The Form Design and Nonlinear Behavior of Shell Structures

Kanggeun Park[1] & Jin Kim[2]
[1]I'ST Institute, Seoul, South Korea
[2]Sungkyul University, Anyang, South Korea

Abstract- In the nonlinear structural analysis, the popular way to trace the load-deflection path for the whole deformation history of a structure is to use the nonlinear incremental iterative method with following down loading and snap through phenomenon. The purpose of this study is to analyze the mechanical behavior of shell structures considering the geometric or material non-linearity. The nonlinearities considered in the analysis of general structures are material, geometric, combined non-linearity, and boundary non-linearity for gap and contact problems. The material nonlinear has the elastoplastic behavior, creep, and hyper-elastic material behavior. The geometric non-linearity should be considered large displacements, large rotations and finite strains. The faster iteration method should be used to a powerful and efficient methodology for solving nonlinear structural problem. The research will be investigated the load deflection curve of overall structures considering the geometric, material and combined nonlinearity using NISA software.
Keywords – Nonlinear Structural Analysis, Load-deflection Path, Nonlinear Increment Iterative method, Snap through Phenomenon, Geometric and Material Nonlinearity, Faster Iteration Method

I. INTRODUCTION

Most of structures have nonlinear behavior beyond a particular level of loading. Nonlinear analysis can be classified as geometric nonlinearities, material nonlinearities, combined nonlinearities and boundary nonlinearities. In geometric nonlinearities the changed shape cannot be neglected and its deformed configuration should be considered. Material nonlinearities behave nonlinearly and linear Hooke's law cannot be used. The more complicated material should be used nonlinear elastic model for material like rubber and elastoplastic model for Huber-von Mises for metals and Drucker-Prager model to simulate the behavior of granular soil material. The most boundary nonlinearities are encountered in contact problems. The nonlinear behavior of a structure can be caused by a single structure element or a nonlinear force-deformation relation in whole structures. If a structure includes nonlinear elements such as cables, unilateral supports, and material plasticity, the analysis applying the incremental method are performed.

The overall behavior of many structures can be characterized by a load-deflection or force-displacement response. If the response plot is nonlinear, the structural behavior is nonlinear. The response combines linear, hardening and softening. The linear response until fracture is characteristic of pure crystal, glassy and high strength material. The response with geometric nonlinear stiffening and hardening is typical behavior of cable, cable net, pneumatic structures and inflatable structures which may be called tensile structures. These tensile structures come from geometry adaptation to the applied load. The response with softening is followed by a softening regime that may occur suddenly yield, slip or gradually. The combination response is snap-through response, snap-back response, bifurcation of buckling and bifurcation combined with limit points and snap-back. The snap-through response combines softening with hardening follows the second limit point, the response has a negative stiffness and is therefore unstable. The snap-through response is typical of slightly curved structures such as shallow arch and shells. The snap-back response is an exaggerated snap-through, in which the curve turns back with consequence appearance of turning points. The snap-back curve is exhibited by trussed dome, folded structures and thin shell structures in which moving arch effects occur following the first limit point. Bifurcation point may occur in thin structures that experiences compressive stresses. The response with bifurcation, limit and turning points may occur in many combinations, an example can find thin cylindrical shells under axial compression [1-15].

II. THEORY OF GEOMETRIC AND MATERIAL NONLINEAR ANALYSIS

A. Geometric nonlinear analysis–

The geometric nonlinearity can be accounted for the analysis by updating the global stiffness matrix and the global geometric stiffness matrix on every step based on the deformation position. The equilibrium equations must be written with respect to the deformation geometry. To describe large displacements and large rotations, it is necessary to calculate the equilibrium with respect to the deformed configuration. In incremental method for nonlinear analysis the load vector is divided into n equal increments. A load increment is applied to the structures in the state of equilibrium for the previous increment is achieved. The norm of unbalanced force is specified for each step, allowing for searching the structure-deformation relationship. The load increment is used when dividing a load into smaller segments. The number of load increments influences the number of calculation iterations. The greater the number of increments, the greater the probability for the calculations to reach the point of converge. The displacement increments and unbalance force are sufficiently small in comparison with the tolerance, the iteration process stops if divergence occurs. The number of load increments can be increased, which usually helps the process to converge. The process for increment method is the initial stress method, the arch length method, the modified Newton-Rapson method, the full Newton-Rapson method, Crisfield method, etc.

The geometrical nonlinear procedure is as follows by NISA software. The entire load history for a typical nonlinear analysis may be divided into the step event. An event may have increment load steps. Step size (load increments or time increments) may be determined automatically or user definition. In each increment analysis, it is defines parameters for step (or time increment) size, number of steps, equilibrium checks, tolerances for convergence, iterative procedure and other control parameters. The incremental solution is performed in a step-by-step manner until the full specified loads are applied. In each increment, the above iterative scheme is performed until convergence is achieved or maximum iterations are reached. During each increment, the tangent stiffness matrix may be updated for each iteration, or kept constant in all iterations of the pertinent increment. The modified Newton-Raphson method has the option of updating the tangent stiffness matrix at the first iteration, or every specified number of iterations. Although the use of the modified Newton-Raphson may be more economical in some specific material nonlinearity applications, the utilization of the method in general for material and geometrical nonlinearities is not always successful. Performance of the Newton-Raphson iterative method can be improved significantly by using line search technique. Line search is done to find a scalar multiplier for the iterative displacement. The optimum value for this scalar is found such that it minimizes the residual forces. The search for the optimum value involves recalculation of stresses and residual forces without reforming stiffness or solving the equilibrium equations. Hence, line search is computationally inexpensive and generally it improves the performance of the Newton-Raphson method. Line search is a powerful and efficient methodology for solving nonlinear structural problems. In mathematical terms they are continuation methods capable of solving problems with strong nonlinearities and passing extremes or limit points. Line searches (with very few extra residual calculations) can significantly improve the performance of the modified Newton-Raphson method. A line search algorithm based on secant-type interpolations (or extrapolations) as proposed by Crisfield and implementation described by Patel. To follow the structural response beyond the critical point presents a challenging task to the analyst. The major difficulty is to overcome the singularity of the incremental tangent stiffness matrix when the structure reaches its stability limit. In addition, snap-through and snap-back buckling phenomena pose some of the most difficult problems in nonlinear structural analysis. The arc length automatic stepping can be applied to overcome the problems of stiffness singularity and post-buckling. The general goal of arc length procedure is the control of iteration in the numerical solution of complex nonlinear problems. The main idea of this method is based on the concept of constraining the length of the incremental displacement. The incremental displacement length in each iteration is constrained by the length of the previous iteration. Correspondingly, the load is adjusted in order to satisfy the global equilibrium requirement of the system [1, 2, 3].

Three convergence criteria are displacement, force, and energy criterion. In the displacement criterion, a load step will be assumed converged when the ratio of the maximum absolute iterative displacement to the maximum absolute displacement at the first iteration is less than or equal to a specified tolerance value. The force criterion assumes a load step converged when the ratio of the Euclidean norm of the residual force vector to the Euclidean norm of the incremental force vector is less than the specified tolerance. Finally, in the energy criterion, a load step will be assumed converged when the ratio of the iterative energy to the energy at the first iteration is less than the specified tolerance. Tolerances for each criteria may be specified independently. If all tolerances are specified, a step will be assumed converged when any one criterion is satisfied. If the maximum number of iteration limit is reached, before the satisfaction of any criteria, the step is not converged and the program stops. It should be noted that loose tolerance values may lead to inaccurate results, whereas tight tolerance values may be uneconomical from the computational point of view. In some applications of elastoplastic problems with zero or very small strain hardening modulus, the force criterion may be easily satisfied while the displacement one is in significant error [1, 2, 3].

Stress stiffening effect means that the stiffness characteristics of the structure are dependent on the state of stress in the structure, in addition to the material and geometric properties of the model. Stress stiffening is automatically included in the geometric nonlinear analysis capability. In the geometric nonlinear static analysis, both stress stiffening and large displacement effects should be included. The stress stiffening effect is important for thin flexible structures subjected to loadings causing high tensile or compressive stresses. The combined stiffness of the structure increases (or decreases) as the membrane stress increases (or decreases). This directly affects the capability of the

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structure to carry lateral loads. Stress stiffening effect may be important also in solid structures with high normal stresses [1, 2, 3] .

B. Material nonlinear analysis –

The material nonlinear analysis has developed specialized techniques and material model to simulate these behaviors. The material nonlinear models are the elastoplastic model, the hyper elastic model, viscoelastic model, creep model, super elastic model, etc. The elastoplastic models by Von Mises or Tresca yield theory work well for material with stress-strain curve before reaching the ultimate stress. Most engineering metal are well-characterized by the elastoplastic model. The elastoplastic model by Drucker-Prager theory works for soils and granular materials. In elastic-perfectly plastic material model the maximum stress magnitude cannot exceed the limit of plastic yield stress. The elastic-perfectly material model has lost all ability to return to its original shape after deformation. Hyper elastic model by Mooney-Rivlin and Osden works well for incompressible elastomers such as rubber. Blatz-Ko model works for compressible rubber. Viscoelastic model works for hard rubber or glass. Creep is a time-dependent strain produced under a state of constant stress. Shape memory alloy such as Nitinol present the super elastic effect. This super elastic material undergoes large deformations in loading-unloading cycles without showing permanent deformations [1, 2, 3].

The isotropic elastoplastic material model is defined the stress-strain curve for the elastic, perfectly plastic model, and the elastic, work hardening model. The work hardening model has the option of specifying linear hardening or piecewise linear hardening. The Ramberg-Osgood stress-strain curve is available, which may be more appropriate for Aluminum and other alloy materials. In the anisotropic materials these curves are needed in all the principal material directions (axial and shear). For the modified Hill's theory, all these curves are required for both tension and compression directions. The yield criterion determines the stress level or the stress intensity level at the onset of plastic deformations. Two common yield criteria are Hill's anisotropic yield criterion and Modified Hill's criterion.

The yield criterion determines the stress level or stress intensity at the onset of plastic deformations. Various yield criteria or yield conditions are available to represent various material behaviors The Tresca condition is a maximum shear criteria which is also independent of the hydrostatic stress. Both Mises and Tresca conditions are suitable for modeling material behavior in which yielding is not appreciably affected by hydrostatic pressure, as in most metals and alloys with moderate hydrostatic pressures. The Mohr-Coulomb and the Drucker-Prager conditions are, on the other hand, dependent on the hydrostatic pressure and may be used for modeling concrete, rock, and soil material behaviors[1,2,3] . The hardening rule determines how the yield function changes during plastic deformation. Besides the no hardening assumption (perfect plasticity), three work-hardening models are available in the program. These are the isotropic, kinematic, and mixed hardening. In the perfect plasticity model, it is simply assumed that the yield surface does not change during plastic deformation. The isotropic hardening model assumes that the yield surface grows in size only while its shape is not changing. This assumption is usually applicable to monotonic proportional loadings with no Bauschinger effect. In kinematic hardening the yield surface is assumed to have a constant size and shape, but moves in the stress space. This is more appropriate for materials with pronounced Bauschinger effect, and cyclic loading applications. A combination of the two models (isotropic and kinematic) is available in the mixed hardening model which usually gives results that are closer to experimental data [1, 2, 3].

The hyper elastic or the rubber-like material model may be used in geometric nonlinear analysis with total Lagrangian formulation. A hyper elastic material is an elastic material for which a strain-energy function. The second Piola-Kirchhoff stress tensor and the constitutive matrix will be the first and the second derivatives, respectively, of the strain energy function with respect to the Green-Lagrange strain tensor. The incompressibility condition is expressed in terms of the third strain invariant, and is handled as an average constraint throughout the volume of the element. This method allows a finite compressibility analysis to be performed which may be important in some actual behavior of rubber-like materials. This option is obtained by adjusting the value of Poisson's ratio, to be lower than the near incompressible value of 0.499. Higher Poisson's ratio value is closer to 0.5, Various forms of strain energy functions are available to account for the behavior of various rubber-like materials. These range from the simplest Neo-Hookean form to a generalized Mooney-Rivlin and some exponential forms [1,2,3].

III. ANALYSIS RSULTS OF NONLINEAR BEHAVIOR FOR SHELL AND SPATIAL STRUCTURES

A. Geometric nonlinear behavior of a spherical shell –

A spherical shell is subjected to a concentrated load of the center. The element type is 3-D four nodes shell element. The load is applied gradually by incrementing the displacement (displacement control). The hinged boundary conditions are applied along the edges. The objective is compare load deflection curve. The modulus of elasticity is 2000 MPa. Poisson's ratio values are 0.3. Various The radius of a shell is 2540 mm, the width is 1570 mm. The thickness is 10 and 12 mm. The tolerance is 0.001. Number of displacement increments is 30 equal increments. Maximum number of iterations per increment allowed 100. Full Newton-Raphson is employed in all runs.

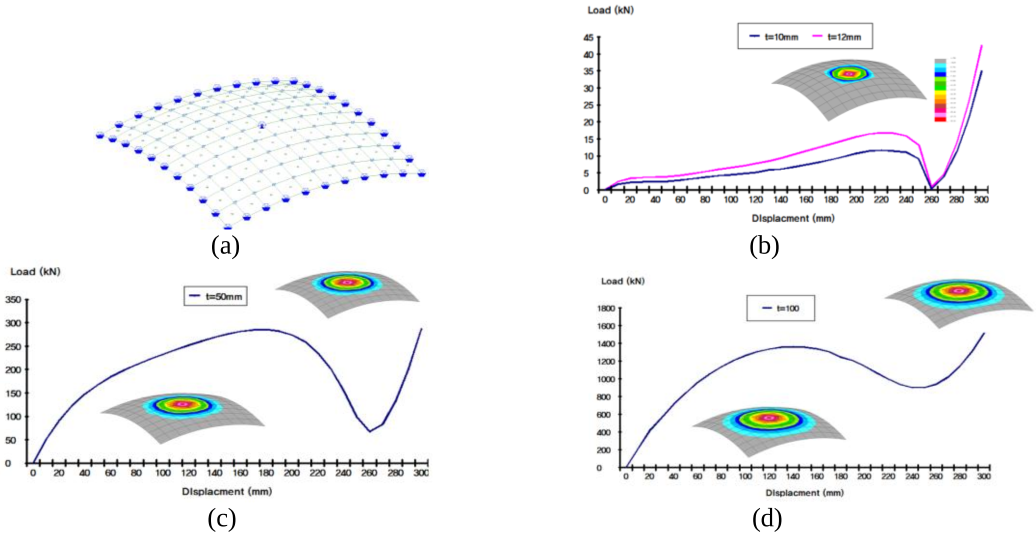


Figure 1. Geomerical nonlinear analysis of a spherical shell under a concenterated load at the center (a) Analytical modeling (b) Load deflection curve and deformed contour (t=10mm) (c) Thickness=50 mm (d) Thickness=100 mm

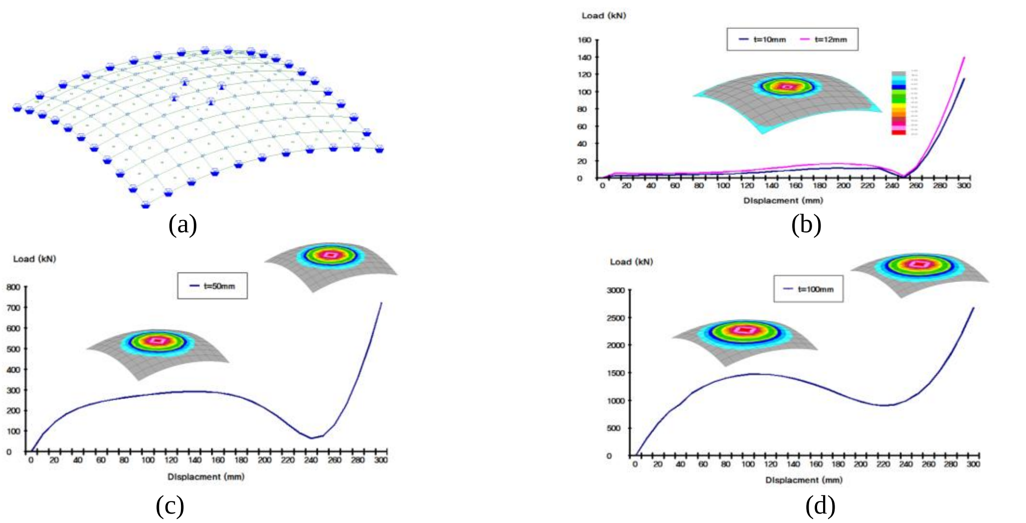
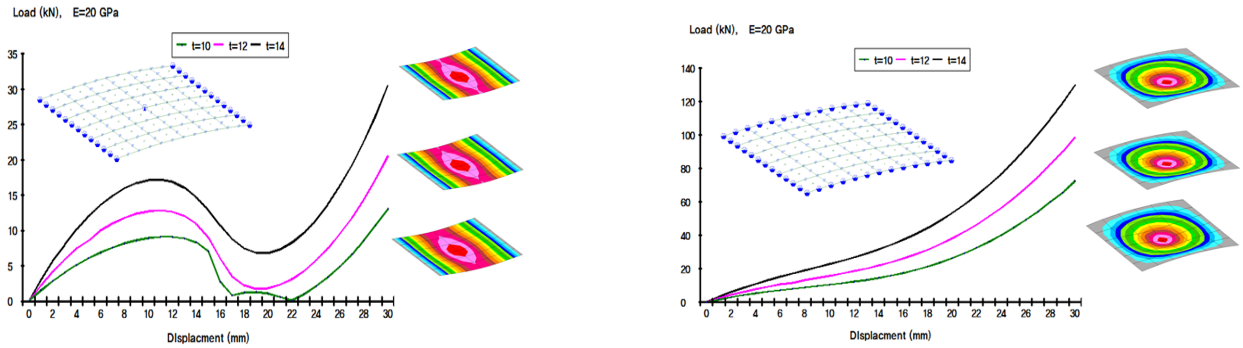


Figure 2. Geometrical nonlinear analysis of a spherical shell under 4 concentrated loads (a) Analytical modeling (b) Load deflection curve and deformed contour (t=12mm) (c) Thickness = 50mm (d) Thickness=100mm

B. Geometric nonlinear behavior of a cylindrical shell –

A cylindrical shell is subjected to a centeral point load. Radius is 2.54m. The 3 cases of thickness are 10, 12 and 14 mm. Modulus of elasticity is 30 GPa. Poisson's ratio is 0.3. The load is applied by incrementing the central point displacement. The objective of this analysis is to obtain the load deflection curve. The central displacement is incremented in equal steps. Number of displacement increments is 30 equal increments. Maximum number of iterations allowed per increment is 100. All tolerances is 0.001. Full Newton-Raphson method is employed in all runs.



(a) (b)

Figure 3. Geometrical nonlinear analysis of a cylindrical shell by boundary conditions (a) Straight edges hinged boundary condition (b) Straight edges hinged and circular edges hinges boundary condition

C. Material nonlinear behavior of a quadratic shell –

The material nonlinear analysis is the elastoplastic analysis of a quadratic shell is subjected to a central point load. The hinged boundary conditions are applied to the outer edges. The width is 6.0m. Thickness is 0.02m. The modulus of elasticity is 30 GPa. Shear modulus is 11.54 GPa. Poisson's ratio is 0.3. Initial yield stress in material direction is 30 MPa. Initial yield stress in shear is 20 MPa. Work hardening parameter is 300 MPa. Material nonlinear is used full newton-Raphson iteration technique. The increments are 30 equal increments. Maximum iterations are 100. Tolerance is 0.0001. The load deflection curve of the central point is observed as shown in Figure 4.

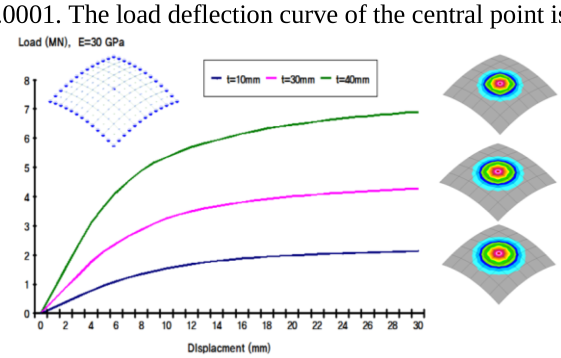


Figure 4. Material nonlinear analysis

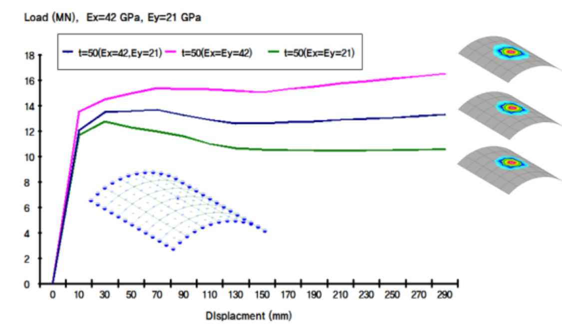


Figure 5. The combined material and geometrical nonlinear analysis

D. The combined material and geometric nonlinear behavior of a quadratic shell –

This objective is large deflection perfectively plastic analysis of an anisotropic cylindrical shell as shown in Figure 5. The shell is modeled by 3-D eight node laminated composite shell element. The shell with hinged supports is subjected to a point load at center. The radius is 7.6m. The width is 15.2m. Seven equal layers are taken through the thickness. The modulus of elasticity is Ex=42 GPa and Ey=21 GPa. Shear modulus is Gxy=Gxz=Gyz=10.5 GPa. The initial yield stress in the material direction is 4.1 MPa. The initial shear yield stress is 2.36 MPa. The load increments are 30 equal increments. Maximum number of iterations is 200. Tolerances are applied to 0.01. Full newton-Raphson method is employed in all runs.

IV.CONCLUSION

The purpose of this study is to analyze the mechanical behavior of shell structures considering the geometric or material non-linearity. The overall behavior of shell structures can be characterized by a load-deflection or force-displacement curve. If the curve is nonlinear, the structural behavior is nonlinear. The response combines linear, hardening and softening. . The snap-through response combines softening with hardening follows the second limit point, the response has a negative stiffness and is therefore unstable. The snap-through response is typical of slightly curved structures such as shallow shells. To describe large displacements and large rotations, it is necessary to calculate the equilibrium with respect to the deformed configuration. In incremental method for nonlinear analysis the load vector is divided into n equal increments. The load increment is used when dividing a load into smaller segments. The number of load increments influences the number of calculation iterations. The greater the number of increments, the greater the probability for the calculations to reach the point of converge. The displacement increments and unbalance force are sufficiently small in comparison with the tolerances, the iteration process stops if divergence occurs. The isotropic elastoplastic material model is defined the stress-strain curve for the elastic, perfectly plastic model, and the elastic, work hardening model. The hardening rule determines how the yield function changes during plastic deformation.

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