Multi-item Economic Production-Policy for Repairable Defective Product with Volume Flexibility

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Abstract: In this paper an economic production quantity model has been developed for multi-item by considering imperfect production process. At the end of the cycle backorder is considered. It is assumed that imperfect products produced during production process are repaired with an extra repairing cost. We assumed that unit production cost of the items are the function of raw material cost, labor cost, tool/die cost. Here, production rate of items considered as decision variable and its value is obtained in such a way that total inventory cost of the system is minimum. Finally, developed model is illustrated with the help of numerical example. Sensitivity analysis is also carried out with respect to key parameters.
Keywords: Volume flexibility, Multi-item, Imperfect production process, Backorder

I. INTRODUCTION

In literature, it is observed that in most of the economic production quantity model it is assumed that production rate of item is regarded as pre-determined and inflexible parameter (**Hax and Candea, 1984**). **Schweitzer and Seidmann (1991)** were the first who adopted the concept of flexibility in production rate. It is obvious that in case of flexible manufacturing system, production rate is considered as decision variable and the unit production cost as the function of the production rate. **Khouja and Mehrez (1994)** reformulated inventory model by considering variable production rate and by taking production cost as function of production rate. They observed that the optimal production rate is smaller than the production rate which minimized unit production cost. **Sana and Chaudhuri (2003)** developed an inventory model for a deteriorating item with stock dependent demand. They assumed that unit production cost is a function of the finite production rate which is treated as a decision variable. **Sana *et al.* (2007)** developed an inventory model for imperfect production/inventory system where the items were a mixture of perfect and imperfect quality. **Sane *et al.*** considered that the production cost to be a function of the finite production rate which is treated as a decision variable. **Mandal *et al.* (2009)** developed the finite replenishment inventory model with imperfect production process by taking unit cost function as function which depends on raw material, labour, replenishment rate and others factors. **Roy *et al.* (2010)** developed an inventory model of a volume flexible manufacturing (VFM) system for a deteriorating item with randomly distributed shelf life, continuous time-varying demand and shortages over a finite time horizon. **Das *et al.* (2011)** developed an economic production lot size model for an item with imperfect quality by considering machine failure. They developed the model as profit maximization in stochastic and fuzzy-stochastic environments by considering some inventory parameters as imprecise in nature. **Qingguo *et al.* (2013)** developed an economic production quantity model with deteriorating items over a finite planning horizon. They assumed that production cost per unit, production rate and demand rate are known and continuous function of time. **Kumar *et al.* (2015)** investigated the supply chain inventory model for a retailer under two level of trade credit. **Tayal et al. (2015)** developed an inventory model for seasonal products with Weibull rate of deterioration having two potential markets say primary market and alternate market.

Under increased competition, inventory related businesses are focused for the coordination between their procurement and marketing decisions to avoid over stocks when sales are low or demand are high. An effective means of such coordination is to conduct the inventory control and manufacturing decision jointly. A manager requires that every employee–operators, analysts, quality inspectors, salesman, purchasing agents, or planners-is thoroughly and strictly disciplined about feeding updates into the system. The main task in doing so is to determine the optimal rate of production and inventory policy.

In this paper, multi-item production-inventory model for a volume flexible production rate allowing shortages which are completely backlogged is considered. During the production-run-time, the production process may shift to an 'out-of-control' state. In 'out-of-control' state, certain percent of produced items are defective. The defective items are reworked immediately at a cost. Total inventory cost function is minimized by considering production rate as decision variables.

This paper is organized as follows. In section 2 notations and assumptions are given which are used to develop the proposed model. Sections 3 presents mathematical model. Section 4 provides the solution procedure to obtain optimal solution. Section 5 provides the numerical illustration of the proposed model and the sensitive analysis is also carried out in this section. Section 6 summarizes the work done in this paper.

II. ASSUMPTIONS AND NOTATIONS

Assumptions:

Formulation of mathematical model is carried out on the basis of the following assumptions:

1) Inventory system involves 'n' items.
2) P_i is the production rate and considered as a decision variable.
3) Shortages are allowed.
4) The production cost per unit item is a function of the production rate.
5) Shortages considered which fully backlogged.

Notations:

Following notations has been used for the development of proposed model.

D_i Total demand over the planning time period [0,T] for the i^{th} item

R_i Daily demand for the i^{th} item

B_i Daily production rate for the i^{th} item

I_i Opportunity cost percentage for the i^{th} item

K_i Setup cost for the i^{th} item

E_i Cost incurred by repairing defective items for the i^{th} item

C_{Si} Backorder cost for the i^{th} item

H_i Daily storage cost per unit for the i^{th} item

S_i Shortage quantity for each cycle for the i^{th} item

p_i The probability that the production process can go 'out of control' for the i^{th} item

a_{pi} Investment is required to reduce the 'out of control probability with p_i for the i^{th} item

P_i Production rate for the i^{th} item

$C_i(P_i)$ Unit production cost $(=r_i + \frac{g_i}{P_i} + \beta_i P_i)$ for the i^{th} item

Where r_i, g_i, β_i are all positive constants. These costs are based on the following factors:

1. The material cost r_i per unit item is fixed.

2. As the production rate increases, some costs like labour and energy costs are equally distributed over a large number of units. Hence the production cost per unit $\left(\frac{g_i}{P_i}\right)$ decreases as the production rate (P_i) increases.

3. The third term ($\beta_i P_i$), associated with tool/die, is proportional to the production rate.

III. MATHEMATICAL FORMULATION

Fig.1 shows the inventory level for i^{th} item. It shows that production process starts at t=0 with production rate P_i and continues upto the time t_{1i}. During this period, inventory level of items increases due to production after adjusting the demand of the market. After time t_{1i}, production process stop and after that inventory level decrease due to demand of the market. Inventory level is positive till t_{2i} and after that shortages occur which is backorder completely.

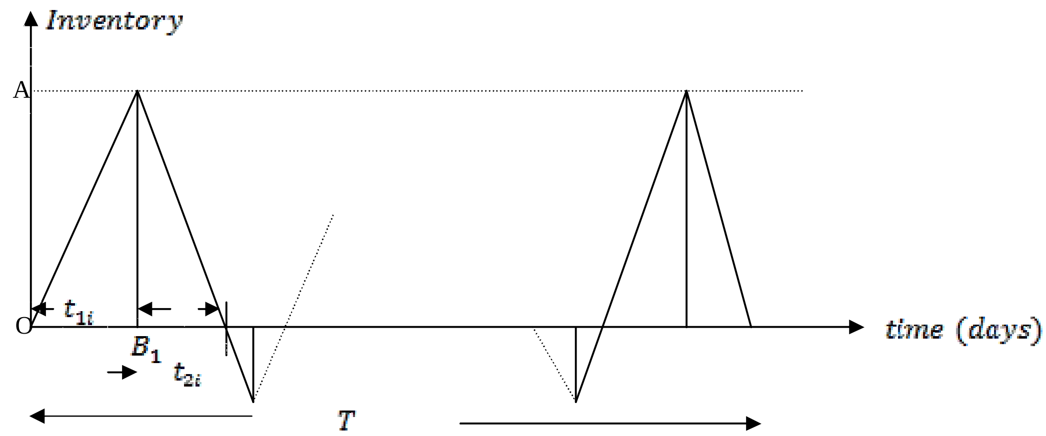


Fig.1 Inventory Level for the ith item

From the figure, it is observed that

The length of the production run for the ith item in days $(t_{1i}) = \frac{P_i}{B_i}$

Sales quantity for the production run for the ith item = $R_i \frac{P_i}{B_i}$

Inventory level at the end of the production run for the ith item $= P_i - R_i \frac{P_i}{B_i} = P_i \left(1 - \frac{R_i}{B_i}\right)$

Average inventory for the ith item $= \frac{P_i}{2}\left(1 - \frac{R_i}{B_i}\right)$

Now, we calculate different cost associated with inventory step by step.

Production Cost for the ith item = $C_i(P_i)D_i$

Investment cost required for fixed process for the ith item $= I_i a_{pi} T$

Setup cost for the ith item $= K_i \frac{D_i}{P_i}$

Storage cost for the ith item $= (I_i C_i(P_i) + H_i) \frac{P_i}{2}\left(1 - \frac{R_i}{B_i}\right)$T

Repairing cost for ith defective item = $D_i p_i E_i$

Annual backordering cost for the ith item = $C_{Si} \frac{S_i^2}{2P_i}$

Total inventory cost per cycle for the ith item is the sum of production cost, investment cost required for fixed process, setup cost, storage cost, repairing cost and backordering cost

$$TC_i(P_i) = C_i(P_i)D_i + I_i a_{pi} T + K_i \frac{D_i}{Q_i} + (I_i C_i(P_i) + H_i) \frac{P_i}{2}\left(1 - \frac{R_i}{B_i}\right)T + D_i p_i E_i + C_{Si} \frac{S_i^2}{2P_i}$$

Thus, total inventory cost of the system is

$$TC(P_i) = \sum_{i=1}^{n} TC_i(P_i)$$

$$TC(P_i) = \sum_{i=1}^{n} \left[C_i(P_i)D_i + I_i a_{pi} T + K_i \frac{D_i}{Q_i} + (I_i C_i(P_i) + H_i) \frac{P_i}{2}\left(1 - \frac{R_i}{B_i}\right)T + D_i p_i E_i + C_{Si} \frac{S_i^2}{2P_i}\right]$$

…. (1)

IV. SOLUTION PROCEDURE

Here, objective function is the function of production rate. Thus, we have to find the optimal value of production rate so that the total inventory cost of the system is minimum. For this, equating to zero the derivative of $TC(P_i)$ with respect to P_i i.e.,

$$\frac{dTC(P_i)}{dP_i} = 0$$

Thus, we get

$$\left(-\frac{g_i}{P_i^2}+\beta_i\right)D_i - K_i\frac{D_i}{P_i^2} + \left(I_i\left(-\frac{g_i}{P_i^2}+\beta_i\right)+H_i\right)\frac{P_i}{2}\left(1-\frac{R_i}{B_i}\right)\mathrm{T} + \frac{(I_iC_i(P_i)+H_i)}{2}\left(1-\frac{R_i}{B_i}\right)\mathrm{T} - C_{Si}\frac{S_i^2}{2P_i^2} = 0$$

…. (2)

On solving the above equation, we can get the optimal value of P_i. Now

$$\frac{d^2TC(P_i)}{dP_i^2} = \left(\frac{2g_i}{P_i^3}+\beta_i\right)D_i + K_i\frac{D_i}{P_i^3} + \left(I_i\left(\frac{2g_i}{P_i^3}+\beta_i\right)+H_i\right)\frac{P_i}{2}\left(1-\frac{R_i}{B_i}\right)\mathrm{T} + \left(I_i\left(-\frac{g_i}{P_i^2}+\beta_i\right)+H_i\right)\left(1-\frac{R_i}{B_i}\right)\mathrm{T} + C_{Si}\frac{S_i^2}{P_i^3} > 0$$

…. (3)

From above it is clear that total inventory cost function is convex with respect to production rate.

V. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

ABC manufacturing company produces three types of refrigerators. The values of different parameters associated with the model are as follows in appropriate units.

Item	H_i	K_i	E_i	I_i	p_i	a_{pi}	R_i	B_i	C_{Si}	S_i	r_i	g_i	β_i
1st	1.00	90000	1000	0.2%	0.02	100000	25	30	10	500	900	9000	0.01
2nd	0.95	89000	990	0.2%	0.02	100090	23	28	9	490	900	9000	0.01
3rd	1.05	91500	1010	0.2%	0.02	100000	28	32	12	510	900	9000	0.01

By using the above data, the optimal values of production rate are

$P_1^* = 1987, P_2^* = 1907, P_3^* = 2087,$

Sensitivity Analysis:

In order to analyze the effect of different key parameters on the optimal solution, we carried out the sensitive analysis. Sensitive analysis is carried out by using above data. The values of P_i^* is obtained, when one of the parameters increases or decreases by 5%, 10%, 15% and 20% while all other parameters remain unchanged.

Fig.2 reflects the change in production rate with respect to demand rate. From Figure it is observed that production rate is very sensitive with respect to demand parameter. As the demand rate increases, production rate is also increases to cope up the demand.

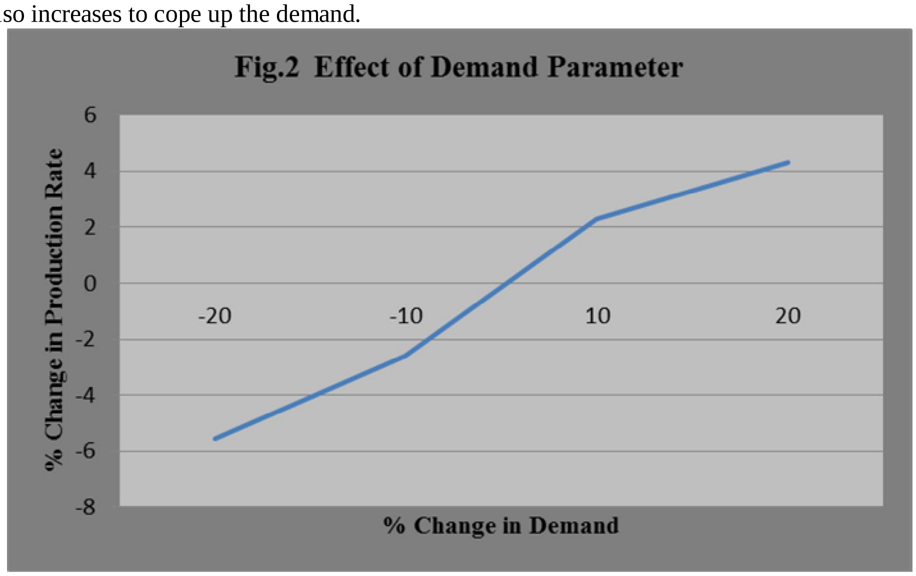
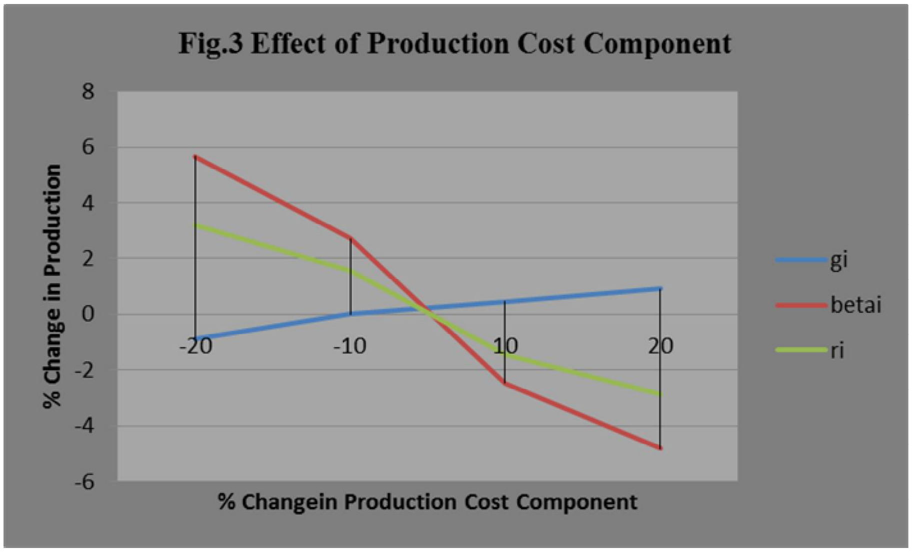


Fig.3 shows the effect different cost component of production cost on the optimal values of production rate. From figure it is observed that as the raw material cost increases production rate decreases whereas it increases as the labour and energy cost increases. On increasing the value of tool/die cost production rate also decreases.



VI. CONCLUSION

In this paper, an economic production quantity model for multi-item is developed by assuming that production system is not perfect. During production process, production system goes out of control and produces imperfect items which reworked simultaneously. Shortages occur at the end of cycle which backlogged completely. From literature it is observed that many inventory practitioners assumed that the rate of production as constant parameter while some others take it as decision variable. It observed that production rate depends on many factor such as cost of raw materials, labour, power, fuel and many more. In general, to avoid the stock-out situation it is assumed that production rate is higher than demand rate. This leads to rapid accumulation of inventories resulting in higher holding costs and other inventory related problems. To avoid this problem we take production rate as decision variable support the real phenomena that the production increases to adjust the increasing demand and it helps to decline the unit production cost.

The sensitivity analysis is also carried out to obtain the optimal solution without re-solving the problem. Sensitive analysis provided the direction to the decision maker that special attention is given to demand parameters and different component cost of production cost. As future research work, the present analysis can be applied to other inventory situation and the present model can be extended to consider different demand pattern, different pattern of deterioration rate, and using budget constraint.

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