W_l\frac{\sin(a_lh)}{a_l^2}-24hW_l\frac{\cos(a_lh)}{a_l^3}+24W_l\frac{\sin(a_lh)}{a_l^4}\right]$

For Aluminium plate

Let K = 0.5330, $\alpha = 23.8\times10^{-6}$, E=$0.675\times10^{11}$, a=4cm, b=1cm, h=0.2cm

$$\sigma_{xx} = \sum_{m=n=l=1}^{\infty}\left\{\left[\left(\frac{ym^2\pi^2}{4^2}+\frac{2m\pi}{4}\right)\left(\frac{4-m\pi}{2m\pi^3}\times(-1)^{n+2}e^{\frac{-m\pi}{4}}e^{\frac{m\pi y}{4}}\right)+\left(\frac{ym^2\pi^2}{4^2}-\frac{2m\pi}{4}\right)\times\left(\frac{4+m\pi}{2m\pi^3}\times(-1)^{n+1}e^{\frac{m\pi}{4}}e^{\frac{-m\pi y}{4}}\right)-n^2\cos(n\pi y)\right]\frac{4\times0.675\times10^{11}\times4\times23.8\times10^{-6}}{(m^4+16n^4)}\emptyset\sin\left(\frac{m\pi x}{4}\right)\right\}$$

Where

$$\emptyset = e^{-23.8\times10^{-6}At}\left[\frac{23.8\times10^{-6}}{\pi}S(1+2(-1)^{m+1})(3(-1)^n-1)\left(\frac{m}{n^2}+\frac{16}{m}\right)\left\{\frac{te^{23.8\times10^{-6}At}}{23.8\times10^{-6}A}-\frac{e^{23.8\times10^{-6}At}}{(23.8\times10^{-6}A)^2}+\frac{1}{(23.8\times10^{-6}A)^2}\right\}+\frac{23.8\times10^{-6}}{0.5330}e^{23.8\times10^{-6}At'}p_l(0)\sin\left(\frac{m\pi x'}{4}\right)\cos\left(\frac{n\pi y'}{1}\right)\right]\frac{p_l(z)}{\lambda_l}$$

And $S=\left[-0.64W_l\frac{\sin(0.2a_l)}{a_l^2}-24hW_l\frac{\cos(0.2a_l)}{a_l^3}+24W_l\frac{\sin(0.2a_l)}{a_l^4}\right]$

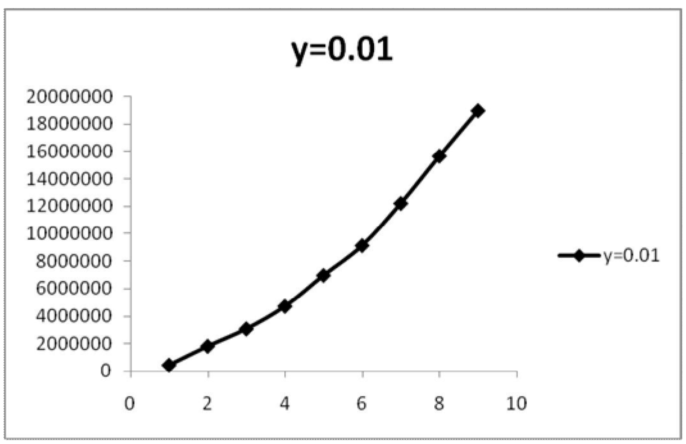

Figure 1: σ_{xx} vs t $(time)$

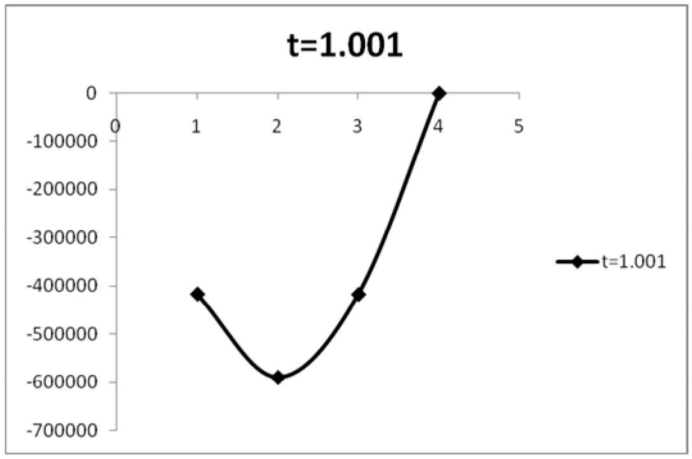

Figure 2: σ_{xx} vs x

Figure1 show the distribution of stress component σ_{xx} for different values of time. It shows stress increases with time.

Figure2 show the distribution of stress component σ_{xx} corresponding to x. stress in x direction is compressive in the region $0 \le x \le 4$.It shows minimum stress occur in middle of the plate and then it increases along x-axis.

VI.CONCLUSION

In this paper temperature has been determined using Finite Marchi-fasulo transform, Finite cosine transform and finite sine transform with internal moving point heat source for thin rectangular plate. The solutions are present in the form of infinite series. Also special case is considered and thermal stresses are determined numerically and graphically for it. It gives the large variation in stresses with respect to time.

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