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Estimating coefficient bounds with respect to a generalized starlike functions on symmetric points

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Abstract - The purpose of the present paper is to estimate the Certain Coefficient for generalized Starlike functions with respect to symmetric points defined on the open unit disk for which $R^{k}_{\lambda,\delta}(\phi)$ of normalized analytic functions $f(z)$ lies in a region with respect to 1 and symmetric with respect to the real axis.
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1. INTRODUCTION

Let $\mathcal{A}$ denote the class of all analytic function $f(z)$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (1.1)$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and satisfy the condition $f(0) = 0,\ f'(0) = 1.$ We also denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of all functions which are univalent in $\mathbb{U}$. For functions $f(z)$ and $g(z)$ analytic in $\mathbb{U}$, we say that the functions $f(z)$ is said to subordinate to $g(z)$ if there exist a schwarz function $\omega(z)$, analytic in $\mathbb{U}$ with

$$\omega(0) = 0 \quad \text{and} \quad |\omega(z)| < 1 \quad (z \in \mathbb{U}),$$

such that

$$f(z) = g(\omega(z)) \quad (z \in \mathbb{U}).$$

We denote this subordination by

$$f \prec g \quad \text{or} \quad f(z) \prec g(z) \quad (z \in \mathbb{U}).$$

In particular, if the function $g(z)$ is univalent in $\mathbb{U}$, the above subordination is equivalent to

$$f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

Let $\phi(z)$ be an analytic function in $\mathbb{U}$ with $\phi(0) = 1$, $\phi'(0) > 0$ and $Re\{\phi(z)\} > 0$, $z \in \mathbb{U}$ which map the open unit disk $\mathbb{U}$ onto a region starlike with respect to 1 and is symmetric with respect to the real axis. Then by $S^*(\phi)$ and $C(\phi)$, respectively, we denote the subclasses of the normalized analytic function class $\mathcal{A}$, which satisfy the following subordination relations:

$$\frac{zf'(z)}{f(z)} \prec \phi(z),\ z \in \mathbb{U}$$

and

$$1+\frac{zf''(z)}{f'(z)}\prec\phi(z),\ z\in\mathbb{U}.$$

These function were introduced and studied by Ma and Minda [9]. In particular case, when

$$\phi(z)=\frac{1+(1-2\alpha)z}{1-z},\ z\in\mathbb{U},0\le\alpha<1,$$

these function reduce respectively to the well-known classes $S^*(\alpha),(0\le\alpha<1)$ of starlike functions of order α in $\mathbb{U}$ and $C(\alpha),(0\le\alpha<1)$ of convex functions of order α in $\mathbb{U}$. Ma and Minda [9], the Fekete-Szegö inequality for the functions in the class $C(\phi)$ was derived and in view of the Alexander result relating the function classes $S^*(\phi)$ and $C(\phi)$.

For a brief history of the Fekete-Szegö problems for the starlike, convex and other various subclasses of the normalized analytic function in $\mathcal{A}$, we refer the reader to the work done by Srivatsava et al [20] and Ramachandran et al [14]. of course the main result shall refer back to Fekete and szegö [2] in the year 1933. After 30 years or so, Keogh and Merkes [4] solved the problem for certain subclasses of univalent functions.

Koepf [6,7], gave excellent results for the class of close-to-convex functions. These articles [2,4,6,7] gave valuable results which have to solve problems for other extended classes.

Recently Shanmugam et al [17] have studied the Fekete-Szegö problem for subclasses of starlike functions with respect to symmetric points.

Motivated essentially by the aforementioned works, we prove the Fekete-Szegö inequality in Theorem 2.1 below for a more general class of normalized analytic functions.

For $\lambda,\delta\in N$, $k\in N_0$ the authors Darus [1] introduced the operator $D^k_{\lambda,\delta}$ defined by

$$D_{\lambda,\delta^k}f(z)=z+\sum_{n=2}^{\infty}[1+(n-1)\lambda]^kC(\delta,n)a_nz^n \tag{1.2}$$

In the present paper, we obtain the Fekete-szegö inequality for the function $f\in\mathcal{A}$ in the class $R^k_{\lambda,\delta}(\phi)$ defined as follows:

Definition: 1.1 Let $D^k_{\lambda,\delta}:\mathcal{A}\to\mathcal{A}$ is a linear operator and $D^k_{\lambda,\delta}$ is analytic in $f(\mathbb{U})$. Let

$$D^k_{\lambda,\delta}f(z)==z+\sum_{n=2}^{\infty}[1+(n-1)\lambda]^kC(\delta,n)a_nz^n$$

where

$$C(\delta,n)=\frac{\Gamma(n+\delta)}{\Gamma(n)\Gamma(\delta+1)}$$

when $\lambda=1,\ \delta=0$ we get the sălăgean differential operator, $k=0$ or $\lambda=0$ gives Ruscheweyh operator, $\delta=0$ gives Al-oboudi differential operator of order k

$$D^0_{1,0}f(z)=f(z),\ D^1_{1,0}f(z)=zf'(z)$$

Definition: 1.2 Let $\phi(z)$ be a univalent starlike function with respect to 1 which map the unit disk $\mathbb{U}$ onto a region in the right half plane which is symmetric with respect to the real axis $\phi(0)=1$ and $\phi'(0)>1$. A function $f\in\mathcal{A}$ is in the class $R^k_{\lambda,\delta}(\phi)$ if

$$\frac{(s-t)z\left[D^k_{\lambda,\delta}f(z)\right]'}{D^k_{\lambda,\delta}[f(sz)]-D^k_{\lambda,\delta}[f(tz)]}\prec\phi(z),\quad(\lambda,\delta\in\mathbb{N},k\in\mathbb{N}_0).$$

In order to prove our main results, we need the following lemma.

Lemma: 1.3 [9] If $p_1(z)=1+c_1z+c_2z^2+...$ is an analytic function with positive real part in $\mathbb{U}$, then

$$c_2-vc_1^2|\le\begin{cases}-4v+2, & if\quad v\le0\\ 2, & if\quad 0\le v\le1\\ 4v-2, & if\quad v\ge1.\end{cases}$$

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when $v<0$ or $v>1$, the equality holds true if and only if $p_1(z)=\frac{1+z}{1-z}$ or one of its rotations. If $0<v<1$, then the equality holds true if and only if $p_1(z)=\frac{1+z^2}{1-z^2}$ or one of its rotation. If $v=0$, then the equality holds true if and only if

$$p_1(z)=\left(\frac{1}{2}+\frac{1}{2}\gamma\right)\frac{1+z}{1-z}+\left(\frac{1}{2}-\frac{1}{2}\gamma\right)\frac{1-z}{1+z}\quad (0\le\gamma\le 1),$$

or one of its rotations. If $v=1$, the equality holds if and only if p_1 is the reciprocal of one of the functions such that the equality holds in the case of $v=0$. Also the above upper bound is sharp, it can be improved as follows when $0<v<1$:

$$|c_2-vc_1^2|+v|c_1^2|\le 2,\ 0<v\le\frac{1}{2}$$

and

$$|c_2-vc_1^2|+(1-v)|c_1^2|\le 2,\ \frac{1}{2}<v\le 1$$

We also need the following result in our investigation.

Lemma: 1.4 [15] If $p_1(z)=1+c_1z+c_2z^2+...$ is a function with positive real part in $\mathbb{U}$, then

$$|c_2-vc_1^2|\le 2.max\left(1, |2v-1|\right).$$

The result is sharp for the functions $p_1(z)$ given by

$$p_1(z)=\frac{1+z^2}{1-z^2}$$

and

$$p_1(z)=\frac{1+z}{1-z}.$$

2. FEKETE-SZEGö PROBLEM FOR THE FUNCTION OF THE CLASS $R_{\lambda,\delta}^k(\phi)$.

By making use of Lemma 1.4, we prove the Fekete-szegö Problem for the class $R_{\lambda,\delta}^k(\phi)$.

*Theorem: 2.1***.** Let $\phi(z)=1+B_1z+B_2z^2+\cdots$. If $f(z)$ given by (1.1) belongs to the class $R_{\lambda,\delta}^k(\phi)$, then

$$|a_3-\mu a_2^2|\le\begin{cases}\Lambda, & \mu\le\sigma_1\\ \eta, & \sigma_1\le\mu\le\sigma_2\\ -\Lambda, & \mu\ge\sigma_2.\end{cases}$$

where

$$\sigma_1=\frac{(\delta+1)(s+t-2)(1+\lambda)^{2k}}{2(\delta+2)(1+2\lambda)^k[(s^2+st+t^2)-3]}\left[\frac{2(B_2-B_1)(s+t-2)-B_1^2(s+t)}{B_1^2}\right],$$

$$\sigma_2=\frac{(\delta+1)(s+t-2)(1+\lambda)^{2k}}{2(\delta+2)(1+2\lambda)^k[(s^2+st+t^2)-3]}\left[\frac{2B_2(s+t-2)-(s+t)B_1^2}{B_1^2}\right],$$

$$\sigma_3=\frac{(\delta+1)(s+t-2)(1+\lambda)^{2k}}{2(\delta+2)(1+2\lambda)^k[(s^2+st+t^2)-3]}\left[\frac{(2B_2-B_1)(s+t-2)-B_1^2(s+t)}{B_1^2}\right],$$

$$\Lambda=\frac{4(\delta+1)(\delta+2)(1+2\lambda)^k[3-(s^2+st+t^2)]}{B_1^2}$$
$$\times\left[(B_2-B_1^2)-\frac{B_1^2}{2}\left(\frac{s+t}{s+t-2}+\frac{2\mu(1+2\lambda)^k(\delta+2)[(s^2+st+t^2)-3]}{(\delta+1)(1+\lambda)^{2k}(2-s-t)^2}\right)\right],$$

$$\eta=\frac{2B_1}{(1+2\lambda)^k(\delta+1)(\delta+2)(3-[s^2+st+t^2])}.$$

Further, If $\sigma_1\le\mu\le\sigma_3$, then

$$|a_3-\mu a_2^2|+\frac{(\delta+1)(s+t-2)(1+\lambda)^{2k}}{(\delta+2)(1+2\lambda)^k[(s^2+st+t^2)-3]B_1^2}\left\{(B_1-B_2)(s+t-2)+\sigma_4B_1^2\right\}|a_2|^2\le\eta.$$

If $\sigma_3 \le \mu \le \sigma_2$, then

$$|a_3 - \mu a_2^2| + \frac{(\delta+1)(s+t-2)(1+\lambda)^{2k}}{(\delta+2)(1+2\lambda)^k[(s^2+st+t^2)-3]B_1^2}\left\{B_2(s+t-2)-\sigma_4 B_1^2\right\}|a_2|^2 \le \eta.$$

Where

$$\sigma_4 = \left[\frac{(s+t)(s+t-2)(\delta+1)(1+\lambda)^{2k} + \mu(\delta+2)(1+2\lambda)^k[(s^2+st+t^2)-3]}{(\delta+1)(s+t-2)(1+\lambda)^{2k}}\right].$$

The result is sharp.

Proof: If $f \in R^k_{\lambda,\delta}(\phi)$, then there exists a Schwarz function $w(z)$, analytic in $\mathbb{U}$ with $w(0)=0$ and $|w(z)|<1$ in $\mathbb{U}$ such that

$$\frac{(s-t)z\left[D^k_{\lambda,\delta} f(z)\right]'}{D^k_{\lambda,\delta}[f(sz)] - D^k_{\lambda,\delta}[f(tz)]} = \phi(w(z))$$

Define a function $p_1(z)$ by

$$p_1(z) = \frac{1+w(z)}{1-w(z)}.$$

since $w(z)$ is a Schwarz function, we see that $Re\{p_1(z)\} > 0$ and $p_1(0)=0$. Define a function $p(z)$ by

$$p(z) = \frac{(s-t)z\left[D^k_{\lambda,\delta} f(z)\right]'}{D^k_{\lambda,\delta}[f(sz)] - D^k_{\lambda,\delta}[f(tz)]} = \phi(w(z)) = 1 + b_1 z + b_2 z^2 + \cdots \quad (2.1)$$

From (2.1), we obtain

$$a_2 = \frac{b_1}{(1+\lambda)^k(\delta+1)(2-s-t)} \quad (2.2)$$

and

$$a_3 = 2\frac{b_2 - (s+t)(s+t-2)(1+\lambda)^{2k}(\delta+1)^2 a_2^2}{[3-(s^2+st+t^2)](1+2\lambda)^k(\delta+1)(\delta+2)}$$

since

$$p_1(z) = \frac{1+\phi^{-1}(p(z))}{1-\phi^{-1}(p(z))}$$

then

$$p(z) = \phi\left[\frac{p_1(z)-1}{p_1(z)+1}\right],$$

and

$$\begin{aligned} 1 + b_1 z + b_2 z^2 + \cdots &= \phi\left(\frac{c_1 z + c_2 z^2 + \cdots}{2 + c_1 z + c_2 z^2 + \cdots}\right) \\ &= \phi\left[\frac{1}{2}c_1 z + \frac{1}{2}(c_2 - \frac{1}{2}c_1^2)z^2 + \cdots\right] \end{aligned} \quad (2.3)$$

Equating the coefficients of z and z^2 , we obtain

$$b_1 = \frac{1}{2}B_1 c_1 \quad \textit{and} \quad b_2 = \frac{1}{2}B_1(c_2 - \frac{1}{2}c_1^2) + \frac{1}{4}B_2 c_1^2 \quad (2.4)$$

From (2.2) and (2.4), we get

$$a_2 = \frac{B_1 c_1}{2(1+\lambda)^k(\delta+1)(2-s-t)}$$

and

$$a_3 = \frac{B_1}{(1+2\lambda)^k(\delta+1)(\delta+2)[3-(s^2+st+t^2)]}\left\{c_2 - c_1^2\left(1 - \frac{1}{2}\frac{B_2}{B_1} + \frac{1}{2}B_1\left(\frac{s+t}{s+t-2}\right)\right)\right\}.$$

Therefore we have

$$|a_3-\mu a_2^2| = \frac{B_1}{[3-(s^2+st+t^2)](1+2\lambda)^k(\delta+1)(\delta+2)}\left\{c_2 - c_1^2\left(1-\frac{1}{2}\frac{B_2}{B_1}+\frac{1}{2}B_1(\frac{s+t}{s+t-2})\right)\right\} - \frac{\mu B_1^2 c_1^2}{(\delta+1)^2(1+\lambda)^{2k}(2-s-t)}$$

$$= \frac{B_1}{[3-(s^2+st+t^2)](1+2\lambda)^k(\delta+1)(\delta+2)}\{c_2 - vc_1^2\}$$

where

$$v = \left[1-\frac{1}{2}\frac{B_2}{B_1}+\frac{B_1}{2}(\frac{s+t}{s+t-2}) - \frac{\mu B_1(\delta+2)(1+2\lambda)^k[3-(s^2+st+t^2)]}{(\delta+1)(1+\lambda)^{2k}(2-s-t)^2}\right]$$

If $\mu \le \sigma_1$, then by Lemma 1.3 and Lemma 1.4, we obtain

$$|a_3-\mu a_2^2| \le \frac{4(\delta+1)(\delta+2)(1+2\lambda)^k[3-(s^2+st+t^2)]}{B_1^2}\left[(B_2-B_1)-\frac{B_1^2}{2}\left(\frac{s+t}{s+t-2}+\frac{2\mu(\delta+2)(1+2\lambda)^k[(s^2+st+t^2)-3]}{(\delta+1)(1+\lambda)^{2k}(2-s-t)^2}\right)\right]$$

which is the first part of Theorem 2.1.

Similarly, if $\mu \ge \sigma_2$, then by Lemma 1.3 and Lemma 1.4, we obtain

$$|a_3-\mu a_2^2| \le \frac{4(\delta+1)(\delta+2)(1+2\lambda)^k[3-(s^2+st+t^2)]}{B_1^2}\left[(B_1-B_2)+\frac{B_1^2}{2}\left(\frac{s+t}{s+t-2}+\frac{2\mu(\delta+2)(1+2\lambda)^k[(s^2+st+t^2)-3]}{(\delta+1)(1+\lambda)^{2k}(2-s-t)^2}\right)\right]$$

If $\sigma_1 \le \mu \le \sigma_2$, we see that

$$|a_3-\mu a_2^2| = \frac{2B_1}{2(1+2\lambda)^k(\delta+1)(\delta+2)(3-[s^2+st+t^2])}\{c_2 - vc_1^2\}$$
$$\le \frac{2B_1}{\delta+1(\delta+2)(1+2\lambda)^k[3-(s^2+st+t^2)]}$$

Further, If $\sigma_1 \le \mu \le \sigma_3$, then

$$|a_3-\mu a_2^2| + (\mu-\sigma_1)|a_2^2| \le \frac{2B_1}{\delta+1(\delta+2)(1+2\lambda)^k[3-(s^2+st+t^2)]}$$

Finally,we see that If $\sigma_3 \le \mu \le \sigma_2$, then

$$|a_3-\mu a_2^2| + (\sigma_2-\mu)|a_2^2| \le \frac{2B_1}{\delta+1(\delta+2)(1+2\lambda)^k[3-(s^2+st+t^2)]}.$$

To show that the bounds are sharp, we define functions $k_n^\phi (n=2,3,...)$ by

$$\frac{z(D_{\lambda,\delta}^k k_n^\phi(z))'}{D_{\lambda,\delta}^k k_n^\phi(z)} = \phi(z^{n-1}),\ k_n^\phi(0)=0=(k_n^\phi(0))'-1,$$

and the function F_γ and $G_\gamma (0 \le \gamma \le 1)$ by

$$\frac{z(D_{\lambda,\delta}^k F_\gamma(z))'}{D_{\lambda,\delta}^k F_\gamma(z)} = \phi(\frac{z(z+\gamma)}{1+\gamma z}),\ F_\gamma(0)=0=(F_\gamma(0))'-1$$

and

$$\frac{z(D_{\lambda,\delta}^k G_\gamma(z))'}{D_{\lambda,\delta}^k G_\gamma(z)} = \phi(-\frac{z(z+\gamma)}{1+\gamma z}),\ G_\gamma(0)=0=(G_\gamma(0))'-1$$

Clearly the functions k_n^ϕ, F_γ and $G_\gamma \in R_{\lambda,\delta}^k(\phi)$. We also write $K^\phi = K_2^\phi$.

If $\mu < \sigma_1$ or $\mu > \sigma_2$, then the equality in Theorem 2.1 holds true if and only if f is K^ϕ or one of its rotations. When $\sigma_1 < \mu < \sigma_2$, then the equality holds true if and only if f is K_3^ϕ or one of its rotations. If $\mu = \sigma_1$, then the equality holds true if and only if f is F_γ or one of its rotations. If $\mu = \sigma_2$, then the equality holds true if and only if f is G_γ or one of its rotations.

By making use of Lemma 1.4, we can easily obtain the following theorem.

Theorem: 2.2. Let $\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots$, where the coefficients B_n are real with $B_1 > 0$ and $B_2 \geq 0$. If $f(z)$ given by (1.1) belongs to $R^k_{\lambda,\delta}(\phi)$, then

$$a_3 - \mu a_2^2| \leq \left(\frac{4B_1}{(1+2\lambda)^k (\delta+1)(\delta+2)[3-(s^2+st+t^2)]} \right) \times \max\left\{1, \left|1 - \frac{2B_2}{B_1} + B_1 \left(\frac{s+t}{s+t-2} - \frac{2\mu(\delta+2)(1+2\lambda)^k [3-(s^2+st+t^2)]}{(\delta+1)(2-s-t)^2 (1+\lambda)^{2k}} \right)\right|\right\}, \quad \mu \in \mathbb{C}$$

The result is Sharp.

Remark: 2.3. The coefficient bounds for $a_2|$ and $a_3|$ are special cases of those asserted by Theorem 2.1.

*Remark: 2.4***.** In its special case when $\lambda = 1$, $\delta = 0$ and $k = 0$, we arrive at a known result due to Ma and Minda [9]

*Remark: 2.5***.** In its special case when $\lambda = 1$, $\delta = 0$ and $k = 0$, $s = 1$, $t = -1$, we arrive at a known result due to T.N Shanmugam et al [17]

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