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Fourth-Order Binder cumulant to characterize the first-order phase transition of equilibrium and nonequilibrium systems

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Abstract- Through Monte Carlo simulations, we study fourth-order Binder cumulant of the order parameter of nonequilibrium systems. Our results show that the fourth-order Binder cumulant is a useful tool in the identification of first-order transitions.

Keywords – phase transition, nonequilibrium, cumulants

I. INTRODUCTION

Distinguishing the order of the phase transition is one of several problems of Monte Carlo simulation studies for spins systems. The higher difficulty arises when the correlation length is finite and more significant than the size of the system. In such situations, common transition order identification tools, such as examining the minimum free energy [1,2] by considering the probability distribution of energy [3] may not be a good indicator.

Shan et al. [4] analyzed the behavior of fourth-order cumulant of physical quantities as the order parameter and the energy associated with Blume-Capel model (S=1) [5]. They illustrated the action of the fourth-order cumulant of magnetization and energy across first and second-order transitions in the phase diagram of the system.

In this work, we investigate the fourth-order Binder cumulant of the order parameter of the SLR model [6] as a tool for the indication of first-order phase transitions.

II. MODEL AND SIMULATIONS

We consider a simple spin-like model SLR [6], on directed random lattice by a set of spin variables S_i taking the values ± 1 situated on every site i of a directed random lattice with N sites. In this random lattice, similar to Sánchez et al. [6], we start from a two-dimensional random lattice consisting of sites linked to their k (where 3<k<17 and different for each site of network) nearest neighbors by both outgoing and incoming links. Then, with probability p we replace nearest neighbor outgoing links by new outgoing links to different sites chosen at random. This process is performed for every link. Therefore, with this procedure, every site will have k outgoing links and vary (random) number of incoming links. Then, the spins or agents are then placed at the network sites. Then, any agent is connected by k outgoing links to other agents or playmates and can be in one up (+1) or down (-1) states.

The time evolution of the system is performed by a single spin-flip dynamics with a probability P_i described by

$$P_i = e^{-G_i/T} \tag{1}$$

where T is the temperature and G_i is the "energy" of the configuration obtained from

$$G_i = -2S_i \sum_{j=1}^{k} S_j \tag{2}$$

where the sum is carried out over the k neighbors of site i . The simulations have been performed on different networks comprising a number N=250, 500, 1000, 2000, 4000, 8000 and 16000 of sites of random lattices. For each system size quenched averages over the connectivity disorder are approximated by averaging over R=50 independent realizations. The simulation has started with a uniform configuration of spins. We ran $4x10^5$ Monte Carlo steps (MCS) per spin with $2X10^5$ configurations discarded to reach steady state.

III. Results and discussions

From the magnetization per spin, $m = \frac{1}{N}\sum_{i=1}^{k} S_i$, we can derive the average the magnetization,

the susceptibility, and the fourth-order magnetic cumulant,

$$m(T) = [\langle |m| \rangle]_{av} \tag{3}$$

$$\chi(T) = \frac{N}{T}\left([\langle m^2 \rangle]_{av} - [\langle m \rangle^2]_{av}\right), \tag{4}$$

$$U_4(T) = 1 - \frac{\langle m^4 \rangle_{av}}{3[\langle m \rangle]^2_{av}}, \tag{8}$$

In the above equations <...> stands for thermodynamic averages and $[...]_{av}$ for averages over different realizations.

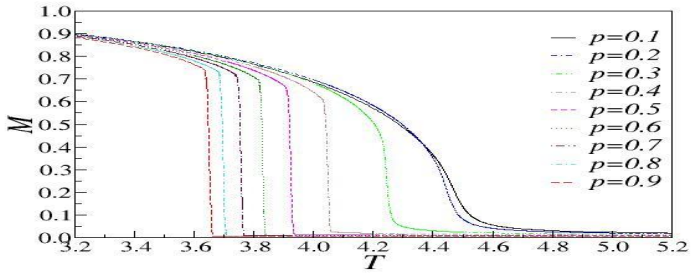

Figure 1. Magnetization as a function of T for N=16000 and rewiring probabilities p=0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.

In Figure 1, we show the magnetization as a function of T for N=16000 sites and various rewiring probabilities from a range of p=0.1 to 0.9. These results we can estimate the location of the tricritical point separating the first-order from the second-order transition. As for $p<0.35$ one still has a second-order behavior and for p=0.4 one has already a first-order transition, we could say that the tricritical point is located at p=0.35(5). Accordingly, the corresponding tricritical temperature is given by $T_t \approx 4.18$.

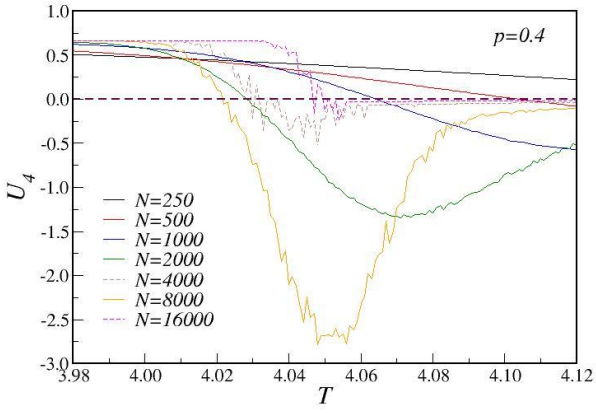

Figure [2] . Displays the fourth- order Binder cumulant (U_4) versus temperature for p=0.4.

In Figure 2 we find that there is a crossing point and the (U_4) shows a negative dip for the system sizes (N=250 and 160000 sites). These are typical indications of a first-order phase transition [13]

III. CONCLUSION

In summary, the SRL social model on directed random lattice presents a second-order phase transition for $p \leqslant 0.35$, otherwise it is a first-order phase transition with a nonequilibrium tricritical point at $p_c = 0.35$. The fourth-order Binder cumulant of the order parameter of the SLR model For p=0.4 (see Figure 2) presents negative values for the network sizes studied here. This is a strong indication that the phase transition is first order.

IV. REFERENCE

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