http://dx.doi.org/10.21172/ijiet.103.09

# Coefficient of Consolidation: Simplified One Point Method

Binu Sharma[1]
[1]*Civil Engineering Department, Assam Engineering College, Guwahati, Assam, India*

**Abstract: Estimation of the time required for a specified percentage of consolidation or the rate of consolidation settlement is very important for civil engineering construction practice. For this purpose selecting a proper value for the coefficient of consolidation, $C_v$ is necessary. To obtain $C_v$ by the consolidation test requires a great amount of time and it is also dependent upon the method adopted of obtaining the same. One point method of obtaining co-efficient of consolidation has been proposed in the literature to optimize the effort and time of the geotechnical person. In this paper the one point method of obtaining the coefficient of consolidation has been further simplified. This method is devoid of any curve fitting procedure and no assumptions are made except that the time - deformation behavior could be treated as a well accepted rectangular hyperbola. The proposed method requires very few time versus deformation readings for a short time of about less than 30 minutes from which one can get the final deformation assuming a rectangular hyperbola behavior of time – deformation. The coefficient of consolidation can be determined knowing time taken for 50% consolidation.**
**Keywords: Consolidation, coefficient of consolidation, one point method, rectangular hyperbola**

## I. INTRODUCTION

An area of soil mechanics of profound importance is the estimation of the time required for a specified amount of consolidation or the probable consolidation settlement at any point of time. The key for estimating the rate of consolidation settlement or the rate of dissipation of pore pressure lies in selecting a proper value for the coefficient of consolidation, $C_v$. To obtain $C_v$ by the consolidation test requires a great amount of time and effort and it is also dependent upon the method adopted of obtaining the same.

Several researchers have proposed methods for obtaining a more reliable value of $C_v$ from laboratory oedometer tests. The usual technique is to compare some characteristics features of Terzaghi's theoretical time factor, $T_v$ – degree of consolidation, U relationship with the compression and time data obtained from the laboratory test. This forms the basis for most of the graphical techniques that are available in literature ( [1- 9]to name a few ). Additionally there are several curve fitting methods which have been proposed [10 – 16].

Pandian et al. [13] showed by examination of Terzaghi's theoretical degree of consolidation (U) and theoretical time factor ($T_v$) curve, that log $U/T_v$ versus log $T_v$ plot exhibits a bilinear behaviour. The intersection point corresponds to a theoretical time factor of 0.793 and a degree of consolidation of 88.5%. Using this property, the following equation (1) for the coefficient of consolidation was proposed

$$C_v = 0.793\, d^2/t_{88.5} \quad (1)$$

where, time t corresponds to 88.5% consolidation which can be obtained from the log δ/t versus log t plot.

There are several research workers who conclude that the time - compression data can be represented by a rectangular hyperbola. According to Sridharan and Sreepada Rao [17] and Sridharan et al.[12], the time factor $T_v$ and degree of consolidation U from Terzaghi's theory can be shown to be a rectangular hyperbola over the degree of consolidation range 60% to 90%. They observed that when time factor $T_v$ and degree of consolidation U are plotted in the form $T_v/U$ versus $T_v$, a curve is obtained upto about U = 60% and a straight line for $60\% \leq U \leq 90\%$ for double drainage. The similarities between theoretical $T_v/U$ versus $T_v$ and experimental t/δ versus t plots are used to present a simple method to determine $C_v$. After plotting t/δ versus t, identification of the initial straight line part in this plot is made. The values of slope 'm' and the intercept 'c' of the straight line part are measured. The thickness of the sample is 'h'. The value of $C_v$ is then computed from the following relation (2)

$$C_v = 0.24 m\, h^2/c \quad (2)$$

Thus this method requires a plot of t/δ versus t until a clear straight line is obtained. After obtaining m and c, $C_v$ can be obtained.

## II. ONE POINT METHOD

Sridharan and Prakash [19] had observed that when theoretical U-T relationship, laboratory δ – t data and the same δ – t data corrected for initial and secondary compression effects are expressed in the form of $\log_{10} H^2/t$ versus U curve, the effects of primary and secondary compressions are least in the range $40\% < U < 60\%$. The $C_v$ values obtained in the range $40\% < U < 60\%$ may be considered to represent the soil for which δ – t is obtained reasonably

well. The very observation that experimental behavior of soil without correction for initial and secondary compression effects match well with the theory in the range 40% < U < 60% lead to the development of the one point method of $C_v$ determination by Sridharan and Prakash [18]. To determine $C_v$ by the one point method the value of final compression $\delta_f$ is considered at the end of the loading period. The duration of loading period for each increment of loading is normally taken as 24 hours. Thus one has to wait for 24hrs or more to get the value of final compression. Next, the compression corresponding to 50% consolidation i.e. 0.5 $\delta_f$ is determined along with corresponding time $t_{50}$. $C_v$ is calculated corresponding to the noted time $t_{50}$ as $C_v = T_v H^2/ t_{50}$. Thus it can be seen that the one point method is very simple, however one need to carry out the consolidation test for a particular load increment for about 24 hrs and need to note the time taken for 50% consolidation after getting $\delta_f$.
The first part of the present article aims to see the efficacy of the one point method corresponding to time $t_{50}$. $C_v$ is also determined corresponding to time $t_{45}$ and $t_{55}$ in this study as these values of $C_v$ will also lie in the range 40% < U < 60% and will give least effects of primary and secondary compressions. Secondly the present article also attempts to determine $C_v$ by a simple and quick method of obtaining $\delta_f$ and $t_{50}$ following observations from Sridharan et al. [18], where the entire range of time compression readings need not be taken.

## III. TEST RESULTS AND DISCUSSION

The experimental data used in the following illustrations were obtained from the conventional one dimensional consolidation tests carried out. Twelve soil samples with liquid limits ranging from 33.8% to 78% were tested to determine their consolidation and other relevant physical properties. The physical properties of the soils are shown in Table1.

Table-1 Physical properties of the soils tested

| Sl. No. | Site Location | Liquid Limit $w_L$ (%) | Plastic Limit $w_P$ (%) Thread rolling method | Plasticity Index P.I. (%) | Specific Gravity $G_s$ | Unified Soil Classification (USCS) | $t_{50}$ (min) For stress range of 160kN/m$^2$ to320kN/m$^2$ |
|---|---|---|---|---|---|---|---|
| **1.** | **V.I.P.** | **38.5** | **18.5** | **21.5** | **2.63** | **CL** | **4.00** |
| **2.** | **Noonmati Sample-2** | **69.0** | **25.6** | **45.0** | **2.72** | **CH** | **6.25** |
| **3.** | **A.E.C. Sample-3** | **61.0** | **23.9** | **38.5** | **2.68** | **CH** | **6.00** |
| **4.** | **Sonapur** | **66.0** | **23.2** | **42.0** | **2.71** | **CH** | **4.00** |
| **5.** | **I.I.T.B.H.-I-5.7m** | **60.0** | **25.5** | **35.8** | **2.71** | **CH** | **6.25** |
| **6.** | **Noonmati Forest Gate** | **77.0** | **31.0** | **49.0** | **2.68** | **CH** | **5.00** |
| **7.** | **Numaligarh Marketing Terminal-3** | **42.0** | **20.5** | **22.0** | **2.65** | **CL** | **4.50** |
| **8.** | **Down Town Hospital Site-I** | **44.0** | **18.0** | **28.0** | **2.70** | **CL** | **2.00** |
| **9.** | **Ajara** | **52.5** | **21.5** | **31.5** | **2.71** | **CH** | **7.00** |
| **10.** | **Bansar** | **33.8** | **17.5** | **17.8** | **2.68** | **CL** | **2.75** |
| **11.** | **A.E.C. Hostel-3** | **46.0** | **17.5** | **29.8** | **2.65** | **CL** | **6.25** |
| **12** | **A.E.C. Sample2** | **78.0** | **27.0** | **48.5** | **2.71** | **CH** | **5.50** |

The coefficients of consolidation based on consolidation tests were determined by the Taylor's square root of time fitting method, Casagrande's logarithm of time fitting method, Pandian's bilinear method and Rectangular hyperbola method. These $C_V$ values which happen to be within the range of $C_V$ values of remoulded soils recorded in NAVFAC- DM 7.1 [20], show their characteristic inconsistency usually observed [21,22]. Using the one point

method, the coefficient of consolidation $C_v$ corresponding to $t_{50}$, was determined for the twelve soil samples. Fig. 1 to Fig. 4 show the comparison between $C_v$ corresponding to $t_{50}$ and Taylor's, Casagrande's, Rectangular Hyperbola and the Pandian's Bilinear $C_v$ values respectively. The mean deviation between the one point method and the Taylor's and Casagrande's method is 16.2% and 14.2% respectively. Fig.5 shows the comparison between the Taylor's method and the Casagrande's method with the experimental data of this work. The average percentage deviation between the two methods is 12.18%. Sridharan and Prakash [18] reports the variation between these popular methods is more than 25% in many cases.
Similarly the mean deviation between the one point method and the Rectangular hyperbola method and Bilinear method are 10.7% and 15%. These observations are of importance in the context that the variation in the values of $C_v$ obtained by Taylor's and Casagrande's method is much larger as reported in literature.

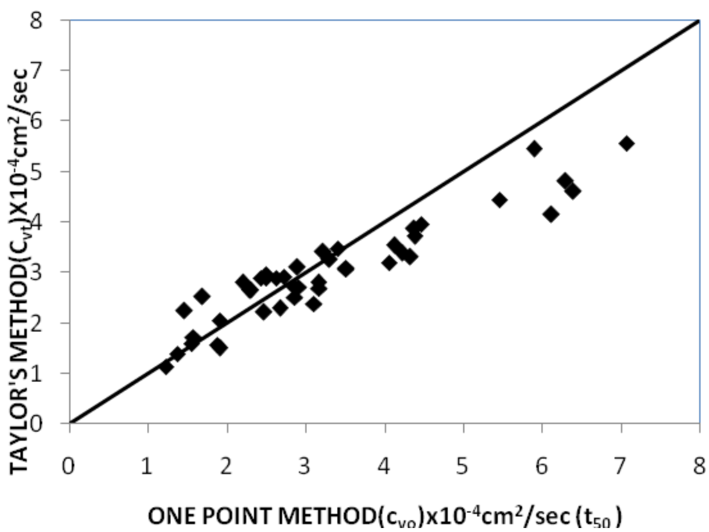


Figure.1Comparison of $C_v$ between Taylor's method and the One point method.

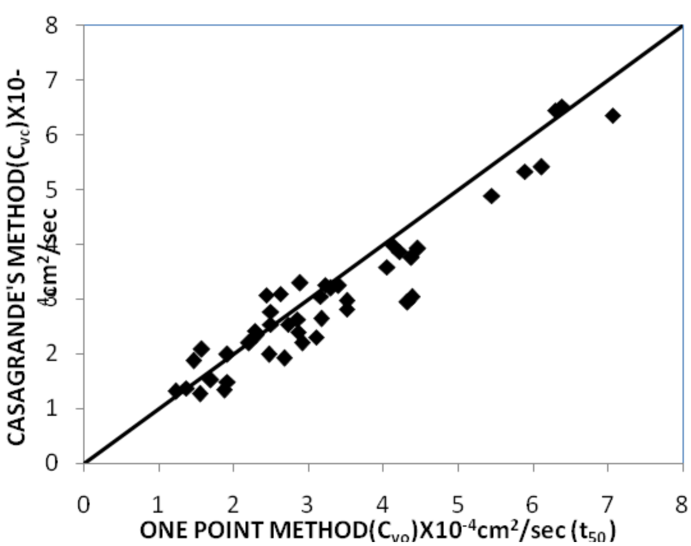


Figure.2Comparison of $C_v$ between Casagrande's method and the One point method.

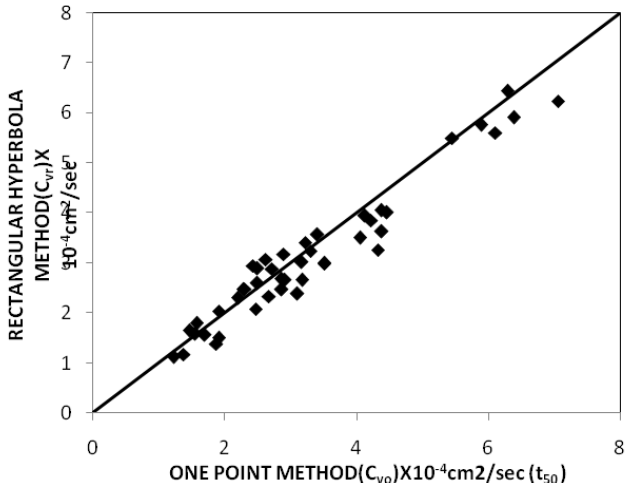


Figure.3Comparison of $C_v$ between Rectangular Hyperbola method and the One point method.

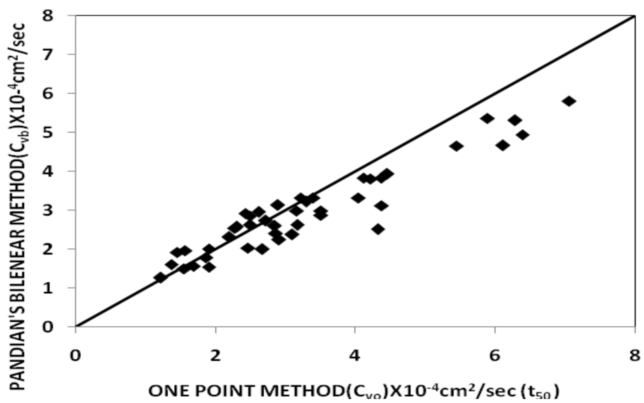


Figure.4Comparison of $C_v$ between Pandian's Bilinear method and the One point method.

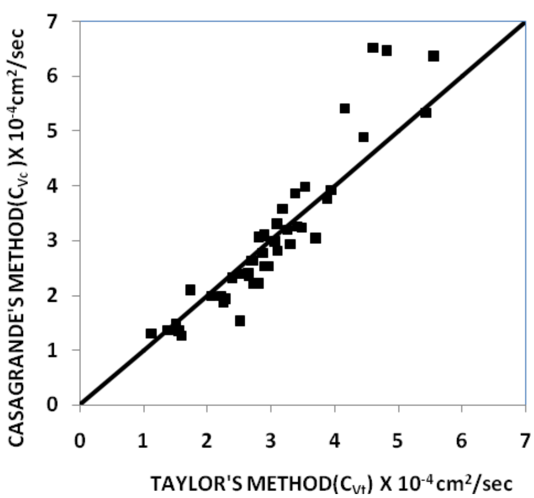


Figure.5 Comparison of $C_v$ between Taylor's method and Casagrande's method.

Using the one point method, the compression corresponding to 45% consolidation i.e. 0.45 $\delta_f$ is determined along with corresponding time $t_{45}$ and the compression corresponding to 55% consolidation i.e. 0.55 $\delta_f$ is determined along with corresponding time $t_{55}$. Using $t_{45}$ and $t_{55}$ the corresponding $C_v$ values are calculated. $C_v$ values are determined corresponding to time $t_{45}$ and $t_{55}$ in this study as these values of $C_v$ will also lie in the range 40% < U < 60% and will give least effects of primary and secondary compressions. It is also intended to see whether better values of $C_v$ can be obtained corresponding to $t_{45}$ and $t_{55}$ compared to $t_{50}$. These values of $C_v$ are found to have a similar variation as that of $C_v$ obtained corresponding to time $t_{50}$ if not better. Hence one point method from $t_{50}$ and $\delta_{50}$, is well acceptable. The average $C_v$ or $C_{av}$ values obtained from averaging time corresponding to time $t_{45}$, $t_{50}$ and $t_{55}$ is also found to have good agreement with the above four methods with mean deviation being 16.7, 15.1, 11.6 and 17% respectively. These values of mean deviation are shown in Table 1.

*3.1. Determination of $C_v$ by the Simplified One Point Method (SOPM)*

Sridharan et al. (1987) showed that the U-$T_v$ relationship can be approximated as rectangular hyperbola in the range of U = 60 to 90%. It is possible to obtain a clear straight line only beyond U = 60%. Figs. 6a to 6d, clearly show this aspect. To get the straight line portion to obtain the slope of the U-$T_v$ relationship after 60% consolidation , very few readings are to be taken. We need not wait upto 24hrs. Again Sridharan et al. [12] have shown that the rectangular hyperbola method provides a simple method to predict the magnitude of final compression, $\delta_f$. The slope (m) of the time (t)/compression ($\delta$) versus time (t) plot can be determined and final compression ($\delta_f$) can be predicted by calculating one divided by the slope. One divided by the slope has been shown to be a useful parameter in predicting the magnitude of total compression $\delta_f$. One need to take time – compression readings only till to get a well defined slope.

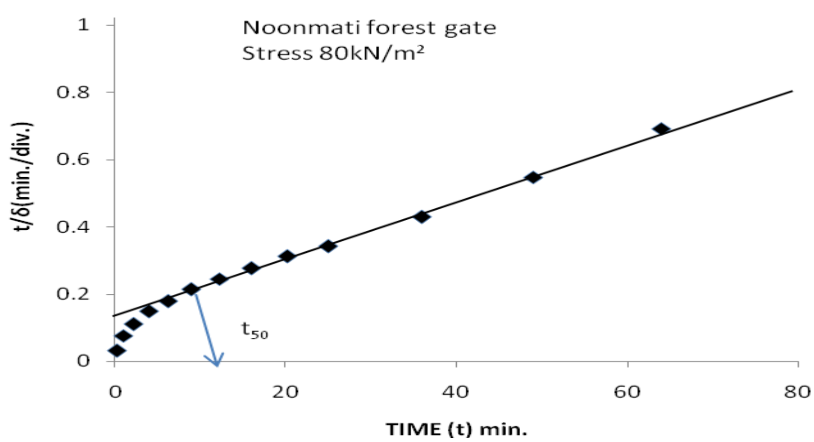


FIGURE.6a

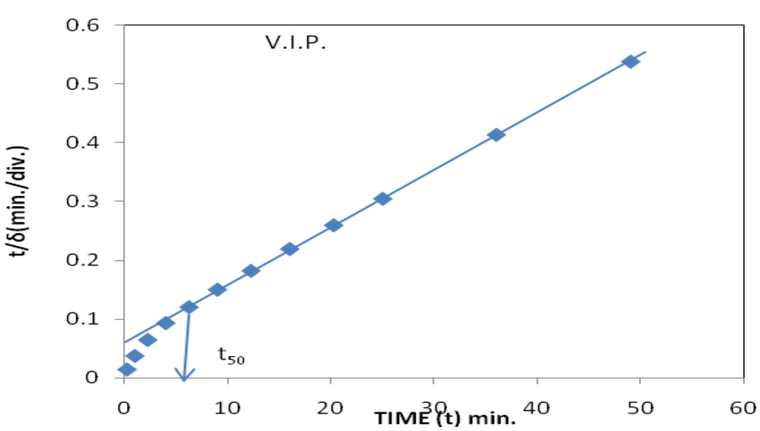

FIGURE.6b

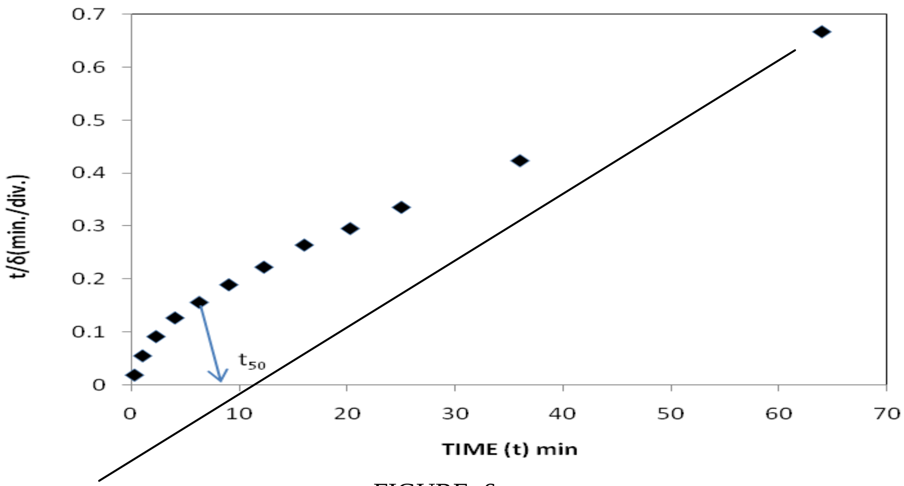


FIGURE. 6c

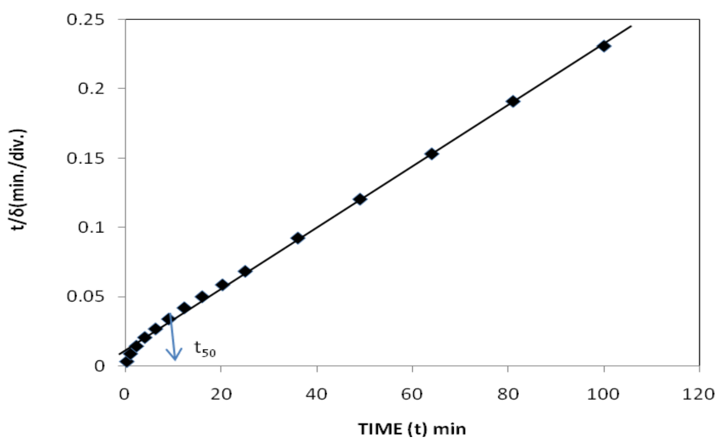


FIGURE.6d

Figure. 6a to 6d show typical plots of t/δ versus t plot of four soils.

Having got $\delta_{f,}$, one can get the time taken for 50% consolidation i.e. $t_{50}$ by taking the reading upto 50% of consolidation. Knowing $t_{50}$, the coefficient of consolidation can be obtained. Time taken to obtain the well defined slope is quite less and varies from 10 minutes to 30 minutes for the different load increments, for the 12 soils tested. Time taken to obtain 50% degree of consosolidation for load increment 160kN/m$^2$ to 320kN/m$^2$ are shown in Table 1. This method, henceforth, will be called the simplified one point method. The advantage of the simplified one point method is that one need not wait till total consolidation is over to get $\delta_f$. Only eight to nine readings are to be taken to get the straight line of the t/δ versus t plot and hence the slope. Typical plots of the t/δ versus t plot of four soils are shown from Fig. 6a to Fig.6d. The plot of ultimate compression or $\delta_f$ versus actual total compression from experiments of the twelve samples are shown in Fig.7. It is observed that comparatively good agreement is seen between $\delta_f$ and actual compression obtained after 24 hours of laboratory consolidation.

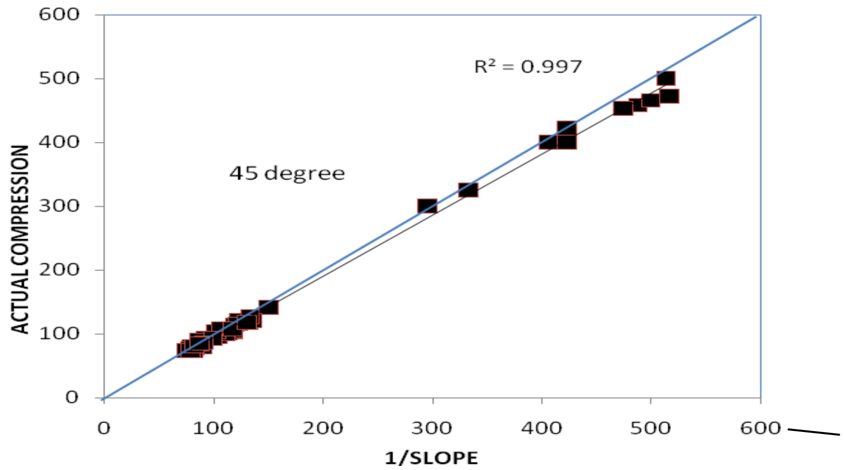


Figure.7 Plot of ultimate compression or $\delta_f$ versus actual total compression.

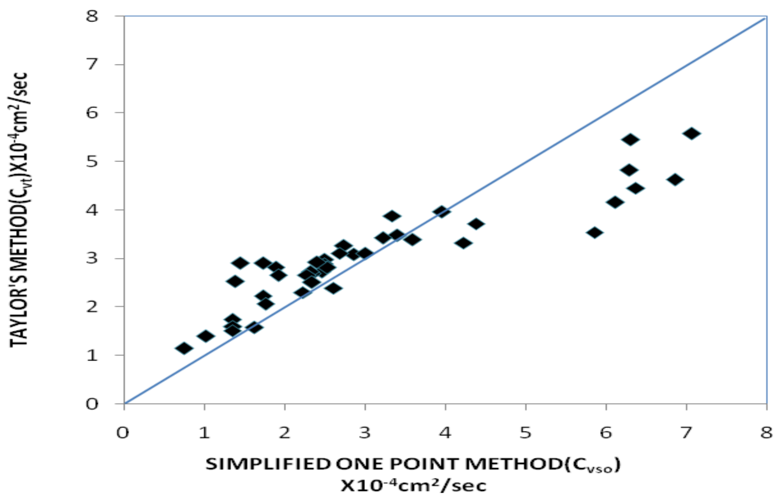


Figure.8 Comparison of $C_v$ between Taylor's method and the Simplified one point method.

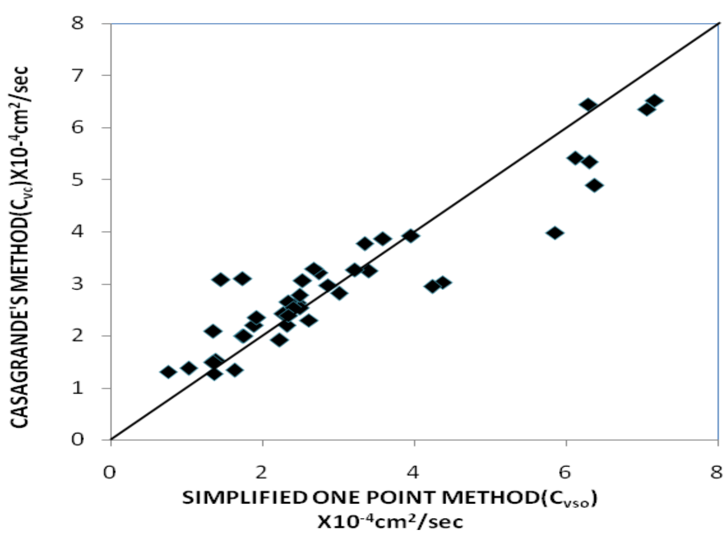


Figure.9 Comparison of $C_v$ between Casagrande's method and the Simplified one point method.

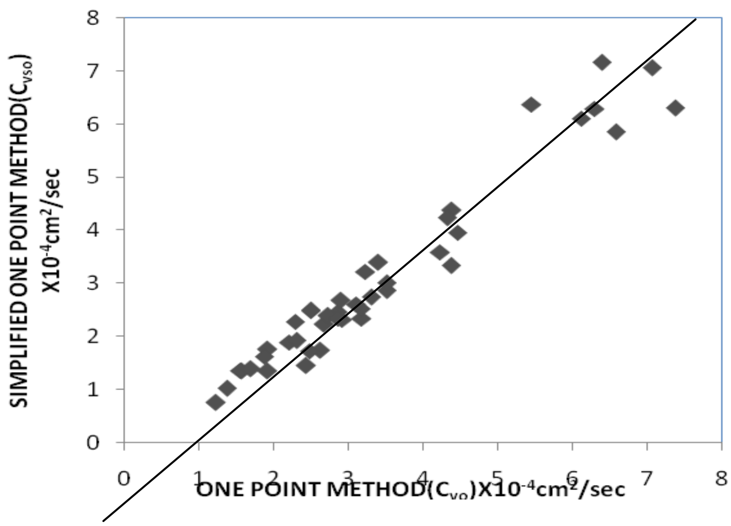


Figure.10 Comparison of $C_v$ between the One point method method and the Simplified one point method.

$C_v$ corresponding to $t_{50}$ obtained from the simplified one point method have been determined for the twelve soil samples. The relationship between these values of $C_v$ called $C_{vso}$ and $C_v$ determined by the Taylors ($C_{vt}$), and Casagrande ($C_{vc}$), methods are shown in Fig. 8 and Fig. 9. Similarly the relationship determined from the simplified one point method ($C_{vso}$) and the one point method is shown in Fig.10. The mean deviation between these two methods is 13.96%. In most of the cases there is good agreement between the methods. The mean deviations as shown in Table 2 are 20.66%, 15.52%, 18.67% and 13.27% with the Taylor method, Casagrande method, Bilinear method and the Rectangular hyperbola method respectively. The variations obtained comparing these various methods is in the same order of variations obtained between different well accepted methods of getting $C_v$. Sridharan and Prakash [18] had shown the mean deviation of $C_v$ between Taylor's method and Casagrande's method to be 49.1%. It is also reported in literature that the values obtained by Taylor's and Casagrande's methods can differ from each other very widely. The advantage with the proposed method is that one can take 8 or 10 readings of time versus settlement while doing the consolidation test to get a straight line in the plot between t/δ versus t. Getting the slope from the plot the total compression or $\delta_f$ can be determined. From $\delta_f$, 50% compression value is obtained and from it time $t_{50}$ and hence $C_v$ corresponding to $t_{50}$ can be determined.

Table- 2 Average % deviation

| Method to determine $C_v$ | Average % deviation with $C_v$ determined from $t_{50}$ of One point method | Average % deviation with $C_v$ determined from $t_{av}$ of One point method | Average % deviation with $C_v$ determined from $t_{50}$ from SOPM |
|---|---|---|---|
| Taylor's Method | 16.2 | 16.7 | 20.66 |
| Casagrande's Method | 14.2 | 15.1 | 15.52 |
| Rectangular Hyperbola Method | 10.7 | 11.6 | 13.27 |
| Pandian Bilinear Method | 15 | 17 | 18.67 |
| One Point Method ($t_{50}$) | | 10 | 13.96 |

While using the one point method or the other methods as discussed above, the entire range of time compression readings are to be taken. However using the simplified one point method, the coefficient of consolidation, $C_v$ can be obtained very quickly as $\delta_f$ can be obtained by knowing a few data points till the slope of the straight line portion of t/δ versus t is obtained. Time taken upto $t_{50}$ is very less.

Again considering the limitations and the assumptions involved in the curve fitting methods, the simplified one point method, which is devoid of any curve fitting procedure can be considered to be a quick, simple and acceptable method to determine the coefficient of consolidation.

## IV. CONCLUSIONS

Simplified one point method of obtaining the coefficient of consolidation has been proposed. The proposed method requires (i) Very few time versus deformation readings for a short time of about less than 30 minutes (ii) One can get the final deformation assuming a rectangular hyperbola behavior of time – deformation behavior. (iii) The coefficient of consolidation can be determined knowing time taken for 50% consolidation. Comparison of results with time consuming popular methods show the proposed method is very much valid. No assumptions are involved except that the time – deformation behavior could be treated as a rectangular hyperbola to obtain final deformation for any load increment.

## V. REFERENCE


[1] Gilboy,G. 1936. Improved soil testing methods. Engng News REc. 116: 732 – 734.

[2] Casagrande, A and Fadum, R.E. 1940. Notes on soil testing for Engineering purposes. Harvard Soil Mechanics, Series No.8, Cambridge,Mass.

[3] Naylor, A.H. and Doran, I.G. 1948. Precise determination of primary consolidation. Proc. 2nd Int. Conf. Soil Mech. Fdn. Engng, Rotterdam 1: 34 – 40.

[4] Su, H.L. 1958. Procedure for rapid consolidation test. Journal Soil Mech. Fdns Div. ASCE. 84(3): 1-13.

[5] Scott, R.F. 1961. New method of consolidation coefficient evaluation. Journal of Soil Mechanics and Foundation Division. Proceedings of ASCE, vol.87,No.SM1: 29-39.

[6] Madhav, M.R. 1964. Evaluation of coefficient of consolidation – a Numerical method. J Instn.Engrs.India 44,No.11, Part C16: 679-684.

[7] Rao, K.S.S. 1975. Curve fitting method in the analysis of consolidation. Proc.5th Asian Regional Conference, Soil Mech. Fdn.Engng, Bangalore.

[8] Sivaram, B. and Swamee, P.K. 1977. A computational method for consolidation coefficient. Soils Foundation.17,No.2 : 48-52.

[9] Parkin,A.K. 1978. Coefficient of consolidation by the velocity method. Geotechnique, Vol.28, No.4: 472 – 474.

[10] Taylor, D.W. 1948. *Fundamentals of Soil Mechanics*. John Wiley and Sons. Newyork.

[11] Sridharan, A and Sreepada Rao, A. 1981. Rectangular Hyperbola Fitting Method for one dimensional consolidation. Geotech testing journal.4, No.4: 161-168.

[12] Sridharan, A.; Murthy, N.S.and Prakash, K. 1987. Rectangular hyperbola method of consolidation analysis. Geotechnique, Vol.37, No.3. : 355-368.

[13] Pandian, N.S., Sridharan, A. and Kumar, K.S. 1992. A new method for the determination of coefficient of consolidation, Geotechnical Testing Journal, GTGODJ, Vol 15,No.1: 74 – 79.

[14] Robinson, R.G. and Allam,M.M. 1996. Determination of coefficient of consolidation from early stage of log t plot. Geotechnical testing journal, 19(3): 316 – 320.

[15] Mesri, G., Feng,T.W. and Shahien, M. 1999. Coefficient of consolidation by inflection point method. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 125(8):716-718.

[16] Tao- Wei Feng and Yi- Jiuan Lee. 2001. Coefficient of consolidation from the linear segment of the $t^{1/2}$ curve. Canadian Geotechnical Journal, 38: 901- 909.

[17] Sridharan, A. and Sreepada Rao. 1982. Rectangular hyperbola fitting method for one dimensional consolidation. Geotech. Test. J., 4(4): 161-168.

[18] Sridharan, A. and Prakash, K. 1998. Determination of coefficient of consolidation: a user friendly approach. Ground Engineering: 30-32.

[19] Bowles, J.E., 1996. Foundation Analysis and Design, fifth ed. McGraw-Hill Inc., New York.

[20] NAVFAC. (1985). “*Soil Mechanics*”, DM 7.1, Naval Facilities Engineering Command, Alexandria, VA, page 144.

[21] Terzaghi, K. and Peck, R.B. 1967. Soil mechanics in Engineering practice. John Wiley and sons, 729p.

[22] Lambe , T.W., and Whitman, R.V. 1969. *Soil Mechanics.* John Wiley and Sons, New York.