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Optimal Allocation Of End-To-End Path Capacity By Decentralization

Seung Hyong Rhee[1]

[1] *Department of Electronics Convergence Engineering, Kwangwoon University, Seoul, Korea*

Abstract- We study the capacity allocation problem of end-to-end paths in general topology networks, in which competitive users make independent decisions to optimize their performance measures. Adopting a game-theoretic approach for decentralization, we formulate a model that differs from the previous ones in two points. First, each user has its own performance objective that is simply assumed to be concave and smooth. Also, its service requirement is restricted by two numbers, e.g., a minimum demand or a system capacity. Adopting this extended model for end-to-end paths in multi-hop networks, we prove that there exists a unique equilibrium which is a network operating point that attains user optimality.[1]

Keywords – End-to-end path, Bandwidth allocation, Decentralized control

I. INTRODUCTION

Until recently, many researchers have studied the problem of allocating optimal capacity for virtual path in communication networks. Given a network topology, link capacity, and the shortest paths, the basic question in the end-to-end path establishment is how one can allocate the available bandwidth of the network to each path. Many works have been done so far to establish the system of virtual paths in an optimal way: To achieve some system-wide performance criteria, algorithms that allocate the optimal capacities, have been investigated [1,2]. The problem of capacity allocation of end-to-end paths is not restricted to wired communication networks: wireless networks also have end-to-end routes and the study on their capacity allocation [3] and capacity probing [4] arose many researchers' interest.

Such centralized methods may have a couple of problems. First, the formulations of optimization problems require huge amount of computational work in most cases, and it has been recognized that the system-wide administration may be an impractical paradigm in case of giant networks [5]. Moreover, they may be unfair in terms of user's need for the bandwidth. Until recently, many works employ a different approach for network control problems. This game-theoretic approach achieves decentralization by allowing the users to make independent decisions according to their individual performance objectives without mandated controls. Many network problems, e.g., flow control and routing, have been addressed using this approach. (See [5,6,7,8,9] and references therein.)

In this paper, we consider general topology networks, where all users' end-to-end paths are established on multi-hop basis: Each user has a single route for its virtual path between two end systems, and decides optimal amount of traffic flow along the path. The users are assumed to maintain their paths through the connections. Note that if users perform both routing and capacity allocation, the result of this section may not hold. See counter-examples of multi-path routings in [6]. Adopting a game-theoretic approach for decentralization, we formulate a model that differs from the previous ones in two points. First, each user has its own performance objective that is simply assumed to be concave and smooth. That is, we use no specific formulation of utility function like net-benefit [5,7] or power [8]. Second, we assume that user's strategy (VP bandwidth reservation) is constrained between two numbers. The previous decentralized models assume that the demands of users are unbounded (bounded by the capacity of resources) [5,7,8]; Those may be impractical assumptions in modern communication networks.

Adopting a standard game-theoretic formulation, we prove that there exists a unique network operating point - Nash Equilibrium Point (NEP) - at which point the users compete no more for bandwidth. This paper is organized as follows. In Section 2 we present the network model and formulate the problem. Section 3 explores the optimal operating point of the network with a simple example, and Section 4 summarizes the paper.

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II. MODEL AND PROBLEM FORMULATION

Let $\mathcal{V}$ be a finite set of nodes and $\mathcal{L} \subseteq \mathcal{V} \times \mathcal{V}$ be a set of undirected links whose elements are unordered pairs of distinct nodes. $\mathcal{L}$ can be interpreted as a set of directed links as in [5], where it is assumed that there exists at most one link between each pair of nodes. Each link $\ell \in \mathcal{L}$ has an available bandwidth (link capacity) C_ℓ.

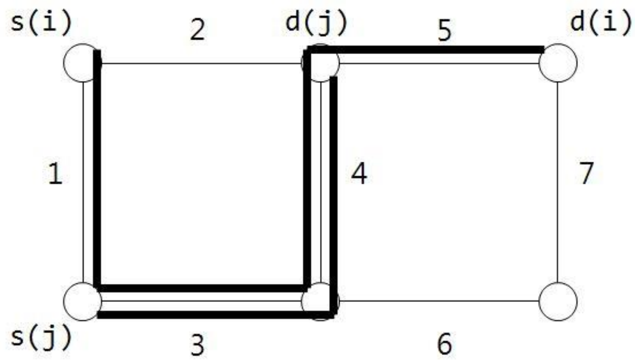

Figure 1. Example of virtual paths with two users i and j: s and d denote source and destination nodes of a user, respectively.

User $i \in \mathcal{K}$ has a set $\mathcal{L}^i \subseteq \mathcal{L}$ of links that it is associated with, and reserves bandwidth f^i through all links in $\mathcal{L}^i$ such that it maximizes its utility function. Thus, the links in $\mathcal{L}^i$ constitute a path between two end points of user i, and f^i represents its optimal capacity. Again, it is assumed that $f^i \in \mathcal{S}^i = [m^i, M^i]$. The set of users which use link ℓ can be defined by

$\mathcal{K}_\ell = \{\, i \in \mathcal{K} \mid \ell \in \mathcal{L}^i \,\}$.

Figure 1 shows a simple example of virtual paths on a general topology network. In the figure, $\mathcal{L}^i = \{1{,}3{,}4{,}5\}$ and $\mathcal{L}^j = \{3{,}4\}$ specify two routes for users i and j, respectively. The set of users on each links are $\mathcal{K}_1 = \mathcal{K}_5 = \{i\}$ and $\mathcal{K}_3 = \mathcal{K}_4 = \{i, j\}$.

For the purpose of virtual path connections, the set of links $\mathcal{L}^i$ should consist a path between the source and destination nodes of the user, which is selected on a multi-hop basis. However, in the following problem formulation, we do not exclude the possibility that the set of links does not constitute a route; i.e., $\mathcal{L}^i$ may be interpreted as just a set of links.

By convention, we assume that a user's *level of satisfaction* in a network can be quantified by *utility* function in the terminology of game theory. The utility function is the performance objective to be optimized by the user, comprising a trade-off between *cost* and *benefit*. Users are assumed to have their own utility functions. Since user i is interested only in the links in $\mathcal{L}^i$, its utility function is given by the summation of utility functions on each links in $\mathcal{L}^i$, i.e.,

$$U^i(f^i, \boldsymbol{F}) = \sum_{\ell \in \mathcal{L}^i} U^i_\ell(f^i, F_\ell), \quad i \in \mathcal{K},$$

where $F_\ell = \sum_{i \in \mathcal{K}_\ell} f^i$ and $\boldsymbol{F} = (F_1, \ldots, F_L)$. User i can optimize $U^i_\ell(f^i, F_\ell)$ by controlling f^i, given the link capacity C_ℓ and the other users' strategy, in particular, their sum $\sum_{j \neq i} f^j_\ell$. The system of optimization problems of users in $\mathcal{K}$ is

$$\begin{aligned} l = \text{maximize}\, & U^i(f^i, \boldsymbol{F}), \quad i \in \mathcal{K} \\ \text{subject to}\, & m^i \le f^i \le M^i. \end{aligned} \tag{1}$$

As a fixed, single path routing is assumed, a user is supposed to reserve a same amount of bandwidth along all the links it is associated with. Thus, note that the *flow conservation constraint* at each node is not necessary in this model. Consult [5] for a case that requires the constraint. Moreover, it is assumed that the minimum bandwidth demand of a user is guaranteed along its route. In other words, a link gives admissions to users such that the summation of their minimum demands does not exceed its capacity, i.e.,

$$C_\ell > \sum_{i \in \mathcal{K}_\ell} m^i , \quad \forall \ell \in \mathcal{L}.$$

In our capacity allocation model, we assume that $U_\ell^i(f^i, F_\ell)$ has the following properties:

[A1] $U_\ell^i(f^i, F_\ell)$ is continuously differentiable with respect to f^i.

[A2] $\frac{\partial}{\partial f^i} U_\ell^i(f^i, F_\ell)$ is strictly decreasing with respect to f^i.

[A3] $\frac{\partial}{\partial f^i} U_\ell^i(f^i, F_\ell)$ is non-increasing with respect to F_ℓ.

The first two assumptions state the strict concavity of U^i with respect to f^i. Let $C_\ell^i = C_\ell - \sum_{j \neq i} f^j$ be the available capacity seen by user i at link ℓ. Given vector $\boldsymbol{C}^i = (C_\ell^i, \ell \in \mathcal{L}^i)$, the response function is defined by

$$R^i(\boldsymbol{C}^i) = \arg\max_{g \in \mathcal{S}^i} \sum_{\ell \in \mathcal{L}^i} U_\ell^i(g, g + C_\ell - C_\ell^i).$$

In other words, $R^i(\boldsymbol{C}^i)$ is the optimal amount of bandwidth, chosen from $\mathcal{S}^i$, which maximizes U^i, if the available capacity for i is $\boldsymbol{C}^i$. The next assumption is a natural one on user's response.

[A4] Given $\boldsymbol{C}^i$ such that $m^i < C_\ell^i \leq C_\ell$ for $\ell \in \mathcal{L}^i$, $R^i(\boldsymbol{C}^i) < \min_{\ell \in \mathcal{L}^i} \boldsymbol{C}_\ell^i$.

A4 easily holds for many situations, where the cost function has a form of penalty (either delay or price) that prevents congestion: $M/M/1$ delay is an immediate example of such a penalty. The above assumptions are natural extensions of previous models studied in the literature [5,7].

III. THE UNIQUENESS OF THE EQUILIBRIUM

The decentralized system given by (1) is a special case of the general concave game considered in [10]. The existence of NEP (Nash Equilibrium Point) can be proved using Kakutani's fixed-point theorem, or by Theorem 1 of [10]. Now we establish uniqueness of the NEP. The following lemma proves that, if there exist two different NEPs $\boldsymbol{f}^*$ and $\hat{\boldsymbol{f}}$, then the corresponding vectors $(F_1^*, \ldots, F_L^*)$ and $(\hat{F}_1, \ldots, \hat{F}_L)$ have a partial ordering in the product space $[0, C_1] \times \ldots \times [0, C_L]$. This lemma is crucial in proving the main result of this section.

Lemma 1. Let $\boldsymbol{f}^*$ and $\hat{\boldsymbol{f}}$ be two different equilibrium points in the end-to-end capacity allocation problem given by (1). Then, except the case of $F_\ell^* = \hat{F}_\ell, \forall \ell$, only one of the following cases holds:

$$i)\ F_\ell^* \geq \hat{F}_\ell, \quad \forall \ell \in \mathcal{L} \quad \text{or} \quad ii)\ F_\ell^* \leq \hat{F}_\ell, \quad \forall \ell \in \mathcal{L},$$

where $F_\ell^* = \sum_{i \in \mathcal{K}_\ell} f^{*i}$ and $\hat{F}_\ell = \sum_{i \in \mathcal{K}_\ell} \hat{f}^i$.

Proof. See 5. □

The following theorem shows that, under the assumption that users do not change their sets of links, the general topology network has a unique equilibrium point, i.e., NEP. The theorem can be proved with Lemma 1 at hand.

Theorem 2. The NEP (Nash Equilibrium Point) of the decentralized path capacity allocation on a general topology network given by (1) is unique.

Proof. Supposing that there are two different NEPs, we have two possibilities in Lemma 1. We examine the first case $F_\ell \geq \hat{F}_\ell$ for all ℓ. If we assume there is a user i such that $f^i > \hat{f}^i$, then by the first order optimality condition (5) and the assumptions A2-A3, $\lambda^i - \mu^i < \hat{\lambda}^i - \hat{\mu}^i$ and (11) gives $f^i \leq \hat{f}^i$; it contradicts our assumption, and thus $f^i \leq \hat{f}^i, \quad \forall i$. Hence, we conclude that $F_\ell \geq \hat{F}_\ell$ for all ℓ means $F_\ell = \hat{F}_\ell$ for all ℓ. The second case of Lemma 1 gives the same result, i.e. $F_\ell \leq \hat{F}_\ell$ for all ℓ means $F_\ell = \hat{F}_\ell$ for all ℓ. Therefore,

$$F_\ell = \hat{F}_\ell, \quad \forall \ell.$$

Now from (5), for all ℓ,

$$\sum_{\ell\in\mathcal{L}^i} -\frac{\partial}{\partial f^i} U_\ell^i(f^i,F_\ell) + \lambda^i - \mu^i = \sum_{\ell\in\mathcal{L}^i} -\frac{\partial}{\partial f^i} U_\ell^i(\hat{f}^i,\hat{F}_\ell) + \hat{\lambda}^i - \hat{\mu}^i. \tag{2}$$

If we suppose that $\lambda^i - \mu^i > \hat{\lambda}^i - \hat{\mu}^i$ for some $i \in \mathcal{K}$, then

$$\sum_{\ell\in\mathcal{L}^i} \frac{\partial}{\partial f^i} U_\ell^i(f^i,F_\ell) > \sum_{\ell\in\mathcal{L}^i} \frac{\partial}{\partial f^i} U_\ell^i(\hat{f}^i,\hat{F}_\ell) = \sum_{\ell\in\mathcal{L}^i} \frac{\partial}{\partial f^i} U_\ell^i(\hat{f}^i,F_\ell).$$

By A2, $f^i < \hat{f}^i$, and this contradicts (10). Hence, $\forall i$, $\lambda^i - \mu^i \geq \hat{\lambda}^i - \hat{\mu}^i$. Similarly, it can be proved that, $\forall i$, $\lambda^i - \mu^i \leq \hat{\lambda}^i - \hat{\mu}^i$. Therefore, $\lambda^i - \mu^i = \hat{\lambda}^i - \hat{\mu}^i$, and as $\frac{\partial}{\partial f^i} U^i(\cdot,F)$ is one-to-one, we have $f^i = \hat{f}^i$, $\forall i$. □

In case a user has different preferences on its set of links, the decentralized optimization model (1) can be generalized as follows:

$$l = \text{maximize} \sum_{\ell\in\mathcal{L}^i} w_\ell^i \cdot U_\ell^i(f^i,F_\ell), \quad i \in \mathcal{K}$$
$$\text{subject to} m^i \leq f^i \leq M^i, \tag{3}$$

where w_ℓ^i is a non-negative real number. From Theorem 2, the corollary below follows without proof.

Corollary 1. The NEP (Nash Equilibrium Point) of the decentralized capacity allocation on a general topology network given by (3) is unique.

We give a simple example of such a unique equilibrium. Let's assume that the users' utility functions are given by the generalized power function [8]

$$U^i(f^i,F) = (f^i)^{\beta^i}(C-F),$$

which is interpreted as benefit $(f^i)^{\beta^i}$ divided by cost $\frac{1}{C-F}$ ($M/M/1$-type delay). With weighting factor $0 < \beta^i \leq 1$, it is easily seen that U^i satisfies A1-A3. To see that A4 is satisfied, consider the corresponding unconstrained optimization problem. Given C^i, the optimality condition $\nabla U^i = 0$ gives

$$arg\, max_g U^i(g,F) = \frac{\beta^i}{\beta^i+1} C^i < C^i. \tag{4}$$

Now suppose that the nodes in Figure 1 are connected via links with capacity 100 each and $\beta^i = \beta^j = 1$. Also, the users' strategy spaces are given by $f^i \in \mathcal{S}^i = [40,100]$ and $f^j \in \mathcal{S}^j = [0,100]$. Then, as $arg\, max_g U^i(g,F) = \frac{1}{2} C^i$ from (4), it is easy to see that $f^{*i} = 40$ and $f^{*j} = 30$, which are the Nash equilibrium point of the network.

IV. CONCLUSION

In a decentralized way, we have considered the problem of end-to-end path capacity allocation in general topology networks where all users have their own routes and fixed-routing is assumed. Our user model has two distinguished features: each user has its own performance objective that is of concave type, and its own strategy space which is bounded by minimum and maximum demands. Adopting the notion of Nash equilibrium, we proved that there exists a unique network operating point which is determined by the users' own objectives. Finally, we note that the decentralized control scheme studied in this paper may be used in a distributed system; i.e., the network provider may choose performance objective(s) for its network and assign them to distributed controllers such that the network is operated by the controllers themselves.

V. PROOF OF LEMMA 1

First, we consider the first case. Assume that $F_\ell^* \geq \hat{F}_\ell$ for some $\ell \in \mathcal{L}$, and let

$$\mathcal{L}_b = \{\, \ell \in \mathcal{L} \mid F_\ell^* \geq \hat{F}_\ell \,\} \quad \text{and} \quad \mathcal{L}_s = \{\, \ell \in \mathcal{L} \mid F_\ell^* < \hat{F}_\ell \,\}.$$

Then $\mathcal{L}_b$ and $\mathcal{L}_s$ are disjoint sets of links such that $\mathcal{L}_b \cup \mathcal{L}_s = \mathcal{L}$. We will show that $\mathcal{L}_s = \emptyset$. Assume that $\mathcal{L}_s$ is not empty and define the new sets of users such as

$\mathcal{K}_b = \{ i \in \mathcal{K} \mid \mathcal{L}^i \subseteq \mathcal{L}_b \}$,
$\mathcal{K}_s = \{ i \in \mathcal{K} \mid \mathcal{L}^i \subseteq \mathcal{L}_s \}$, and
$\mathcal{K}_e = (\mathcal{K}_b \cup \mathcal{K}_s)^C$.

$\mathcal{K}_b$ ($\mathcal{K}_s$) is the set of users whose routes only consists of the links in $\mathcal{L}_b$ ($\mathcal{L}_s$). $\mathcal{K}_e$ is the set of users whose routes are chosen from both of $\mathcal{L}_b$ and $\mathcal{L}_s$. Then, again $\mathcal{K}_b$, $\mathcal{K}_s$, and $\mathcal{K}_e$ are disjoint sets of users such that $\mathcal{K}_b \cup \mathcal{K}_s \cup \mathcal{K}_e = \mathcal{K}$. For any user $i \in \mathcal{K}$, the first order optimality condition at a NEP $\boldsymbol{f}$ is given by

$$-\sum_{\ell \in \mathcal{L}^i} \frac{\partial}{\partial f^i} U_\ell^i(f^i, F_\ell) + \lambda^i - \mu^i = 0. \tag{5}$$

Also note that, from the complementary slackness conditions, we have the following set of conditions, $\forall i$, at any NEP $\boldsymbol{f}$:

$$\lambda^i \geq 0, \quad \mu^i \geq 0 \tag{6}$$

$$\lambda^i (f^i - M^i) = 0, \quad \mu^i (m^i - f^i) = 0 \tag{7}$$

From (6) and (7), we have the following implications:

$$\Rightarrow \quad c@ \quad \Rightarrow \quad c = \lambda^i < \mu^i \mu^i > 0 f^i = m^i \tag{8}$$

$$\Rightarrow \quad c@ \quad \Rightarrow \quad c = \lambda^i > \mu^i \lambda^i > 0 f^i = M^i \tag{9}$$

Now for the two different NEPs, $\boldsymbol{f}^*$ and $\hat{\boldsymbol{f}}$, consider Lagrange multipliers (λ, μ) and ($\hat{\lambda}, \hat{\mu}$). Both sets of vectors $(\boldsymbol{f}^*, \lambda, \mu)$ and $(\hat{\boldsymbol{f}}, \hat{\lambda}, \hat{\mu})$ satisfy (5)-(7), and thus (8)-(9). We can establish the following relations for all $i \in \mathcal{K}$:

$$\lambda^i - \mu^i > \hat{\lambda}^i - \hat{\mu}^i \quad \Rightarrow \quad f^i \geq \hat{f}^i \tag{10}$$

$$\lambda^i - \mu^i < \hat{\lambda}^i - \hat{\mu}^i \quad \Rightarrow \quad f^i \leq \hat{f}^i \tag{11}$$

(10) can be proved by considering two possible values of $\lambda^i - \mu^i$: (i) If $\lambda^i - \mu^i > 0$, then $f^i = M^i$ from (9) and thus $f^i \geq \hat{f}^i$. (ii) Else if $\lambda^i - \mu^i \leq 0$, then $\hat{\lambda}^i < \hat{\mu}^i$. Hence $\hat{f}^i = m^i$ from (8) and $f^i \geq \hat{f}^i$. The same reasoning also proves (11). Now consider an $i \in \mathcal{K}_b$, then $F_\ell^* \geq \hat{F}_\ell$ for all $\ell \in \mathcal{L}^i$. If we assume that $f^{*i} > \hat{f}^i$, then from the assumptions A2 and A3, we have $\lambda^{*i} - \mu^{*i} < \hat{\lambda}^i - \hat{\mu}^i$ and (11) gives $f^{*i} \leq \hat{f}^i$; it contradicts our assumption, and thus

$$f^{*i} \leq \hat{f}^i \quad \text{for all } i \in \mathcal{K}_h. \tag{12}$$

From the definitions, for $\ell \in \mathcal{L}_b$,

$$\mathcal{K}_\ell = (\mathcal{K}_\ell \cap \mathcal{K}_b) \cup (\mathcal{K}_\ell \cap \mathcal{K}_s) \cup (\mathcal{K}_\ell \cap \mathcal{K}_e)$$
$$= (\mathcal{K}_\ell \cap \mathcal{K}_b) \cup (\mathcal{K}_\ell \cap \mathcal{K}_e),$$

since $\mathcal{K}_\ell \cap \mathcal{K}_s = \emptyset$ for $\ell \in \mathcal{L}_b$. Then

$$\sum_{\ell \in \mathcal{L}_h} F_\ell^* = \sum_{\ell \in \mathcal{L}_h} \sum_{i \in \mathcal{K}_\ell} f^{*i} = \sum_{\ell \in \mathcal{L}_h} \sum_{i \in \mathcal{K}_\ell \cap \mathcal{K}_h} f^{*i} + \sum_{\ell \in \mathcal{L}_h} \sum_{i \in \mathcal{K}_\ell \cap \mathcal{K}_e} f^{*i}.$$

Since from (12)

$$\sum_{\ell \in \mathcal{L}_h} \sum_{i \in \mathcal{K}_\ell \cap \mathcal{K}_h} f^{*i} \leq \sum_{\ell \in \mathcal{L}_h} \sum_{i \in \mathcal{K}_\ell \cap \mathcal{K}_h} \hat{f}^i,$$

and from the definition of $\mathcal{L}_b$

$$\sum_{\ell \in \mathcal{L}_h} F_\ell^* \geq \sum_{\ell \in \mathcal{L}_h} \hat{F}_\ell,$$

we have

$$\sum_{\ell\in\mathcal{L}_b}\sum_{i\in\mathcal{K}_\ell\cap\mathcal{K}_s} f^{*i} \geq \sum_{\ell\in\mathcal{L}_b}\sum_{i\in\mathcal{K}_\ell\cap\mathcal{K}_s} \hat{f}^i \quad (13)$$

Now consider an $i \in \mathcal{K}_s$, and assume that $f^{*i} < \hat{f}^i$. Then, by the assumptions A2 and A3, $\lambda^{*i} - \mu^{*i} > \hat{\lambda}^i - \hat{\mu}^i$ and (10) gives $f^{*i} \geq \hat{f}^i$; it contradicts the assumption, and thus

$$f^{*i} \geq \hat{f}^i \quad \text{for all } i \in \mathcal{K}_s. \quad (14)$$

As before, for $\ell \in \mathcal{L}_s$, $\mathcal{K}_\ell$ can be represented by

$\mathcal{K}_\ell = (\mathcal{K}_\ell \cap \mathcal{K}_s) \cup (\mathcal{K}_\ell \cap \mathcal{K}_s)$,

since $\mathcal{K}_\ell \cap \mathcal{K}_b = \emptyset$ for $\ell \in \mathcal{L}_s$. Then, from (13) and (14),

$$\sum_{\ell\in\mathcal{L}_s} F_\ell^* = \sum_{\ell\in\mathcal{L}_s}\sum_{i\in\mathcal{K}_\ell} f^{*i} = \sum_{\ell\in\mathcal{L}_s}\sum_{i\in\mathcal{K}_\ell\cap\mathcal{K}_s} f^{*i} + \sum_{\ell\in\mathcal{L}_s}\sum_{i\in\mathcal{K}_\ell\cap\mathcal{K}_s} f^{*i}$$

$$\geq \sum_{\ell\in\mathcal{L}_s}\sum_{i\in\mathcal{K}_\ell\cap\mathcal{K}_s} \hat{f}^i + \sum_{\ell\in\mathcal{L}_s}\sum_{i\in\mathcal{K}_\ell\cap\mathcal{K}_s} \hat{f}^i$$

$$= \sum_{\ell\in\mathcal{L}_s}\sum_{i\in\mathcal{K}_\ell} \hat{f}^i = \sum_{\ell\in\mathcal{L}_s} \hat{F}_\ell.$$

This contradicts the definition of $\mathcal{L}_s$. Therefore, $\mathcal{L}_s$ is empty and it means that, if $F_\ell^* \geq \hat{F}_\ell$ for some $\ell \in \mathcal{L}$, then $F_\ell^* \geq \hat{F}_\ell$ for all $\ell \in \mathcal{L}$. The second case can be proved in the same way, i.e. if $F_\ell^* \leq \hat{F}_\ell$ for some $\ell \in \mathcal{L}$, then it means $F_\ell^* \leq \hat{F}_\ell$ for all $\ell \in \mathcal{L}$. □

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