Boundary Layer Flow Of Incompressible Fluid past an Inclined Stretching Sheet and Heat Transfer

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Abstract-In the present paper, we derived the boundary layer flow of incompressible fluid over an inclined stretching sheet and heat transfer. We derived the flow boundary layer equation with body force. Defining stream function, the boundary layer equation has been transformed into two-point boundary value problem in an infinite domain. To see the dependence of velocity field on some physical parameters such as angle of inclination and Froude number, we solve flow boundary layer equation numerically using MATLAB. Skin friction has also been discussed. We also solved thermal boundary layer equation for temperature field. The coefficient of convective heat transfer, i.e. Nusselt number has also been obtained. In the end, the results have been discussed with the help of graphs or otherwise.
Key Words:Inclined stretching plate, boundary layer equation, skin friction, thermal boundary layer equation and Nusselt number.

I. INTRODUCTION

The boundary layer flow due to stretching flat plate is of practical interest in fibre technology and in extrusion processes. There are industrial applications too based on stretching sheet technology such as cooling of aninfinite metallic plate in a cooling bath, boundary layer along a.liquid film in condensation process and polymer yarn industry. Sakiadis[1] was the first who considered the boundary layer flow over continuously moving solid surface. Crane, L. J [2] extended the Sakiadis[1] concept to a stretching sheet and he obtained the closed form solution. There are researchers such as K. R. Rajagopal, T. Y. Na and A. S. Gupta [3] , B. S. Dhanpat and A. S. Gupta [4], N. Ahmad, G. S. Patel and B. Siddappa [5], N. Ahmad and K. Marwah[6], N. Ahmad, Z. U. Siddiqui and M. K. Mishra [7], Naseem Ahmad and Kamran Ahmad [8] who investigated the boundary layer flow over stretching plate in various variants with or without heat transfer. These scholars investigated the flow of different type of fluids over linear stretching plate. Apart from linear stretching plate, there are researchers who investigated the flow over non-linear stretching sheet. Some of these authors are K. Vajravelu[9], Javad Alinejad and SinaSamarbakhsh[10], K. Vajravelu and J. R. Cannon[11], KhairiZaimi, AnuarIshak and Loan Pop [12]. Also, there are few scholars who investigated the boundary layer flow over exponentially stretching plate.

We come across the examples where we notice the existence of the boundary layer flow over an inclined stretching plate. Some of the examples are forward motion of a car to climb the over- bridge, take off aeroplane and inclined conveyer belts. Here, our aim is to investigate the boundary layer flow of incompressible fluid over an inclined stretching plate and heat transfer.

II. FORMULATION OF THE PROBLEM

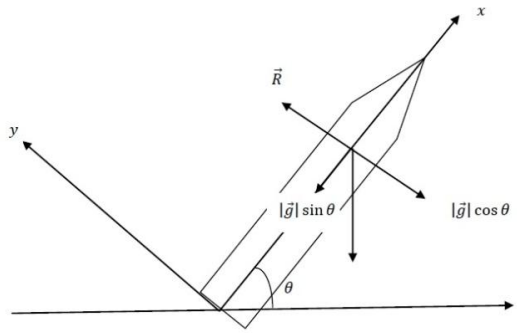

Figure 1: Inclined stretching plate

Refering the following figure, we have a stretching plate inclined to the horizontal base at an angle θ. We consider the geometry by assuming the x-axis along the stretching plate which makes an angle θ with the horizonatal and y-axis as perpendicular to x-axis. Due to inclination of stretching plate with horizontal base, there are two components of gravitational force $\vec{g}$. The y component of the gravitaional force cancelled with the force of reaction $\vec{R}$ while x component of gravitational force opposes the motion of the plate in the upward direction. Therefore the governing equations of the boundary layer flow of incompressible fluid are:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial x}=-|\vec{g}|\sin\theta+\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

The relevent boundary conditions are:

$$y=0,\ u=mx(=U_0), v=0 \tag{3}$$

$$y\to\infty, u=0 \tag{4}$$

$$f'''+ff''-f'^2=-\frac{1}{(F_{r_x})^2}\sin\theta \tag{5}$$

where $[m]=T^{-1}$.

We define dimensionless variable by

$\eta=\left(\frac{m}{\nu}\right)^{\frac{1}{2}}y$ and the stream function $\psi(x,y)=(m\nu)^{1/2}x\,f(\eta)$ so that $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ satisfy the Eq. (1).

With this definition of u and v, we have

$u=mxf'(\eta)$ and $v=-(m\nu)^{\frac{1}{2}}f(\eta)$. Using u and v in the Eq. (2), we have

where $F_{r_x}=\frac{U_0^2}{\sqrt{|\vec{g}|x}}$, the Froude number.

The relavent boundary conditions are:

$$\eta=0, f'=1, f=0 \tag{6}$$

$$\eta\to\infty, f'=0 \tag{7}$$

Here, Eq. (5), (6) and (7) constitute a non-linear boundary value problem of order three solved numerically by using MatLab.

Skin Friction

The wall shear stress at the stretching plate is given by

$$\tau_w=-\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}-\mu\left(\frac{\partial mxf(\eta)}{\partial\eta}\right)_{\eta=0}\left(\frac{m}{\nu}\right)^{\frac{1}{2}}=-\mu U_0\left(\frac{m}{\nu}\right)^{\frac{1}{2}} \tag{8}$$

Thus, the skin friction is

$$C_f=\frac{\tau_\omega}{\rho U_0^2 L}=-R_e^{-1}\left(\frac{m}{\nu}\right)^{\frac{1}{2}} \tag{9}$$

Heat Transfer Problem

The convective heat transfer between surrounding and the inclined sheet is governed byfollowing boundary value problem:

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2} \tag{10}$$

where ρ, C_p and K are density of fluid, specific heat at constant pressure and thermal conductivity, repectively.

The relevent boundary conditions are:

$$y=0,\quad T=T_p \tag{11}$$

$$y\to\infty, T=T_\infty \tag{12}$$

Observing the boundary conditions, we conclude that the temperature field T is independent of abcissa x, so the equation (10) reduces to

$$\rho C_p\left(v\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} \tag{13}$$

Defining dimensionless temperature by

$$\theta(\eta) = \frac{T-T_\infty}{T_p-T_\infty} \tag{14}$$

and using the expression for v and T, we have the following problem

$$\theta'' + P_r f(\eta)\theta' = 0 \tag{15}$$

with boundary conditions

$$\eta = 0, \quad \theta = 0 \tag{16}$$

$$\eta \to \infty, \quad \theta = 1 \tag{17}$$

The solution of the problem posed by the equations (16) and (17) is given as

$$\theta = \frac{\int_0^\eta \exp\left(P_r \int f(\eta)d\eta\right)d\eta}{\int_0^\infty \exp\left(P_r \int f(\eta)d\eta\right)d\eta} \tag{18}$$

Nusselt Number

The coefficient of heat transfer is calculated as

$$q_w = -K\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$= -K\left(T_p - T_\infty\right)\left(\frac{m}{\nu}\right)^{\frac{1}{2}} c \tag{19}$$

Therefore, the Nusselt number is

$$N_u = \frac{q_w}{k\left(T_p - T_\infty\right)}$$

$$= -\left(\frac{m}{\nu}\right)^{\frac{1}{2}} c \tag{20}$$

where c is constant.

III. DISCUSSION AND RESULTS

The problem "Boundary layer flow of incompressible fluid past an inclined stretching sheet and heat transfer", has been investigated thorowly. The governing Eqs. (5), (6) & (7) constitute a non-linear boundary value problem of order three in an infinite domain. Using the MatLab, the flow problem has been solved and the solution has been presented in the following graphs.

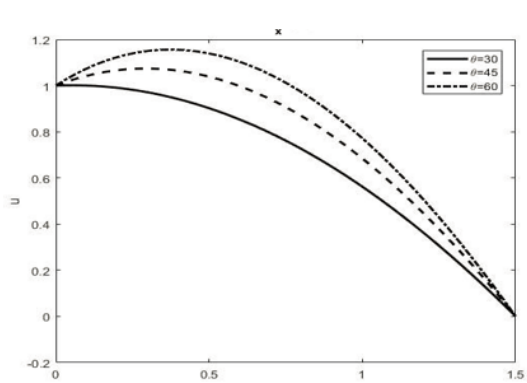

Figure 2: Velocity component u versus η when Froude number is 0.5

Figure 2 exhibits the dependence of longitudinal velocity component u on the angle of inclination θ for Froude number $Fr_x = 0.5$. We observe that as angle of inclination θ increases, the value velocity component u increases within the dynamic domain $[0,1.5]$. The technical cause of this increasing trend of u with increase in θ is to increase the stretching velocity $U_0 = mx$ so that the downward pull $|\vec{g}|sin\theta$ may be overcome to create the boundary layer.

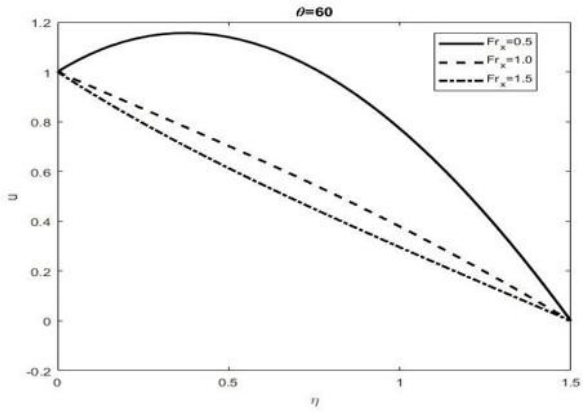

Figure 3: Velocity component u versus η for different F_{r_x} when $\theta = \frac{\pi}{3}$

Figure 3 depicts the behaviour of longitudinal component u for the increasing trend of Froude number F_{r_x} at angle of inclination $\theta = \frac{\pi}{3}$. We observe that as Froude number increases, the velocity component decreases within the dynamic region $[0,1.5]$. Here, low Froude number means low downward pull. Hence u increases.

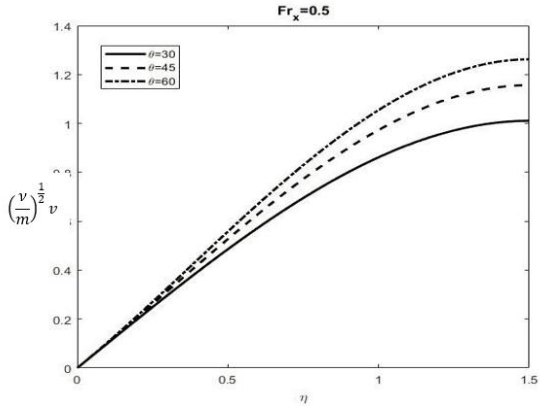

Figure 4. Vertical velocity component v when Froude number 0.5 and angle of inclination varies

Figure 4 shows the behaviour of vertical velocity component v within dynamic domain for a particular value of Froude number F_{r_x} as the angle of inclination varying. As the angle of inclination of stretching sheet increases, the magnitude of v is increasing. This is due to increase in negative pull as θ decreases.

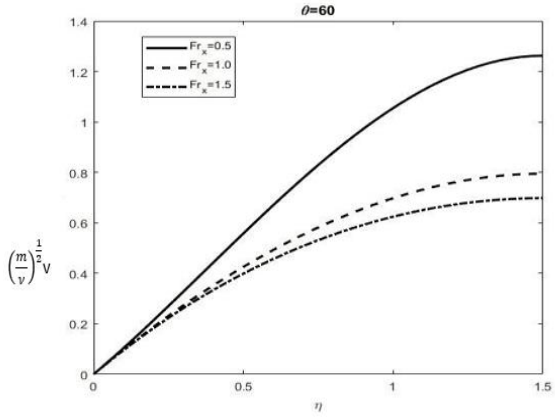

Figure 5 vertical velocity component v
whenFroude number varies and angle of

inclination is $\frac{\pi}{3}$.

Figure 5 shows the behaviour of vertical velocity component v within dynamic domain for a particular value of angle of inclination θ when the Froude number Fr_x varies. As the Froude number Fr_x increases, the magnitude of v is decreasesThe skin friction formula has been obtained as $C_f = -R_e^{-1}\left(\frac{m}{\nu}\right)^{\frac{1}{2}}$. Here $C_f \, \alpha \frac{1}{R_e}$ and the negative sign represents that the skin friction grows opposite to direction of stretching plate movement.

The formula for convective heat transfer has been obtained as $\theta = \frac{\int_0^{\eta} \exp\left(P_r \int f(\eta) d\eta\right) d\eta}{\int_0^{\infty} exp\left(P_r \int f(\eta) d\eta\right) d\eta}$. Here, the function $f(\eta)$ has not been evaluated, hence one has to be tabulated employing the method of interpolation using the boundary conditions.

The formula for the Nusselt number has been obtained. Here, $N_u \alpha \left(\frac{m}{\nu}\right)^{\frac{1}{2}}$. In the formula of Nusselt number negative sign shows that flow of heat if from external edge of boundary layer to stretching plate.

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