Heat Transfer in The Forced Flow of A Non-Newtonian Second-Order Fluid Between Two Infinite Discs of Different Permeability

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Abstract: The problem of heat transfer in the steady forced axisymmetric flow of an incompressible non-Newtonian second-order fluid between two infinite porous discs of different permeability has been discussed. It is assumed that the lower and upper infinite discs are maintained at constant temperature T_a and T_b respectively. Since the governing equations of motion and energy equations are highly non-linear and of sufficiently large order, therefore the series solution method is adopted to solve these equations. The flow of the fluid is taken as laminar and flow Reynold's number is assumed small such that it's third and higher powers are assumed to be very small. Hence the term containing R^3 and higher powers of R are neglected. The dimensionless velocity functions F, G and energy functions ϕ and ψ are expended in the ascending powers of the perturbation parameter R. The variations of the dimensionless temperature T^* with the dimensionless gap-length variable ζ and the Nusselt's number Nu_a (on the lower disc), Nu_b (on the upper disc), with the dimensionless radial variable ξ are shown through the numerical values given in the tables and the graphs (represented by various figures) for different values of parameters m (forced parameter), A (suction parameter), N (permeability ratio parameter) and τ_1 and τ_2 (second-order parameters). The behaviour of the temperature and nature of the heat flux is discussed and given in the conclusion heading of the research paper.

Keywords: Heat transfer, forced-flow, second-order fluid, rotating infinite discs of different permeability.

I. INTRODUCTION

The non-Newtonian fluids play an important role in modern technology and industrial applications. Increasing emergence of non-Newtonian fluids such as molten plastic pulp, emulsions, raw materials in petroleum industries and chemical processes has simulated a considerable amount of interest to study the heat transfer in the flow of such fluid. During the past decades there have been several studies on such fluids. The problem of steady forced flow of a viscous incompressible fluid against a rotating disc was first studied by Schlichting and Truckenbrodt [1]. Jain [2] extended this problem for Reiner-Rivlin fluid. Srivastava and Sharma [3] studied the problem of forced flow in case of second-order fluid against a rotating disc. Sharma and Prakash [4] have discussed the problem of forced flow of a second-order fluid when the disc is subjected to uniform high suction. Sharma and Singh [5] have discussed the problem of forced flow of second-order fluid between two infinite discs of uniform porosity. Singh and Shiva [6] have extended this problem in case of discs of different permeability. There after Singh and Richa [7] have discussed the problem of heat transfer in the forced flow of a non-Newtonian Reiner-Rivlin fluid between two infinite rotating porous discs of different permeability.

The purpose of the present paper is to discuss the heat transfer in the forced flow of non-Newtonian second-order fluid between two infinite discs of different permeability.

II. FORMULATION OF THE PROBLEM

Coleman and Noll [8] suggested the constitutive equation of second-order fluid as :

$$\tau_{ij} = -p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 c_{ij} \qquad (1)$$

Where $\quad d_{ij} = (1/2)\left(u_{i,j} + u_{j,i}\right)$

$$e_{ij} = (1/2)\left(a_{i,j} + a_{j,i}\right) + u^m_{,i} u_{m,j}$$

and $\quad c_i j = d_{im}\, d^m_{,j}$ (2)

p is the hydrostatic pressure, τ_{ij} is the stress-tensor, u_i is the velocity vector, δ_{ij} is the kronecker delta tensor, a_i is the acceleration vector, μ_1 is the coefficient of Newtonian viscosity, μ_2 is the coefficient of elastic-viscosity, μ_3 is the coefficient cross viscosity and $u_{i,j}$ is the covariant derivative with respect to j of the velocity vector u_i.

The equation (1) together with the momentum equation for no extraneous force

$$\rho\left(\partial u_i / \partial t + u^m \; u_{i,m}\right) = \tau^m{}_{i,m} \qquad (3)$$

the equation of continuity for incompressible fluid

$$u^i_{,i} = 0 \qquad (4)$$

the energy equation describing the transport of the thermal energy is

$$\rho c_\upsilon \left(DT / Dt\right) = k \nabla^2 T + \Phi \qquad (5)$$

Where $\quad \Phi = \tau^i_j\, d^j_i$ (6)

ρ is the density of the fluid and $(\,,)$ represents covariant differentiation and τ^i_j is the deviatoric stress tensor from the set of governing equations.

Where c_v is the specific heat at constant volume, Φ be the viscous dissipation function, k is the thermal conductivity, ρ is the density of the fluid, T is the temperature of the fluid at any time t and τ^i_j is the deviatoric stress tensor and $(\,,)$ represents covariant differentiation , form the set of governing equations.

In considering the fluid motion due to rotation of the lower disc $(z = 0)$ with constant angular velocity Ω about z axis and the upper disc $(z = z_0)$ simultaneously creating a symmetrical radial velocity a_r , we use cylindrical polar co-ordinates (r, θ, z) and denote the corresponding velocity components u, v, w respectively. The lower disc is maintained at the constant temperature T_a and the upper disc is maintained at constant temperature T_b .

The relevant boundary conditions of the problem are:

$$z = 0: \quad u = 0 \quad v = r\Omega \quad w = w_0 \quad T = T_a$$

$$z = z_0: \quad u = a_r \quad v = Nr\Omega \quad w = w_0 \quad T = T_b \qquad (7)$$

The velocity components suggested by Srivastava [9] satisfy the continuity equation and that of pressure are as follows:

$$u = r\Omega F'(\zeta)$$

$$v = r\Omega\, G(\zeta) \qquad (8)$$

$$w = -2 z_0 \Omega\, F(\zeta)$$

and $P = \Omega\mu_1\left[-P_1(\zeta) + R\left(r^2 / z_0{}^2\right)\left(2\tau_1 + \tau_2\right)\left(F''^2 + G'^2\right) + \lambda\left(r^2 / z_0{}^2\right)\right]$ (9)

where $F(\zeta)$ and $G(\zeta)$ are non-dimensionless functions of the dimensionless variable $\zeta\left(= z / z_0\right)$, $R\left(= \Omega \rho z_0{}^2 / \mu_1\right)$ is the Reynolds number, $\tau_1 (= \mu_2 / \rho z_0{}^2)$ and $\tau_2 (= \mu_3 / \rho z_0{}^2)$ are the dimensionless parameters representing elastico-viscous and cross-viscous effects respectively, λ is a parameter which depends upon the Reynolds number R and primes denotes differentiation with respect to ζ .

Substituting the expression (8) and (9) in the equation of motion (3) and using the constitutive equation (1) we obtain:

$$R\left(F^{'2}-2FF^{''}-G^{2}\right)=F^{'''}-R\left[\left(\tau+\tau_{1}\right)F^{''2}+\left(3\tau+\tau_{1}\right)G^{'2}+2\tau_{1}FF^{iv}+2\tau_{2}FF^{'''}\right]-2\lambda \quad (10)$$

$$2R\left(F^{'}-FG^{'}\right)=G^{''}+2R\left[\tau F^{''}G^{'}-\tau_{1}FG^{'''}-\tau_{2}F^{'}G^{''}\right] \quad (11)$$

$$4RFF^{'}=P_{1}^{'}-2F^{''}+4R\left\{\left(7\tau+4\tau_{1}\right)F^{'}F^{''}+\tau_{1}FF^{'''}\right\} \quad (12)$$

Where $\tau=\tau_1+\tau_2$ represents total second-order effects.

Energy equation (5) together with the expression (8) of velocity profile suggest the form of temperature distribution as follows:

$$T=T_{a}+\left(\nu_{1}\Omega/C_{v}\right)\left[\phi(\zeta)+\xi^{2}\psi(\zeta)\right]$$

(13)

Using (13) in energy equation (5) and equating the coefficient of ξ^2 and terms of independent of ξ^2 on the both side of the resulting equation, we obtain:

$$\psi^{''}=P_{r}R\left(2F^{'}\psi-2F\psi^{'}-F^{''2}-G^{'2}\right)+2P_{r}R^{2}\tau_{1}\left(F^{'}F^{''2}+F^{'}G^{'2}+FF^{'}F^{'''}+FG^{'}G^{''}\right)$$
$$+4P_{r}R^{2}\tau_{2}\left\{(3/4)\left(F^{'}F^{''2}+F^{'}G^{'2}\right)\right\} \quad (14)$$

$$\phi^{''}=-P_{r}R\left(2F\phi^{'}+12F^{'2}\right)-4\psi+2P_{r}R^{2}\tau_{1}\left(12F^{'3}+12FF^{'}F^{''}\right)+24P_{r}R^{2}\tau_{2}F^{'3} \quad (15)$$

Where $P_r(=\mu C_v/k)$ is the Prandtl's number, $\xi(=r/d)$ is the dimensionless radius and prime $(')$ denotes the differentiation with respect to ζ.

The expression of temperature distribution in the dimensionless form can be written as:

$$T^{*}=\frac{T-T_{a}}{T_{b}-T_{a}}=E\left[\phi(\zeta)+\xi^{2}\psi(\zeta)\right]$$

(16)

Where $E\left[=\upsilon_{1}\Omega/\left(T_{b}-T_{a}\right)C_{\upsilon}\right]$ is the Eckert number.

The boundary conditions (7) transform to as :

$$\zeta=0: \quad F=-A \quad F^{'}=0 \quad G=1 \quad \phi=0 \quad \psi=0$$
$$\zeta=1: \quad \phi_{0}=S \quad F^{'}=m \quad G=0 \quad \phi=1/S=E \quad \psi=0$$

(17)

Where $m(=a/\Omega)$ is the dimensionless forced parameter assumed to be small $m\leq 1$ and $A(=w_0/2z_0\Omega)$ is the suction parameter.

III. SOLUTION OF THE PROBLEM

A regular perturbation technique is developed by expanding F, G, λ, ϕ and ψ in ascending powers of Reynolds number R (assumed small). The terms containing R^3 and higher powers of R are neglected.

$F(\zeta)=-A+\sum R^{n}F_{n}(\zeta)$, $G(\zeta)=\sum R^{n}G_{n}(\zeta)$, $\lambda(\zeta)=\sum R^{n}\lambda_{n}(\zeta)$, $\phi(\zeta)=\sum R^{n}\phi_{n}(\zeta)$ and $\psi(\zeta)=\sum R^{n}\psi_{n}(\zeta)$. (18)

Substituting the series in (18) into the equation (10), (11), (12),(14) and (15) and equating the terms independent of R, coefficient of R and R^2 we obtain the partial differential equation as :

$$F_{0}^{'''}=2\lambda_{0}$$

$$F_{1}^{'''}=F_{0}^{'2}-2F_{0}F_{0}^{''}-G_{0}^{2}+\left(\tau+\tau_{1}\right)F_{0}^{''2}+\left(3\tau+\tau_{1}\right)G_{0}^{'2}+2\tau_{1}\left(F_{0}-A\right)F_{0}^{iv}+2\tau_{2}F_{0}^{'}F_{0}^{'''}+2\lambda_{1}$$

$$F_2''' = 2\begin{bmatrix} F_0'F_1' - (F_0 - A)F_1'' - F_1F_0'' - G_0G_1 + (\tau+\tau_1)F_0''F_1'' \\ +(3\tau+\tau_1)G_0'G_1' + \tau_1\left\{(F_0 - A)F_1^{iv} + F_1F_0^{iv} + \tau_2\left(F_0'F_1''' + F_1'F_0'''\right) + \tau_2\right\} \end{bmatrix}$$

(19)

$$G_0'' = 0$$

$$G_1'' = 2\left[F_0'G_0 - (F_0 - A)G_0' - \tau F_0''G_0' + \tau_1(F_0 - A)G_0''' + \tau_2 F_0'G_0''\right]$$

$$G_2'' = 2\begin{bmatrix} F_0'G_1 + F_1'G_0 - (F_0 - A)G_1' - F_1G_0' - \tau\left(F_0''G_1' + F_1''G_0'\right) \\ +\tau_1\left\{(F_0 - A)G_1''' + F_1G_0'''\right\} + \tau_2\left(F_0'G_1'' + F_1'G_0''\right) \end{bmatrix}$$

(20)

$$\psi_0'' = 0$$

$$\psi_1'' = P_r\left[2F_0'\psi_0 - 2(F_0 - A)\psi_0' - F_0''^2 - G_0'^2\right]$$

$$\psi_2'' = P_r\left[2F_1'\psi_0 + 2F_0'\psi_1 - 2F_1'\psi_0' - 2(F_0 - A)\psi_1' - 2F_0''F_1'' - 2G_0'G_1'\right]$$

$$+2P_r\tau_1\left[F_0'F_0''^2 + F_0'G_0'^2 + (F_0 - A)F_0''F_0''' + (F_0 - A)G_0'G_0''\right]$$

$$+3P_r\tau_2\left(F_0'F_0''^2 + F_0'G_0'^2\right)$$

(21)

$$\phi_0'' = -4\psi_0$$

$$\phi_1'' = -P_r\left\{2(F_0 - A)\psi_0' + 12F_0'^2\right\} - 4\psi_1$$

$$\phi_2'' = -P_r\left\{2(F_0 - A)\psi_1' + 2F_1\phi_0' + 24F_0'F_1'\right\} - 4\psi_2 + 2P_r\tau_1\left\{12F_0'^3 + 12(F_0 - A)F_0'F_0'' + 24P_r\tau_2F_0'^3\right\}$$

(22)

The boundary conditions (17) in terms of F_0, F_1 etc. can be written as:

$F_n(0) = 0$	$F_n'(0) = 0$	$\forall n = 0,1,2,3,\ldots\ldots\ldots$
$F_0(1) = A(1-N)$	$F_n(1) = 0$	$\forall n \ge 1$
$F_0'(1) = m$	$F_n'(1) = 0$	$\forall n \ge 1$
$G_0(0) = 1$	$G_n(0) = 0$	$\forall n \ge 1$
$G_n(1) = 0$	$\psi_n(1) = 0$	$\forall n \ge 0$
$\phi_n(0) = 0$	$\psi_n(0) = 0$	$\forall n \ge 0$
$\phi_0(1) = S = 1/E$	$\phi_n(1) = 0$	$\forall n \ge 1$

(23)

We observe that the set of equations (21) and (22) of temperature components $\psi_0, \psi_1, \psi_2; \phi_0, \phi_1, \phi_2$ not contain F_2, F_2', F_2'' .... and G_2, G_2', G_2'' .... etc.. Hence there is no need to solve the differential equations of velocity functions F_2 and G_2.

The values of $F_0, F_1, G_0, G_1, \lambda_0$ and λ_1 are obtained by solving the set of equations (19) and (20) subjected to the boundary conditions (23) as:

$$F_0(\zeta) = \alpha_1\zeta^3 - \alpha_2\zeta^2$$

$$F_1(\zeta) = -\frac{1}{70}\alpha_1^{\,2}\zeta^7 + \frac{1}{30}\alpha_1\alpha_2\zeta^6 - \frac{1}{60}\zeta^5 + \frac{1}{12}\zeta^4 - \frac{1}{6}\zeta^3 + (\tau+\tau_1)\left\{\frac{3}{5}\alpha_1^{\,2}\zeta^5 - \alpha_1\alpha_2\zeta^4 + \frac{1}{3}\left(2\alpha_2^{\,2} - \frac{3}{2}\right)\zeta^3\right\}$$
$$+12\tau_2\left(\frac{1}{20}\alpha_1^{\,2}\zeta^5 - \frac{1}{12}\alpha_1\alpha_2\zeta^4\right) + \alpha_3\left(\zeta^3 - \zeta^2\right) + \alpha_4\left(2\zeta^3 - 3\zeta^2\right)$$
(24)

$$\lambda_0 = 3\alpha_1$$
$$\lambda_1 = 6\alpha_4 + 3\alpha_3$$
(25)

Where

$$\alpha_1 = m + 2A(N-1)$$
$$\alpha_2 = m + 3A(N-1)$$
$$\alpha_3 = \frac{1}{10}\alpha_1^{\,2} - \frac{1}{5}\alpha_1\alpha_2 + \frac{1}{4} + (\tau+\tau_1)\left(-3\alpha_1^{\,2} + 4\alpha_1\alpha_2 - 2\alpha_2^{\,2} + \frac{3}{2}\right) + 12\tau_2\left(\frac{1}{3}\alpha_1\alpha_2 - \frac{1}{4}\alpha_1^{\,2}\right)$$
$$\alpha_4 = -\frac{1}{70}\alpha_1^{\,2} + \frac{1}{30}\alpha_1\alpha_2 - \frac{1}{10} + (\tau+\tau_1)\left\{\frac{3}{5}\alpha_1^{\,2} - \alpha_1\alpha_2 + \frac{1}{3}\left(2\alpha_2^{\,2} - \frac{3}{2}\right)\right\} + 12\tau_2\left(\frac{1}{20}\alpha_1^{\,2} - \frac{1}{12}\alpha_1\alpha_2\right)$$
(26)

$$G_0(\zeta) = 1 - \zeta$$
$$G_1(\zeta) = 2\left[-\frac{1}{10}\alpha_1\zeta^5 + \frac{1}{12}(\alpha_2 + 3\alpha_1)\zeta^4 - \frac{1}{3}\alpha_2\zeta^3 - \frac{1}{2}A\zeta^2 + \tau\left(\alpha_1\zeta^3 - \alpha_2\zeta^2\right)\right] + \alpha_5\zeta$$
(27)

Where

$$\alpha_5 = 2\left[\frac{1}{10}\alpha_1 - \frac{1}{12}(\alpha_2 + 3\alpha_1) + \frac{1}{3}\alpha_2 + \frac{1}{2}A + \tau(\alpha_2 - \alpha_1)\right]$$

On substituting the values of F_0, F_1, G_0, G_1 and their derivatives with respect to ζ in the set of differential equations (21) and (22) and integrating these subject to the boundary conditions (23) we get

$$\psi_0(\zeta) = 0$$
$$\psi_1(\zeta) = -P_r\left[3\alpha_1^{\,2}\zeta^4 - 4\alpha_1\alpha_2\zeta^3 + \left(2\alpha_2^{\,2} + \tfrac{1}{2}\right)\zeta^2\right] + \alpha_6\zeta$$

$$\psi_2(\zeta)=2P_r\left[\begin{array}{l} P_r\left\{\begin{array}{l}\frac{3}{56}\alpha_1^3\zeta^8-\frac{1}{7}\alpha_1^2\alpha_2\zeta^7+\frac{1}{30}\left(2\alpha_2^2-1/2\right)\alpha_1\zeta^6\\ -A\left(\frac{3}{5}\alpha_1^2\zeta^5-\alpha_1\alpha_2\zeta^4+\frac{1}{3}\left(2\alpha_2^2+1/3\right)\zeta^3\right)\end{array}\right\}\\ +\alpha_6\left(\frac{1}{10}\alpha_1\zeta^5-\frac{1}{12}\alpha_2\zeta^4+\frac{1}{2}A\zeta^2\right)+\frac{9}{140}\alpha_1^3\zeta^8+\frac{6}{35}\alpha_1^2\alpha_2\zeta^7\\ +\frac{1}{15}\alpha_1\left(1+\alpha_2^2\right)\zeta^6-\frac{1}{10}\left(3\alpha_1+\alpha_2/3\right)\zeta^5+\frac{1}{6}\left(3\alpha_1+\alpha_2\right)\zeta^4-\frac{1}{3}\alpha_2\zeta^3\\ +\left(\tau+\tau_1\right)\left\{-\frac{12}{5}\alpha_1^3\zeta^6+\frac{24}{5}\alpha_1^2\alpha_2\zeta^5-\left(4\alpha_1\alpha_2^2-3\alpha_1/2\right)\zeta^4+\frac{2}{3}\alpha_2\left(2\alpha_2^2-3/2\right)\zeta^3\right\}+12\end{array}\right]$$

(28)

Where

$$\alpha_6=P_r\left(3\alpha_1^2+2\alpha_2^2-4\alpha_1\alpha_2+\frac{1}{2}\right)$$

$$\alpha_7=60\alpha_1\alpha_2^2+3\alpha_1$$

$$\alpha_8=-2P_r\left[\begin{array}{l} P_r\left\{\frac{3}{56}\alpha_1^3-\frac{1}{7}\alpha_1^2\alpha_2+\frac{1}{30}\left(2\alpha_2^2-\frac{1}{2}\right)\alpha_1-A\left(\frac{3}{5}\alpha_1^2-\alpha_1\alpha_2+\frac{1}{3}\left(2\alpha_2^2+\frac{1}{3}\right)\right)\right\}\\ +\alpha_6\left(\frac{1}{10}\alpha_1-\frac{1}{12}\alpha_2+\frac{1}{2}A\right)+\frac{9}{140}\alpha_1^3+\frac{6}{35}\alpha_1^2\alpha_2+\frac{1}{15}\alpha_1\left(1+\alpha_2^2\right)\\ -\frac{1}{10}\left(3\alpha_1+\frac{\alpha_2}{3}\right)+\frac{1}{6}\left(3\alpha_1+\alpha_2\right)-\frac{1}{3}\alpha_2\\ +\left(\tau+\tau_1\right)\left\{-\frac{12}{5}\alpha_1^3+\frac{24}{5}\alpha_1^2\alpha_2-\left(4\alpha_1\alpha_2^2-\frac{3\alpha_1}{2}\right)+\frac{2}{3}\alpha_2\left(2\alpha_2^2-\frac{3}{2}\right)\right\}\\ +12\tau_2\left(-\frac{1}{5}\alpha_1^3+\frac{2}{5}\alpha_1^2\alpha_2-\frac{1}{6}\alpha_1\alpha_2^2\right)-\alpha_1\alpha_3-2\alpha_2\alpha_4\end{array}\right]$$

(29)

$$\phi_0(\zeta)=S\zeta$$

$$\phi_1(\zeta)=-2P_rS\left(\frac{1}{20}\alpha_1\zeta^5-\frac{1}{12}\alpha_2\zeta^4-\frac{1}{2}A\zeta^2\right)-4P_r\left\{\frac{4}{5}\alpha_1^2\zeta^6-\frac{8}{5}\alpha_1\alpha_2\zeta^5+\frac{1}{12}\left(10\alpha_2^2-\frac{1}{2}\right)\zeta^4\right\}$$

$$-\frac{2}{3}\alpha_6\zeta^3+\alpha_9\zeta$$

$$\phi_2(\zeta) = -2P_r\left[-2P_rS\beta_1 - 4P_r\beta_2 + S\beta_3 + 12\beta_4 + 12\tau_2\beta_5\right] + 24P_r\tau_1\beta_6 + 24P_r\tau_2\beta_7$$
$$-8P_r\left[P_r\beta_8 + \beta_9 + (\tau+\tau_1)\beta_{10} + \beta_{11} + 2\beta_{12}\right] - 8P_r\tau_1\beta_{13} - 12P_r\tau_2\beta_{14} - \frac{2}{3}\alpha_8\zeta^3 + \alpha_{10}\zeta$$

(30)

Where

$$\beta_1 = \left\{\frac{1}{288}\alpha_1^2\zeta^9 - \frac{1}{96}\alpha_1\alpha_2\zeta^8 + \frac{1}{126}\alpha_2^2\zeta^7 - A\left(\frac{1}{24}\alpha_1\zeta^6 - \frac{1}{15}\alpha_2\zeta^5 - \frac{A}{6}\zeta^3\right)\right\}$$

$$\beta_2 = \left\{\begin{array}{l} \frac{4}{75}\alpha_1^3\zeta^{10} - \frac{8}{45}\alpha_1^2\alpha_2\zeta^9 + \frac{1}{168}\alpha_1\left(34\alpha_2^2 - \frac{1}{2}\right)\zeta^8 - \frac{1}{42}\left(\frac{10}{3}\alpha_2^3 + \frac{24}{5}A\alpha_1^2 + 2\alpha_1\alpha_6 - \frac{\alpha_2}{6}\right)\zeta^7 \\ +\frac{1}{15}\left(4\alpha_1\alpha_2A + \alpha_2\alpha_6\right)\zeta^6 + \frac{1}{60}\left(10\alpha_2^2 + 3\alpha_1\alpha_9 - \frac{1}{2}\right)\zeta^5 + \frac{1}{12}\left(2A\alpha_6 - \alpha_2\alpha_9\right)\zeta^4 - \frac{1}{2}A\alpha_9\zeta^2 \end{array}\right\}$$

$$\beta_3 = \left\{\begin{array}{l} -\frac{1}{5040}\alpha_1^2\zeta^9 + \frac{1}{1680}\alpha_1\alpha_2\zeta^8 - \frac{1}{2520}\zeta^7 + \frac{1}{360}\zeta^6 - \frac{1}{120}\zeta^5 \\ +(\tau+\tau_1)\left(\frac{1}{70}\alpha_1^2\zeta^7 - \frac{1}{30}\alpha_1\alpha_2\zeta^6 + \frac{1}{60}\left(2\alpha_2^2 - \frac{3}{2}\right)\zeta^5\right) + 12\tau_2\left(\frac{1}{840}\alpha_1^2\zeta^7 - \frac{1}{360}\alpha_1\alpha_2\zeta^6\right) \\ +\alpha_3\left(\frac{1}{20}\zeta^5 - \frac{1}{12}\zeta^4\right) + \alpha_4\left(\frac{1}{10}\zeta^5 - \frac{1}{4}\zeta^4\right) \end{array}\right\}$$

$$\beta_4 = \left\{\begin{array}{l} -\frac{1}{300}\alpha_1^3\zeta^{10} + \frac{1}{90}\alpha_1^2\alpha_2\zeta^9 - \frac{1}{56}\alpha_1\left(\frac{1}{2}\alpha_2^2 + \frac{1}{2}\right)\zeta^8 + \frac{1}{42}\left(\alpha_1 + \frac{1}{6}\alpha_2\right)\zeta^7 \\ -\frac{1}{30}\left(\frac{3}{2}\alpha_1 + \frac{2}{3}\alpha_2\right)\zeta^6 + \frac{1}{20}\alpha_2\zeta^5 \\ +(\tau+\tau_1)\left(\frac{9}{56}\alpha_1^3\zeta^8 - \frac{3}{7}\alpha_1^2\alpha_2\zeta^7 + \frac{1}{30}\alpha_1\left(14\alpha_2^2 - \frac{9}{2}\right)\zeta^6 - \frac{1}{10}\alpha_2\left(2\alpha_2^2 - \frac{3}{2}\right)\zeta^5\right) \end{array}\right\}$$

$$\beta_5 = \left(\begin{array}{l} \frac{3}{224}\alpha_1^3\zeta^8 - \frac{1}{28}\alpha_1^2\alpha_2\zeta^7 + \frac{1}{45}\alpha_1\alpha_2^2\zeta^6 + \frac{3}{10}\left(\alpha_1\alpha_3 + 2\alpha_1\alpha_4\right)\zeta^6 \\ -\frac{3}{10}\left(\alpha_1\alpha_3 + \alpha_2\alpha_3 + 3\alpha_1\alpha_4 + 2\alpha_2\alpha_4\right)\zeta^5 + \frac{1}{3}\alpha_2\left(\alpha_3 + 3\alpha_4\right)\zeta^4 \end{array}\right)$$

$$\beta_6 = \left[\frac{45}{56}\alpha_1^3\zeta^8 - \frac{15}{7}\alpha_1^2\alpha_2\zeta^7 + \frac{29}{15}\alpha_1\alpha_2^2\zeta^6 - \frac{1}{10}\left(6\alpha_2^3 + 9\alpha_1^2A\right)\zeta^5 + \frac{3}{2}A\alpha_1\alpha_2\zeta^4 - \frac{2}{3}A\alpha_2^2\zeta^3\right]$$

$$\beta_7 = \left[\frac{27}{56}\alpha_1^3\zeta^8 - \frac{9}{7}\alpha_1^2\alpha_2\zeta^7 + \frac{6}{5}\alpha_1\alpha_2^2\zeta^6 - \frac{2}{5}\alpha_2^3\zeta^5\right]$$

$$\beta_8 = \left\{\begin{array}{l} \frac{1}{1680}\alpha_1^3\zeta^{10} - \frac{1}{504}\alpha_1^2\alpha_2\zeta^9 + \frac{1}{1680}\left(2\alpha_2^2 - \frac{1}{2}\right)\alpha_1\zeta^8 \\ -A\left(\frac{1}{70}\alpha_1^2\zeta^7 - \frac{1}{30}\alpha_1\alpha_2\zeta^6 + \frac{1}{60}\left(2\alpha_2^2 + \frac{1}{3}\right)\zeta^5\right) \end{array}\right\}$$

$$\beta_9=\alpha_6\left(\frac{1}{420}\alpha_1\zeta^7-\frac{1}{360}\alpha_2\zeta^6+\frac{1}{24}A\zeta^4\right)+\frac{1}{1400}\alpha_1^3\zeta^{10}+\frac{1}{420}\alpha_1^2\alpha_2\zeta^9+$$
$$\frac{1}{840}\alpha_1\left(1+\alpha_2^2\right)\zeta^8-\frac{1}{420}\left(3\alpha_1+\frac{1}{3}\alpha_2\right)\zeta^7+\frac{1}{180}\left(3\alpha_1+\alpha_2\right)\zeta^6-\frac{1}{60}\alpha_2\zeta^5$$

$$\beta_{10}=\left\{-\frac{3}{70}\alpha_1^3\zeta^8+\frac{4}{35}\alpha_1^2\alpha_2\zeta^7-\frac{1}{60}\left(8\alpha_1\alpha_2^2-3\alpha_1\right)\zeta^6+\frac{1}{30}\alpha_2\left(2\alpha_2^2-\frac{3}{2}\right)\zeta^5\right\}$$
$$\beta_{11}=12\tau_2\left(-\frac{1}{280}\alpha_1^3\zeta^8+\frac{1}{105}\alpha_1^2\alpha_2\zeta^7-\frac{1}{180}\alpha_1\alpha_2^2\zeta^6\right)-\frac{1}{10}\alpha_1\alpha_3\left(\zeta^6-\zeta^5\right)+\alpha_2\alpha_3\left(\frac{1}{10}\zeta^5-\frac{1}{6}\zeta^4\right)$$
$$-\alpha_1\alpha_4\left(\frac{1}{5}\zeta^6-\frac{3}{10}\zeta^5\right)+\alpha_2\alpha_4\left(\frac{1}{5}\zeta^5-\frac{1}{2}\zeta^4\right)+\frac{1}{24}\alpha_5\zeta^4$$

$$\beta_{12}=\left\{-\frac{1}{3360}\alpha_1\zeta^8+\frac{1}{2520}\left(3\alpha_1+\alpha_2\right)\zeta^7-\frac{1}{360}\alpha_2\zeta^6-\frac{1}{120}A\zeta^5+\tau\left(\frac{1}{120}\alpha_1\zeta^6-\frac{1}{60}\alpha_2\zeta^5\right)\right\}$$
$$\beta_{13}=\left[\begin{array}{l}\frac{9}{140}\alpha_1^3\zeta^8-\frac{6}{35}\alpha_1^2\alpha_2\zeta^7+\frac{1}{360}\alpha_7\zeta^6-\frac{1}{60}\alpha_2\left(4\alpha_2^2+1\right)\zeta^5\\ +6\alpha_1\left\{\frac{1}{280}\alpha_1^2\zeta^8-\frac{1}{105}\alpha_1\alpha_2\zeta^7+\frac{1}{180}\alpha_2^2\zeta^6-A\left(\frac{1}{20}\alpha_1\zeta^5-\frac{1}{12}\alpha_2\zeta^4\right)\right\}\end{array}\right]$$
$$\beta_{14}=\left[\frac{9}{140}\alpha_1^3\zeta^8-\frac{6}{35}\alpha_1^2\alpha_2\zeta^7+\frac{1}{360}\alpha_7\zeta^6-\frac{1}{60}\alpha_2\left(4\alpha_2^2+1\right)\zeta^5\right]$$
$$\alpha_9=2P_rS\left(\frac{1}{20}\alpha_1-\frac{1}{12}\alpha_2-\frac{1}{2}A\right)+4P_r\left\{\frac{4}{5}\alpha_1^2-\frac{8}{5}\alpha_1\alpha_2+\frac{1}{12}\left(10\alpha_2^2-\frac{1}{2}\right)\right\}+\frac{2}{3}\alpha_6$$
$$\alpha_{10}=2P_r\left[-2P_rS\beta_{15}-4P_r\beta_{16}+S\beta_{17}+12\beta_{18}+12\tau_2\beta_{19}\right]+24P_r\tau_1\beta_{20}-24P_r\tau_2\beta_{21}$$
$$+8P_r\left[P_r\beta_{22}+\beta_{23}+\left(\tau+\tau_1\right)\beta_{24}+\beta_{25}+2\beta_{26}\right]+8P_r\tau_1\beta_{27}+12P_r\tau_2\beta_{28}+\frac{2}{3}\alpha_8$$
$$\beta_{15}=\left\{\frac{1}{288}\alpha_1^2-\frac{1}{96}\alpha_1\alpha_2+\frac{1}{126}\alpha_2^2-A\left(\frac{1}{24}\alpha_1-\frac{1}{15}\alpha_2-\frac{A}{6}\right)\right\}$$

$$\beta_{16}=\left\{\begin{array}{l}\frac{4}{75}\alpha_1^3-\frac{8}{45}\alpha_1^2\alpha_2+\frac{1}{168}\alpha_1\left(34\alpha_2^2-\frac{1}{2}\right)-\frac{1}{42}\left(\frac{10}{3}\alpha_2^3+\frac{24}{5}A\alpha_1^2+2\alpha_1\alpha_6-\frac{\alpha_2}{6}\right)\\ +\frac{1}{15}\left(4\alpha_1\alpha_2A+\alpha_2\alpha_6\right)+\frac{1}{60}\left(10\alpha_2^2+3\alpha_1\alpha_9-\frac{1}{2}\right)+\frac{1}{12}\left(2A\alpha_6-\alpha_2\alpha_9\right)-\frac{1}{2}A\alpha_9\end{array}\right\}$$
$$\beta_{17}=\left\{\begin{array}{l}-\frac{1}{5040}\alpha_1^2+\frac{1}{1680}\alpha_1\alpha_2-\frac{1}{168}+\left(\tau+\tau_1\right)\left(\frac{1}{70}\alpha_1^2-\frac{1}{30}\alpha_1\alpha_2+\frac{1}{60}\left(2\alpha_2^2-\frac{3}{2}\right)\right)\\ +12\tau_2\left(\frac{1}{840}\alpha_1^2-\frac{1}{360}\alpha_1\alpha_2\right)-\frac{1}{30}\alpha_3-\frac{3}{20}\alpha_4\end{array}\right\}$$

$$\beta_{18}=\begin{Bmatrix}-\frac{1}{300}\alpha_1^3+\frac{1}{90}\alpha_1^2\alpha_2-\frac{1}{56}\alpha_1\left(\frac{1}{2}\alpha_2^2+\frac{1}{2}\right)+\frac{1}{42}\left(\alpha_1+\frac{1}{6}\alpha_2\right)\\-\frac{1}{30}\left(\frac{3}{2}\alpha_1+\frac{2}{3}\alpha_2\right)+\frac{1}{20}\alpha_2\\+\left(\tau+\tau_1\right)\left(\frac{9}{56}\alpha_1^3-\frac{3}{7}\alpha_1^2\alpha_2+\frac{1}{30}\alpha_1\left(14\alpha_2^2-\frac{9}{2}\right)-\frac{1}{10}\alpha_2\left(2\alpha_2^2-\frac{3}{2}\right)\right)\end{Bmatrix}$$

$$\beta_{19}=\begin{pmatrix}\frac{3}{224}\alpha_1^3-\frac{1}{28}\alpha_1^2\alpha_2+\frac{1}{45}\alpha_1\alpha_2^2+\frac{3}{10}\left(\alpha_1\alpha_3+2\alpha_1\alpha_4\right)\\-\frac{3}{10}\left(\alpha_1\alpha_3+\alpha_2\alpha_3+3\alpha_1\alpha_4+2\alpha_2\alpha_4\right)+\frac{1}{3}\alpha_2\left(\alpha_3+3\alpha_4\right)\end{pmatrix}$$

$$\beta_{20}=\left[\frac{45}{56}\alpha_1^3-\frac{15}{7}\alpha_1^2\alpha_2+\frac{29}{15}\alpha_1\alpha_2^2-\frac{1}{10}\left(6\alpha_2^3+9\alpha_1^2A\right)+\frac{3}{2}A\alpha_1\alpha_2-\frac{2}{3}A\alpha_2^2\right]$$

$$\beta_{21}=\left[\frac{27}{56}\alpha_1^3-\frac{9}{7}\alpha_1^2\alpha_2+\frac{6}{5}\alpha_1\alpha_2^2-\frac{2}{5}\alpha_2^3\right]$$

$$\beta_{22}=\begin{Bmatrix}\frac{1}{1680}\alpha_1^3-\frac{1}{504}\alpha_1^2\alpha_2+\frac{1}{1680}\left(2\alpha_2^2-\frac{1}{2}\right)\alpha_1\\-A\left(\frac{1}{70}\alpha_1^2-\frac{1}{30}\alpha_1\alpha_2+\frac{1}{60}\left(2\alpha_2^2+\frac{1}{3}\right)\right)\end{Bmatrix}$$

$$\beta_{23}=\alpha_6\left(\frac{1}{420}\alpha_1-\frac{1}{360}\alpha_2+\frac{1}{24}A\right)+\frac{1}{1400}\alpha_1^3+\frac{1}{420}\alpha_1^2\alpha_2+$$

$$\frac{1}{840}\alpha_1\left(1+\alpha_2^2\right)-\frac{1}{420}\left(3\alpha_1+\frac{1}{3}\alpha_2\right)+\frac{1}{180}\left(3\alpha_1+\alpha_2\right)-\frac{1}{60}\alpha_2$$

$$\beta_{24}=\left\{-\frac{3}{70}\alpha_1^3+\frac{4}{35}\alpha_1^2\alpha_2-\frac{1}{60}\left(8\alpha_1\alpha_2^2-3\alpha_1\right)+\frac{1}{30}\alpha_2\left(2\alpha_2^2-\frac{3}{2}\right)\right\}$$

$$\beta_{25}=12\tau_2\left(-\frac{1}{280}\alpha_1^3+\frac{1}{105}\alpha_1^2\alpha_2-\frac{1}{180}\alpha_1\alpha_2^2\right)+\frac{1}{15}\alpha_2\alpha_3+\frac{1}{10}\alpha_1\alpha_4-\frac{3}{10}\alpha_2\alpha_4+\frac{1}{24}\alpha_5$$

$$\beta_{26}=\left\{-\frac{1}{3360}\alpha_1+\frac{1}{2520}\left(3\alpha_1+\alpha_2\right)-\frac{1}{360}\alpha_2-\frac{1}{120}A+\tau\left(\frac{1}{120}\alpha_1-\frac{1}{60}\alpha_2\right)\right\}$$

$$\beta_{27}=\begin{bmatrix}\frac{9}{140}\alpha_1^3-\frac{6}{35}\alpha_1^2\alpha_2+\frac{1}{360}\alpha_7-\frac{1}{60}\alpha_2\left(4\alpha_2^2+1\right)\\+6\alpha_1\left\{\frac{1}{280}\alpha_1^2-\frac{1}{105}\alpha_1\alpha_2+\frac{1}{180}\alpha_2^2-A\left(\frac{1}{20}\alpha_1-\frac{1}{12}\alpha_2\right)\right\}\end{bmatrix}$$

$$\beta_{28}=\left[\frac{9}{140}\alpha_1^3-\frac{6}{35}\alpha_1^2\alpha_2+\frac{1}{360}\alpha_7-\frac{1}{60}\alpha_2\left(4\alpha_2^2+1\right)\right] \tag{31}$$

$$\phi=\phi_0+R\phi_1+R^2\phi_2\,;\ \psi=\psi_0+R\psi_1+R^2\psi_2$$

On substituting the values of

In the expression (13) and (16), we get the temperature and dimensionless

temperature respectively.

The amount of heat transfer from the lower and upper discs are

$$Q_a = \left\{ \frac{1}{\pi\left(\xi^2 - \xi_0^{\ 2}\right)} \right\} \int_{\xi_0}^{\xi} 2\pi\xi q_a d\xi \qquad (32)$$

and

$$Q_b = \left\{ \frac{1}{\pi\left(\xi^2 - \xi_0^{\ 2}\right)} \right\} \int_{\xi_0}^{\xi} 2\pi\xi q_b d\xi \qquad (33)$$

respectively.
Where

$q_a \left\{ = \left(-k / z_0\right)\left(\partial T / \partial \zeta\right)_{\zeta=0} \right\}$ and $q_b \left\{ = \left(-k / z_0\right)\left(\partial T / \partial \zeta\right)_{\zeta=1} \right\}$

are the heat fluxes on the lower and upper discs.

The average Nusselt's numbers on the lower and upper disc are:

$$Nu_a = \frac{Q_a z_0}{\left(k\left(T_b - T_a\right)\right)} \qquad (34)$$

and $Nu_b = \dfrac{Q_b z_0}{\left(k\left(T_b - T_a\right)\right)}$

(35)

Table 1: Variation of temperature T^* with ζ for different values of suction parameter A

ζ	A=0	A=1	A=2
0	0	0	0
0.1	0.119983	0.425786	1.794906
0.2	0.236096	0.755284	3.11079
0.3	0.348655	0.992976	3.954672
0.4	0.457626	1.14003	4.325218
0.5	0.562866	1.201042	4.239888
0.6	0.664131	1.188791	3.756878
0.7	0.76083	1.1266	2.989998
0.8	0.851529	1.047908	2.114474
0.9	0.933166	0.992645	1.361576
1	1	1	1

Table 2: Variation of temperature T^* with ζ for different values of second order parameter τ_2

ζ	τ_2 =0	τ_2 =2	τ_2 =4
0	0	0	0
0.1	1.702449	1.794906	1.887362
0.2	2.949747	3.11079	3.271834
0.3	3.745837	3.954672	4.163507
0.4	4.08955	4.325218	4.560886
0.5	3.999801	4.239888	4.479976
0.6	3.535595	3.756878	3.978161
0.7	2.809324	2.989998	3.170672
0.8	1.991377	2.114474	2.237571
0.9	1.303743	1.361576	1.419409
1	1	1	1

Table 3: Variation of temperature T^* with ζ for different values of suction ratio parameter N

ζ	N=0	N=2	N=4
0	0	0	0
0.1	0.907651	1.794906	19.80954
0.2	1.545662	3.11079	35.33687
0.3	1.938881	3.954672	46.21132
0.4	2.115809	4.325218	51.87082
0.5	2.111399	4.239888	51.92111
0.6	1.967719	3.756878	46.46525
0.7	1.732903	2.989998	36.37069
0.8	1.458811	2.114474	23.43992
0.9	1.19792	1.361576	10.44673
1	1	1	1

Table 4: Variation of temperature T^* with ζ for different values of force parameter M

ζ	M=0	M=0.5	M=1
0	0	0	0
0.1	1.501464	1.641438	1.794906
0.2	2.626762	2.857526	3.11079
0.3	3.378629	3.653156	3.954672
0.4	3.753037	4.025673	4.325218
0.5	3.758279	3.987776	4.239888
0.6	3.430861	3.586278	3.756878
0.7	2.847112	2.91529	2.989998
0.8	2.129361	2.122277	2.114474
0.9	1.445407	1.405328	1.361576
1	1	1	1

Table 5: Variation of Nusselt's number Nu_a with ξ for different values of suction parameter A

ξ	A=0	A=1	A=2
0	-1.07497	-3.61748	-16.8853
1	-2.25088	-12.9074	-45.3909
2	-5.7786	-40.777	-130.908
3	-11.6581	-87.2264	-273.436
4	-19.8895	-152.256	-472.976
5	-30.4727	-235.864	-729.526
6	-43.4077	-338.053	-1043.09
7	-58.6945	-458.822	-1413.66
8	-76.3331	-598.17	-1841.25
9	-96.3236	-756.098	-2325.84
10	-118.666	-932.605	-2867.45

Table 6: Variation of Nusselt's number Nu_a with ξ for different values of second order parameter τ_2

ξ	τ_2 =0	τ_2 =2	τ_2 =4

0	-16.3606	-16.8853	-17.41
1	-40.5414	-45.3909	-50.2404
2	-113.084	-130.908	-148.732
3	-233.988	-273.436	-312.884
4	-403.254	-472.976	-542.697
5	-620.882	-729.526	-838.171
6	-886.871	-1043.09	-1199.31
7	-1201.22	-1413.66	-1626.1
8	-1563.93	-1841.25	-2118.56
9	-1975.01	-2325.84	-2676.68
10	-2434.45	-2867.45	-3300.46

Table 7: Variation of Nusselt's number Nu_a with ξ for different values of suction ratio parameter N

ξ	N=0	N=2	N=4
0	-8.9016	-16.8853	-189.482
1	-21.7542	-45.3909	-429.376
2	-60.3121	-130.908	-1149.06
3	-124.575	-273.436	-2348.53
4	-214.544	-472.976	-4027.8
5	-330.217	-729.526	-6186.85
6	-471.596	-1043.09	-8825.69
7	-638.68	-1413.66	-11944.3
8	-831.469	-1841.25	-15542.7
9	-1049.96	-2325.84	-19621
10	-1294.16	-2867.45	-24179

Table 8: Variation of Nusselt's number Nu_a with ξ for different values of force parameter M

ξ	M=0.0	M=0.5	M=1
0	-14.2502	-15.5033	-16.8853
1	-35.9552	-40.4911	-45.3909
2	-101.07	-115.454	-130.908
3	-209.595	-240.393	-273.436
4	-361.531	-415.307	-472.976
5	-556.876	-640.197	-729.526
6	-795.631	-915.062	-1043.09
7	-1077.8	-1239.9	-1413.66
8	-1403.37	-1614.72	-1841.25
9	-1772.36	-2039.51	-2325.84
10	-2184.75	-2514.28	-2867.45

Table 9: Variation of Nusselt's number Nu_b with ξ for different values of suction parameter A

ξ	A=0	A=1	A=2

0	-0.99315	-5.47329	-19.7085
1	1.614213	4.918947	-7.3994
2	9.436294	36.09567	29.52778
3	22.4731	88.05686	91.07307
4	40.72461	160.8025	177.2365
5	64.19086	254.3327	288.018
6	92.87182	368.6473	423.4177
7	126.7675	503.7465	583.4354
8	165.8779	659.6301	768.0714
9	210.203	836.2981	977.3253
10	259.7429	1033.751	1211.198

Table 10: Variation of Nusselt's number Nu_b with ξ for different values of second order parameter τ_2

ξ	τ_2 =0	τ_2 =2	τ_2 =4
0	-3.09747	-19.7085	-36.3195
1	10.96358	-7.3994	-25.7624
2	53.14674	29.52778	5.908791
3	123.452	91.07307	58.69409
4	221.8794	177.2365	132.5935
5	348.4288	288.018	227.6071
6	503.1004	423.4177	343.7347
7	685.8941	583.4354	480.9765
8	896.8099	768.0714	639.3324
9	1135.848	977.3253	818.8024
10	1403.008	1211.198	1019.387

Table 11: Variation of Nusselt's number Nu_b with ξ for different values of suction ratio parameter N

ξ	N=0	N=1	N=2
0	7.952754	-2.28319	-19.7085
1	14.46228	0.247019	-7.3994
2	33.99085	7.837633	29.52778
3	66.53845	20.48866	91.07307
4	112.1051	38.20009	177.2365
5	170.6908	60.97193	288.018
6	242.2956	88.80418	423.4177
7	326.9193	121.6968	583.4354
8	424.5622	159.6499	768.0714
9	535.2241	202.6634	977.3253
10	658.905	250.7373	1211.198

Table 12: Variation of Nusselt's number Nu_b with ξ for different values of force parameter M

ξ	M=0.0	M=0.5	M=1

0	-10.2067	-14.6344	-19.7085
1	-0.51743	-3.32785	-7.3994
2	28.55029	30.59174	29.52778
3	76.99648	87.12439	91.07307
4	144.8212	166.2701	177.2365
5	232.0243	268.0289	288.018
6	338.6059	392.4007	423.4177
7	464.566	539.3856	583.4354
8	609.9046	708.9835	768.0714
9	774.6216	901.1946	977.3253
10	958.7172	1116.019	1211.198

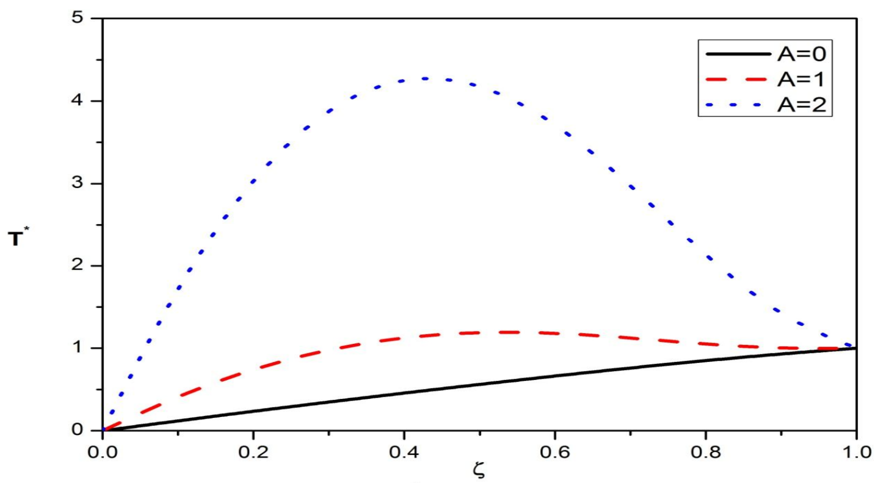

Fig. 1 Variation of temperature T^* with ζ for different values of suction parameter **A**

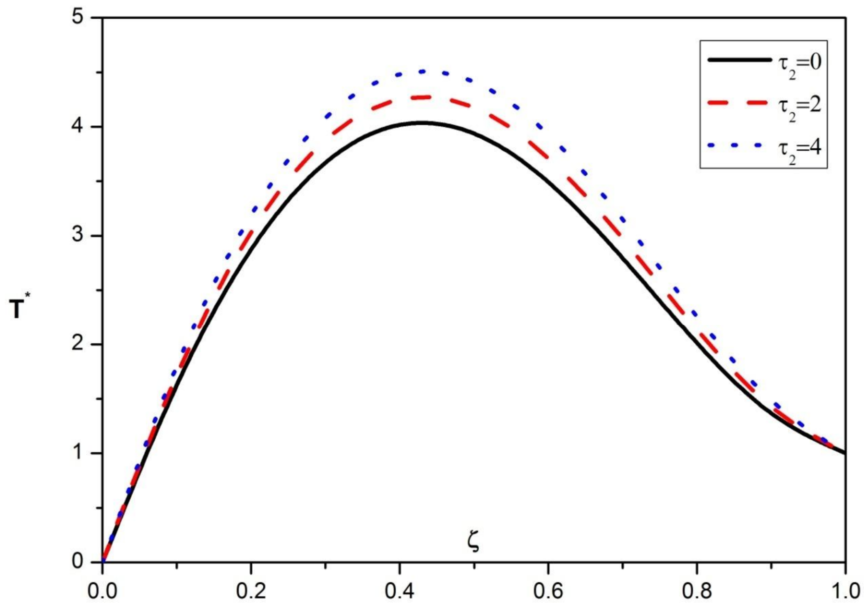

Fig. 2: Variation of temperature $\mathbf{T}^*$ with ζ for different values of second order parameter τ_2

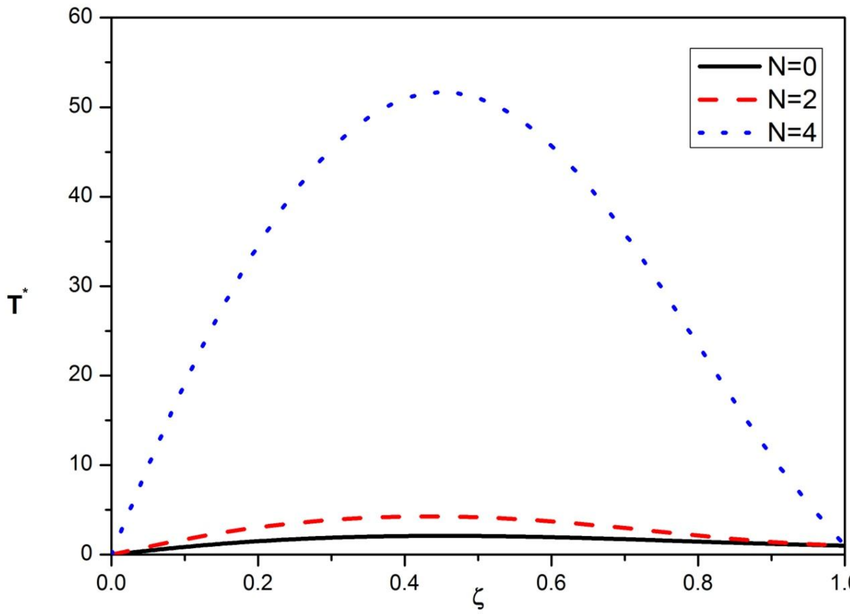

Fig. 3 Variation of temperature **T*** with ζ for different values of suction ratio parameter N

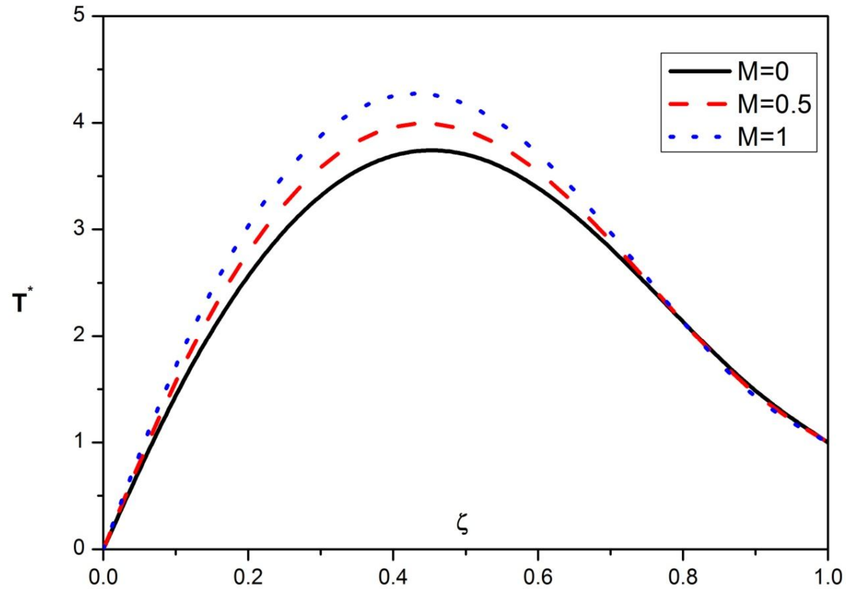

Fig. 4 Variation of temperature T^* with ζ for different values of force parameter **M**

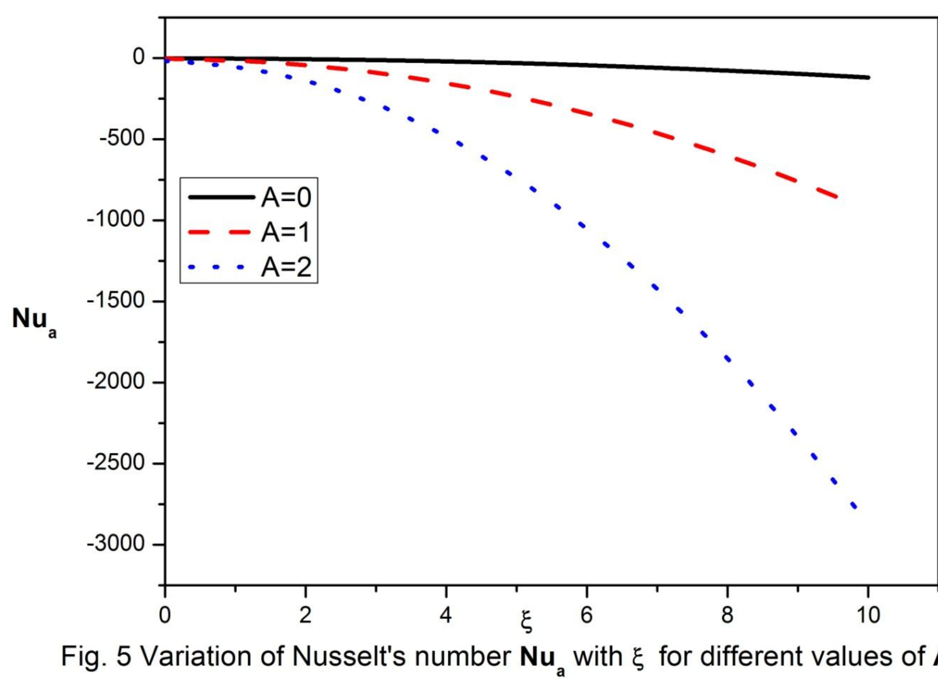

Fig. 5 Variation of Nusselt's number $\mathbf{Nu_a}$ with ξ for different values of **A**

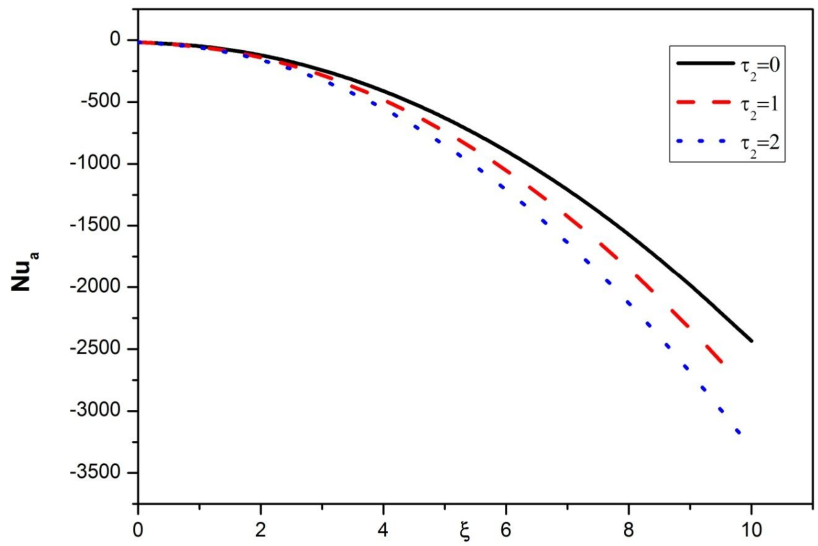

Fig. 6 Variation of Nusselt's number $\mathbf{Nu_a}$ with ξ for different values of τ_2

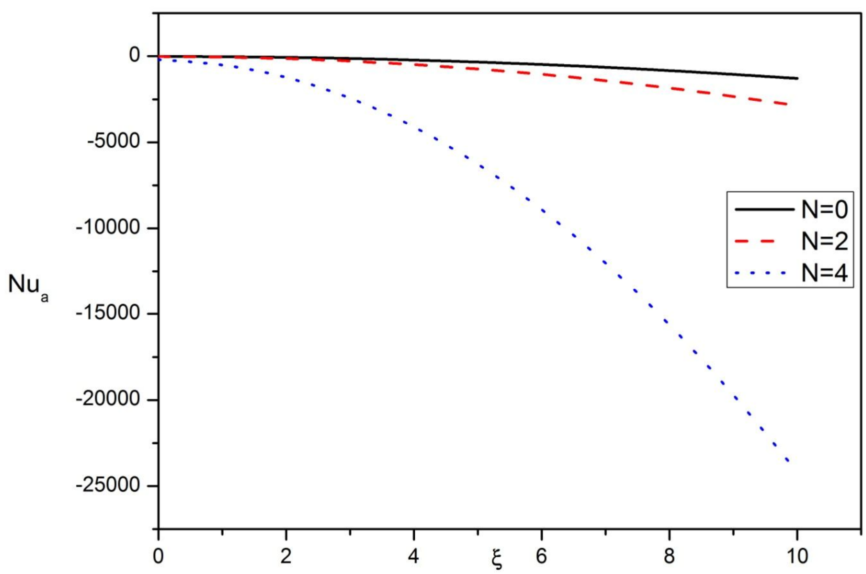

Fig. 7 Variation of Nusselt's number $\mathbf{Nu_a}$ with ξ for different values of **N**

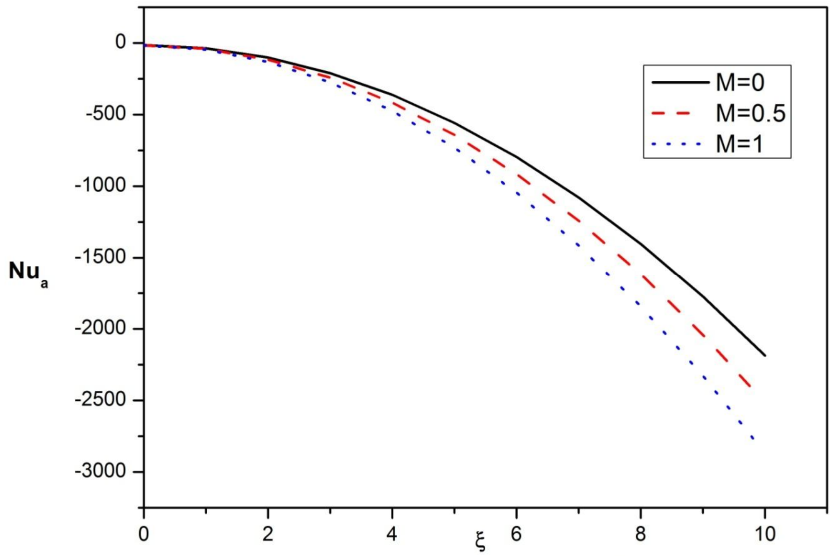

Fig. 8 Variation of Nusselt's number $\mathbf{Nu_a}$ with ξ for different values of **M**

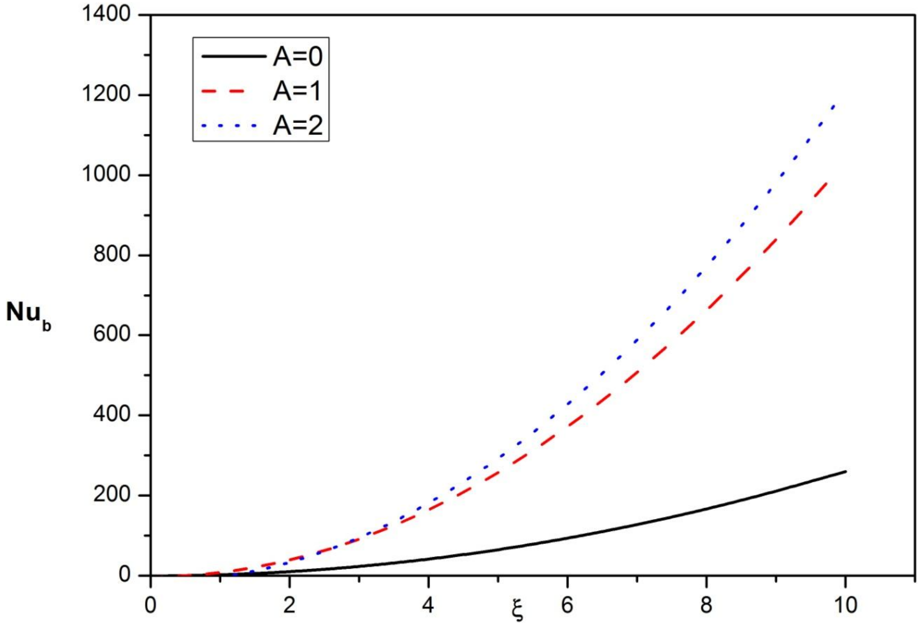

Fig. 9 Variation of Nusselt's number $\mathbf{Nu_b}$ with ξ for different values of **A**

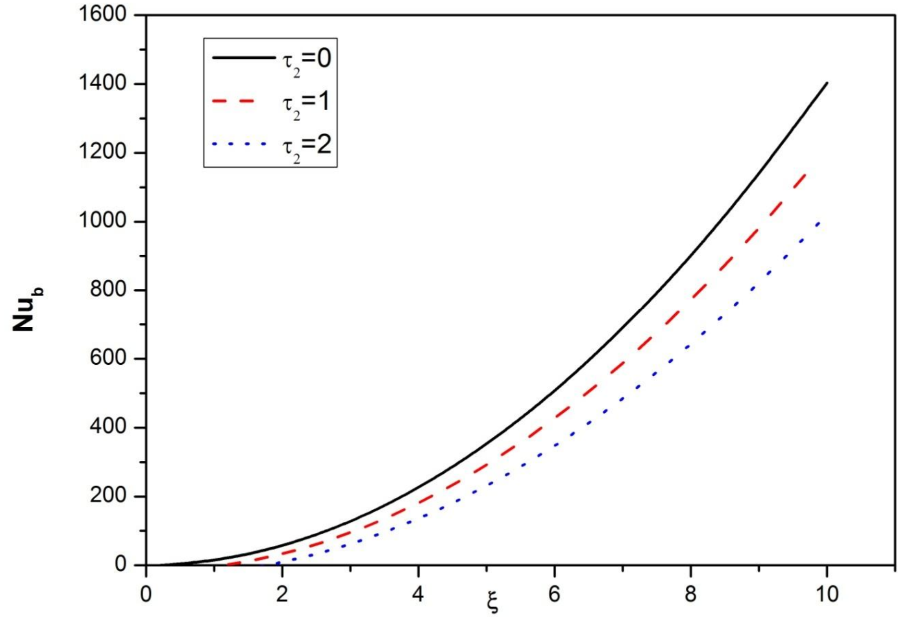

Fig. 10 Variation of Nusselt's number $\mathbf{Nu_b}$ with ξ for different values of τ_2

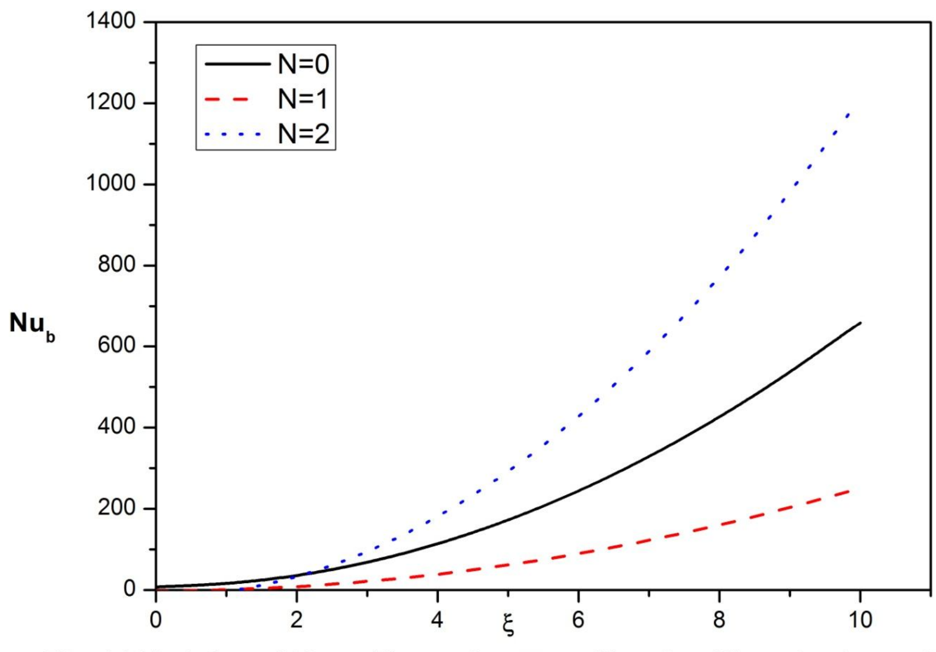

Fig. 11 Variation of Nusselt's number $\mathbf{Nu_b}$ with ξ for different values of **N**

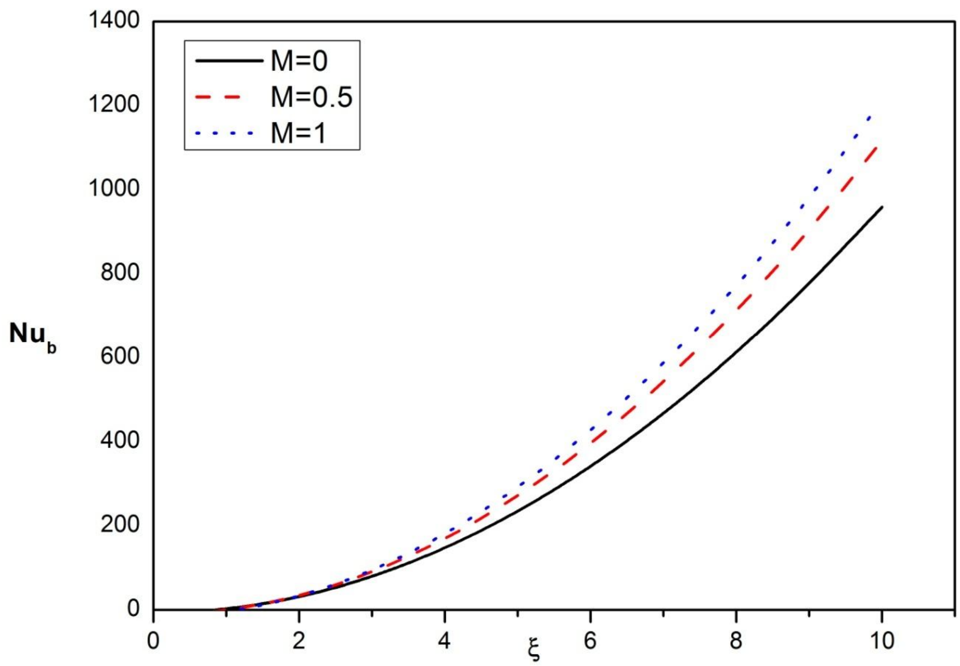

Fig. 12 Variation of Nusselt's number $\mathbf{Nu_b}$ with ξ for different values of **M**

IV. RESULT AND DISCUSSION

The variation of the dimensionless temperature T^* with ζ at $R=0.02,\ \tau_2=2, E=1, P_r=5,\ \xi=1,\ M=1, N=2$ for different values of the parameter $A=0,1,2$ is shown in the fig.1. It is evident from this figure the temperature is minimum at the lower disc and maximum in the middle of gap length approximately. It is also clear from the figure that the temperature increases with an increase in A throughout the gap length. At $R=0.02, E=1, P_r=5,\ \xi=1, M=1, N=2, A=2$ the different branches of fig.2 for respective value of $\tau_2=0,\ 2,\ 4$ represents the behaviour of the temperature with ζ . It is observed that the temperature increases with an increase in τ_2 through the gap length. The temperature is minimum at the lower disc and maximum in the neighbourhood of $\zeta=0.4$. Fig.3 exhibits the behaviour of the dimensionless temperature T^* at $R=0.02, E=1, P_r=5,\ \xi=1,\ \tau_2=2, M=1, A=2$ for different values of $N=0,\ 2,\ 4$.

The variation of the temperature with ζ for $N=0,2,4$ (fig.3) and for $A=0,1,2$ (fig.1) is similar in the whole gap length. The variation of the temperature with ζ at $R=0.02,\ E=1,\ P_r=5,\ \xi=1,\ \tau_2=2,\ N=2,\ A=2$ for different values of $M=0,0.5,1$ is represents through fig.4. The temperature increases with an increase in M in the gap length region $0\le\zeta\le0.7$ with it's reverse behaviour in the region $0.8\le\zeta\le1$.

The behaviour of Nu_a (the average Nusselt's number at the lower disc) with ξ at $R=0.02,\ E=1,\ P_r=5,\ \xi_0=4,\ \tau_2=2,\ N=2,\ M=1$ for different values of $A=0,1,2$ is represented through fig.5. It is observed from this figure that Nu_a decreases with an increase in A throughout the radial region.

Rate of decrement of Nu_a is faster than the increment in A. It is also evident from this figure that heat is flowing from the fluid to the lower disc. The variation of Nu_a with ξ at $R=0.02,\ E=1, P_r=5,\ \xi_0=4$ for $\tau_2=0,2,4(N=2,A=2,M=1)$ for $N=0,2,4(\tau_2=2,A=2,M=1)$ and for $M=0,0.5,1(\tau_2=2,A=2,N=2)$ is represented through fig. 6, 7 and 8 respectively. It is evident from these figures that Nu_a decreases with an increase in τ_2, N and M throughout the radial region. It is also observed that $Nu_a \le 0$ for $0 \le \xi \le 1$. The behaviour of Nu_a with ξ shows that heat flux is flowing from upper disc to fluid and fluid to the lower disc.

The behaviour of Nu_b (the average Nusselt's number at the upper disc) with ξ at $R=0.02, E=1, P_r=5,\ \xi_0=4$ for $A=0,\ 1,\ 2\,(\tau_2=2, N=2, M=1)$ and for $M=0,0.5,1(\tau_2=2,A=2,N=2)$ shown through fig. 9 and fig. 12 respectively. It is evident from these figures that Nu_b increases with an increase in A and M both in the whole of the radial region approximately. It is also seen from these figures that the heat flux is flowing from the fluid to the upper disc in the entire radial region approximately.

Fig 10 exhibits the behaviour of Nu_b with ξ at $R=0.02, E=1, P_r=5,\ \xi_0=4, A=2, N=2, M=1$ for different values of $\tau_2=0,\ 2,\ 4$. It is clear from this figure that Nu_b decreases with an increase in τ_2 in the entire radial region and the heat flux is flowing from fluid to the upper disc. The behaviour of Nu_b with ξ at $R=0.02, E=1, P_r=5, \xi_0=4, A=2, \tau_2=2, M=1$ for different values of $N=0,\ 1,\ 2$ is shown in fig. 11. It is evident from this figure that Nu_b increases with an increase in ξ for all values of N. It is also observed from this figure that the heat flux is flowing from fluid to the upper disc in the entire radial region.

V. CONCLUSION

From the graph of the figures $1,2,3,4$ we conclude that the temperature is minimum at the lower disc and maximum in the middle of the gap length approximately. The temperature is increasing with an increase in the values of all the parameters A, N, τ_2 and M. It is observed from the graph of the figures $5,6,7$ and 8 that the heat flux is flowing from upper disc to the fluid and fluid to the lower disc. Figures $9,10,11$ and 12 shows that the heat flux is flowing from the lower disc to the fluid and fluid to the upper disc in the entire radial region $0 \le \xi \le 10$

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