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Transfer length formula and optimum strand stress curve in precast pre-stressed element transmission zone

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Abstract-The beams, slabs and floor plates, precast pre-stressed, ensure a swift execution of works. Their design by means of standards requires the determination of the transfer length. The calculated values vary from one standard to another, and differ from tests results. Besides, they depend neither on the coating nor the spacing of pre-stressed strands. Based on a theoretical model which has been experimentally validated and that describes physical phenomena, this paper uses the theory of experimental designs to perform a model reduction. It's suggesting, for standards a formula for the calculation of the transfer length depending on the set of parameters. Moreover, it allows measuring the influence of each of them on the transfer length. Another optimum curve of the tendon stress depending on the length is proposed.
Key words: Prestressing, strands, transfer length, coating, design formula, experimental designs, optimum curve.

I. INTRODUCTION

According to standards, the transfer length can be used for obtaining the evolution of the pre-stress load applied to the beam in accordance with the distance to the beam end. However, a dispersion of the transfer length is observed between the European standard and the American standards, then standards with the experimental values (see table 1). According to AASHTO LRFD [1] it totally depends on the diameter of the strand. As for ACI 318[2] it depends on the diameter of the strand and the final prestressed load. The NF EN 1992-1[3] takes into account the diameter of the strand, its stress immediately after relaxation and the concrete tensile strength. The coating and spacing of the strands are not considered in the standards. The influence of the cracking of the concrete around the strand is therefore not properly considered.

The determination of the transfer length, depending on all parameters, can be done by means of a short and smart tests list following an experimental design. Yet, the tests performed at scale 1 do not follow an experimental design. A theoretical model, experimentally validated, allows addressing this blemish. Digital tests are performed instead. The formula of the transfer length obtained is validated with regard to the theoretical model and tests performed at scale 1. The coating to be considered in the case of multiple strands is determined. Likewise, a simplified (piecewise-linear) and optimum evolution of the strand stress in accordance with the distance to the beam end, is proposed.

Tableau 1 : calculation of the transfer length according to standards

Φs = 12.7 mm 2.7 mm ; fcj=35 MPa ; C0=40 mm ; σpi=1392 MPa				
	Tests	NF EN 1992-1	ACI	AASHTO
lpt (mm)	669	840	700	762
Φs = 12.7 mm ; fcj=34 MPa ; C0=50 mm ; a = 3 Φs σpi=1398 MPa				
lpt (mm)	808	852	703	762

Φs, C0, a, fcj, σpi, lpt respectively refer to the diameter of the strand, the coating of the strand, the spacing of the strands, the compressive strength of the concrete while the strand is being relaxed, the strand stress immediately after relaxation and the transfer length.

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II. FACTORS WHICH DEPEND ON THE TRANSFER LENGTH

The precast prestressing consists in stretching a tendon by means of actuators, in casting concrete in a mould around the strand and once the concrete is hardened, the tendon can be cut at its endpoints. Once cut, the strand tends to revert to its untensioned state, but the hardened concrete prevents this move happening. Then occurs a shortening of the strand and the concrete. The radial pressure at the strand-concrete interface (as a result of the shortening of the strand and the swelling of its section by Poisson effect) through the strand-concrete friction, limits the penetration of the strand in its concrete jacket, therefore its sliding. The radial pressure at the interface gives rise to a radial compressive stress in the concrete which is concomitant to a circumferential tensile stress. Over a given radius, the circumferential stress is higher than the tensile strength of the concrete and the it cracks. This cracking entails a relaxation of the pressure at the interface and an additional sliding of the strand. The prestressing force is entirely and adhesively transmitted to the concrete at a given distance called transfer length. The more the strand slides into its concrete jacket, the more the transfer length increases.[4] the tension of the strand, its diameter, the coating, the strand spacing and the compressive strength (therefore the tensile strength) are factors on which depends the transfer length.

III. DESCRIPTION OF TESTS

Kim and Oh [5] have performed tests on 36 precast prestressed beams, of which 24 are single-stranded. (Single Strand. See Fig. 1) and 12 with two strands (double strands. See Fig. 1). They are 200 mm high and 3000 mm long. The strands vary between T12.7 and T15.2. The strands are tensed at 0.75 fpu, fpu being the y stress to the breaking. The target resistance figures to the concrete compression strength, at relaxation of the tendon are 35 MPa and 45 MPa. The horizontal coating is 50mm. For the single strands, the vertical coatings are 30, 40 and 50 mm. For the double strands, the vertical coating is 50 mm and the strands spacing is 3 diameters, 4 diameters, 5 diameters.

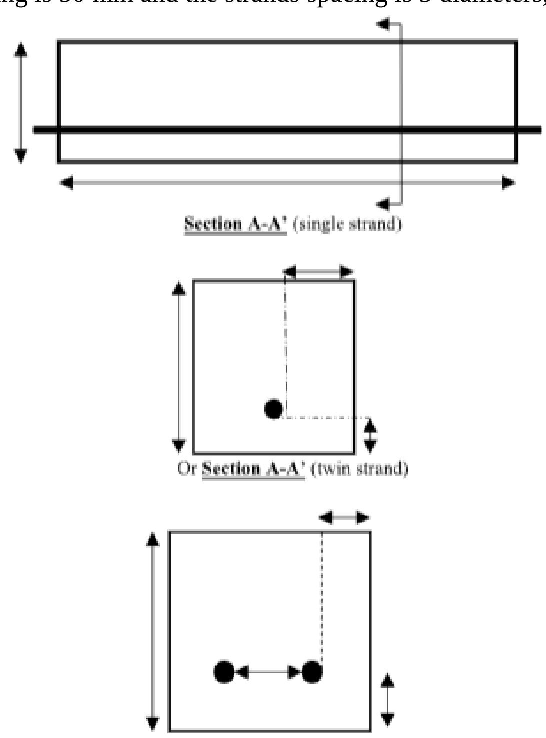


Figure. 1- Precast pre-stressed beam (single or twin strand)

Mechanical gauges have been placed on the concrete at every 50 mm alongside the strand. The precision of the gauges is satisfactory. In order to reduce room for error, 2 persons have to read the indicated displacement value. In theory, the transfer length is the length from which the concrete strain is constant. In reality, the concrete constant strain zone corresponds to a sawtooth zone where it does not vary much. The transfer length is conventionally taken as equal to the one for which the strain reaches to 95% of the average value of the maximal strain zone.

IV. DESCRIPTION OF THEORETICAL MODEL

4.1 Presentation of the model

The radial pressure at the strand-concrete interface develops radial compressive stress concomitant to circumferential tensile stress. When the compressive stress is higher than fctm the concrete cracks. At a given radius, the radial compressive stress disappears. So, here is a localized radial cracking, the study of which does not require the whole height of the beam. So, it's carried out on a strand-concrete biphasic cylinder while remaining compatible with the kinematic of the beam. (See fig. 2). Indeed, the axial stress of the biphasic cylinder concrete is constant and corresponds to the average stress of the section of the beam on the height of the cylinder.

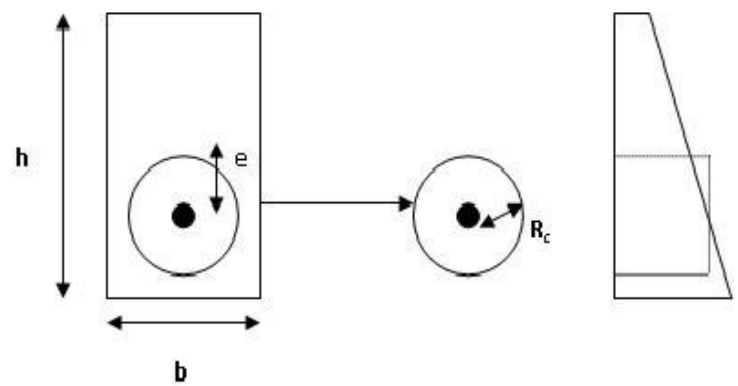


Figure 2 - contrainte axiale du béton du tirant

h, b and e are respectively the height, the width of the beam and the eccentricity of the strand compared with the average fiber.

On the biphasic cylinder, we have in cylindrical coordinates:

$$\sigma_{rr}^{c}\left(R_c, z\right) = 0 \quad \forall z \tag{1}$$

For a single strand:

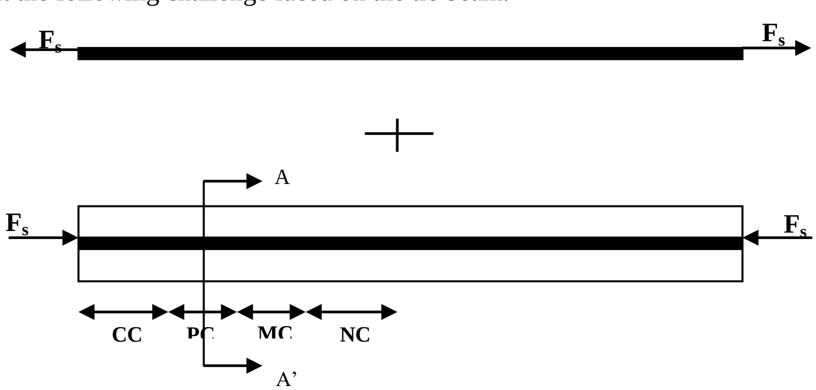
$$R_c = \min\left(C_V, C_h\right) + R_s \tag{2}$$

Pour a multiple strands :

$$R_c = \frac{2 \times \min\left(C_V, C_h\right) + \left(n_s - 1\right) \times \xi a}{2 \times n_s} + R_s \tag{3}$$

ns, a, Rs, CV, Ch, ζ, Rc are respectively the number of strands, the spacing of strands, the radius of the stretched strand, the vertical coating, the horizontal coating, an arbitrary value parameter 1.5 [] that's supposed to consider the impact of adjacent strands on the cracking and the outer radius of the tie beam where the radial compressive stresses cancel each other out.

We are to puzzle out the following challenge faced on the tie beam:



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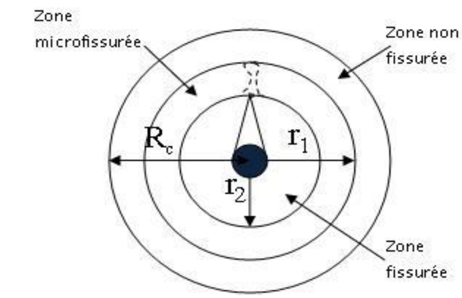
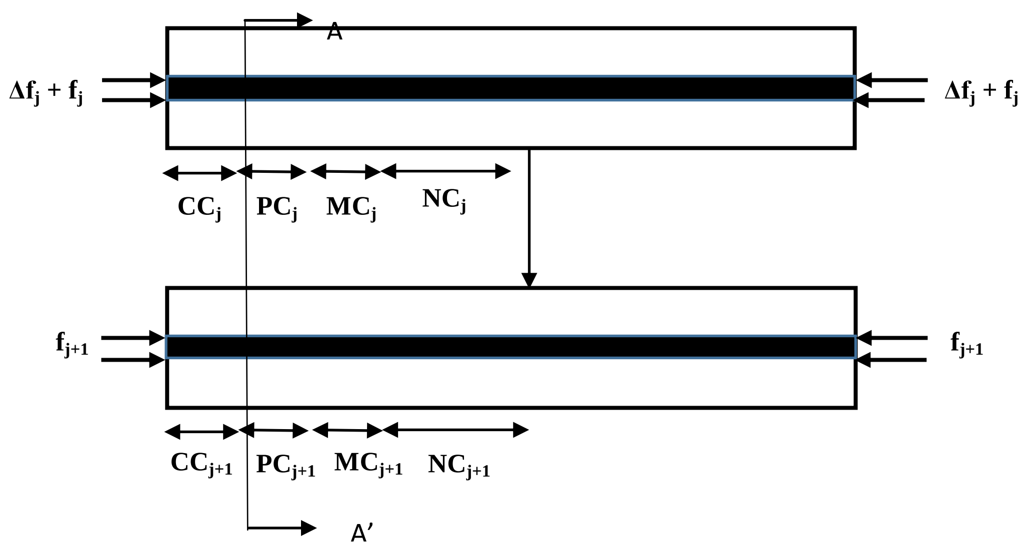


Figure 3- Mechanical problem on the tie beam

Along the tie beam are completely cracked zones (CC), partially cracked zones (PC), micro cracked zones (MC) and non-cracked zones (NC). r2 and r1 respectively refer to the tips of the crack and micro crack.
The shearing at the strand-concrete interface is spelled out as follows in cylindrical coordinates:

$$\tau_{zr}^{c}\left(R_S,z\right)=C+\mu\sigma_{rr}^{c}\left(R_S,z\right)=C+\mu P\left(z\right) \quad (4)$$

where C is the cohesion of the model (pseudo cohesion). $P\left(z\right)=\sigma_{rr}^{c}\left(R_S,z\right)$ and μ are respectively the radial pressure and the friction coefficient at the strand-concrete interface.
The 2010 model code [6] provides constitutive laws that allow considering the micro cracking and the cracking of the concrete. The plastic strains as a result of micro cracking and cracking are taken into account to come up with linear differential equations with non-constant coefficients. The solving of these equations cannot be a closed-form solution. An iterative solving method using the Runge-Kutta Fourth Order Method has been developed (See Fig. 4).
To the prestressed beam in its cracking state under the fj load, is applied a load increment Δfj corresponding to the strand-concrete interface, to a ΔPj increment of elastic radial pressure. This leads to another cracking state and a new radial pressure at the strand-concrete interface[7].



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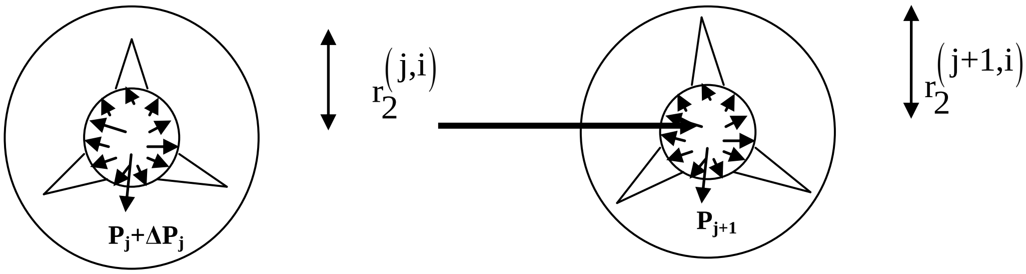

Figure 4- Diagram of solving the mechanical problem

4.2 Experimental validation of model

The table below shows the transfer length obtained by means of elastic calculation lpte and the transfer length obtained by means of elastoplastic calculation (being the subject matter of the preceding iterative method) lpt of various tested samples. The parameters on which depends the transfer length are specified for each test body. M for single strand, 12 for T12.7 and CX for X cm coating.

Tableau 2-Transfer length according with model and experimentation

specimen	fcj (MPa)	σpi(MPa)	lpte(mm)	Test [5]lpt(mm)	Tigri [7] lpt (mm)	Kim [8] lpt (mm)	Ec2 [3] lpt (mm)
M12NC3	33.6	1402.1	780	851	800	739	851
M12NC4	35.0	1391.9	770	669	710	596	836
M12NC5	33.6	1402.7	770	589	630	574	871
M12HC3	44.7	1359.3	730	692	640	653	690
M12HC4	46.3	1375.1	720	568	570	524	687
M12HC5	44.7	1394.7	720	513	560	516	718
M15NC3	35	1377.1	920	1084	1010	955	960
M15NC4	33.6	1392.5	920	839	870	703	1010
M15NC5	35	1393.2	910	698	780	624	994
M15HC3	46.4	1357.5	860	888	850	854	790
M15HC4	44.7	1364.9	860	722	740	645	823
M15HC5	45.6	1384.4	860	574	685	567	831

V. DIGITAL TESTS ACCORDING WITH EXPERIMENTAL DESIGN

5.1 Fields of factors

For digital tests, the factors on which depends the transfer length vary in accordance with the study domain.

Strands diameter: $[\min\emptyset \quad \max\emptyset] = [12{,}7\text{ mm} \quad 15{,}2\text{ mm}]$.

Strands coating: $[\min C \quad \max C] = [30\text{ mm} \quad 50\text{ mm}]$.

Compressive strength of concrete: $[\min R \quad \max R] = [32{,}5\text{ MPa} \quad 47{,}2\text{ MPa}]$.

Tension of strand: During the Kim and Oh tests, the tension corresponded to a 0.75 fpu. Stress. In practice, we also have cases of de 085 fpu. Stress. So, the field study is: $[\min\sigma \quad \max 0{,}85f_{pu}] = [1357{,}4\text{ MPa} \quad 1569{,}89\text{ MPa}]$.

5.2 The complete and reduced factorials design

The complete factorial design comprises 24 tests (See table 3). The 8 tests with a 40 mm coating are the central points[9]. The other 16 tests are the reduced factorial design. The central points will allow validating the Minitab model against the theoretical model. The classification is based on the one proposed by Kim and Oh. f1 = 1357.4 MPa and f2 = 1569.89MPa, N for normal compressive strength (35 MPa targeted), H for high compressive strength (45 MPa targeted), C5for 50 mm coating.

Tableau 3- Digital tests according with complete factorial design

Spécimen	diameter(mm)	strength(MPa)	coating(mm)	Tension(MPa)

M15NC4f2	15,2	32,5	40	1569,89
M15HC4f1	15,2	47,2	40	1357,4
M15HC3f2	15,2	47,2	30	1569,89
M15NC4f1	15,2	32,5	40	1357,4
M12HC4f2	12,7	47,2	40	1569,89
M12HC4f1	12,7	47,2	40	1357,4
M15NC3f1	15,2	32,5	30	1357,4
M15HC3f1	15,2	47,2	30	1357,4
M12NC3f1	12,7	32,5	30	1357,4
M12HC3f2	12,7	47,2	30	1569,89
M15HC4f2	15,2	47,2	40	1569,89
M12NC4f2	12,7	32,5	40	1569,89
M12HC5f2	12,7	47,2	50	1569,89
M12HC3f1	12,7	47,2	30	1357,4
M12NC5f1	12,7	32,5	50	1357,4
M12NC4f1	12,7	32,5	40	1357,4
M15NC5f2	15,2	32,5	50	1569,89
M12NC3f2	12,7	32,5	30	1569,89
M15HC5f1	15,2	47,2	50	1357,4
M15NC3f2	15,2	32,5	30	1569,89
M12NC5f2	12,7	32,5	50	1569,89
M15HC5f2	15,2	47,2	50	1569,89
M15NC5f1	15,2	32,5	50	1357,4
M12HC5f1	12,7	47,2	50	1357,4

VI. THE SCALE MODEL OF TRANSFER LENGTH

6.1 Validation of scale model with respect to theoretical model

the Pareto chart on impacts reveals that the most significant impacts are those of the diameter, coating, tension and interaction [diameter and coating]. If the scale model is strictly limited to these impacts, then we have the following table:

Tableau 4- effects of factors taking into account the Pareto diagram

Facteurs et interactions	impacts
constant	781,3
Diameter (D)	108,7
Strength (R)	-72,5
Coating (E)	-93,75
Tension (T)	46,25
diameter*coating (D*E)	-31,25

The factors and their interactions are expressed in standardized values and they multiply their respective coefficients (impacts) to get the value of the transfer length.

Let A be a factor with the following field of study $[\min A \max A]$. The A standardized value is got by means of this formula: $x = \frac{A-A_0}{T}$ with $T = \frac{\max A - \min A}{2}$ and $A_0 = \frac{\max A + \min A}{2}$. Besides, knowing the Unicode x, we would have $A = x * T + A_0$ to get the original value.

For example

specimen	diameter (mm)	strength (MPa)	coating (mm)	Strand stress (MPa)	Standardized values D	R	E	T
M15NC4f2	15,2	32,5	40	1569,9	1	-1	0	1

Tableau 5-Standardized values of beam M15NC4f2

The calculation of beam M15NC4f2 transfer length is obtained in the following way:

$$L_{-redu} = 781{,}3 + 108{,}7 \times 1 - 72{,}5 \times (-1) - 93{,}75 \times 0 + 46{,}25 \times 1 \ - 31{,}25 \times (1) \times (0) = 1009 \text{ mm}$$

The validation of the scale model against the theoretical model is achieved by means of central points, i.e. those with a 40 mm coating. It's all about confirming the fidelity of the scale model against the theoretical model. The average error value is 5.2%. The error is calculated as follows: ERROR = $\left|\frac{L_{redu} - L\_theo}{L\_theo}\right|$. It is concluded that the reduction of the model is validated against the theoretical model.

6.2 Impacts analyse

While observing the multiplying factors, it clearly appears that the transfer length increases in a significant way with the increment of the strand diameter. The increment of the strand diameter entails an increment of swelling, hence increment of radial and orthoradial stress in the concrete which is likely to get cracked. The cracking of the concrete increases the sliding of the strand into its concrete jacket, hence the transfer length. The increment of the strand diameter has a higher impact on the increment of the transfer length than the increment of the relaxed prestressed load. In order to reduce the transfer length, it is more efficient to increase the coating rather than increasing the compressive strength of concrete.

The most significant interactions are those existing between [diameter and coating] and [resistance and coating]. This is not surprising. Indeed, the cracking of the concrete due to the swelling of the strand, according to the thickness of the coating, can partially or completely crack the concrete. The more the concrete is cracked, the more the strand slides into its concrete jacket and the more the transfer length increases.

6.3 Transfer length formula

In the case of Kim and Oh's singled-stranded beams, the transfer length has been calculated by means of scale model before being compared with tests. For beams with a 3 cm coating, the transfer length obtained by means of scale model is poor, about ten cm compared with the experimental value. If one decides to adjust these values by multiplying them by 1.1, the average error after adjustment is 7.1 %. The ACI 318, AASHTO and NFEN1992-1 average error is respectively 30, 24,3 and 24,4%.The scale model is well validated through experience in the case of single strands.

In the case of multiple strands, the cracking of the concrete around a strand is hampered by the adjacent strand. The possible cracking zone is more difficult to determine (See Fig. 5). Let's propose an inverse analysis based on the tests and scale model of the transfer length in order to get the outer radius of the tie beam.

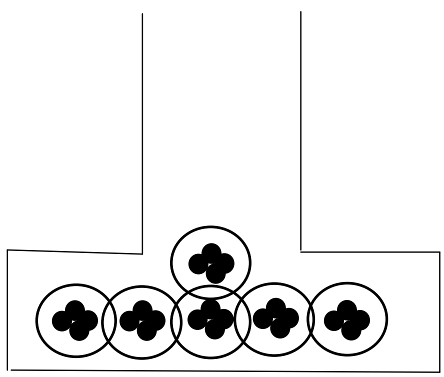

Figure 5-cracked zones of multiple strand

For example :

Tableau6- Standardized values of beam T12NS4

Specimen	diameter	Strength	spacing	tension	Standardized values			
T12NS4	Φ = 12,7	35,5	4Φ	1418	D	R	E	T
					-1	-0,59	?	-0,43

$L = 781{,}3 + 108{,}7D - 72{,}5R - 93{,}75E + 46{,}25T - 31{,}25DE$. Connaissant la valeur expérimentale de L, nous avons :

$$E_{redu} = \frac{L_{exp} - 781{,}3 - 108{,}7D + 72{,}5R - 46{,}25T}{-93{,}75 - 31{,}25D}$$

We obtain the standardized value of coating. The real value of coating is :

$E_{reel} = 40 + 10 * E_{redu}$ then $\alpha = \frac{E_{reel}}{\min(C_V; C_h; \min(S))} = 0{,}86$.

By proceeding the same way for all the tests with double strands, we get the average of the coefficient α which is 0.87. Let's also suggest a coating formula likely to allow calculating the transfer length by means of a scale model. $E_{reel} = 0{,}87\min(C_v; C_h; \min(S))$.

The average value of error for a theoretical model is 6.6%. The average value for a scale model is 8.9 %. The ACI 318, AASHTO and NFEN1992-1 average error is respectively 28,8, 19,6 and 18,1%.

We note from the previous results that:

Si $E > 3D$ $L_{redu} = 781{,}3 + 108{,}7D - 72{,}5R - 93{,}75E - 31{,}25DE$ (mm)

Si $E \leq 3D$ $L_{redu} = 859{,}4 + 119{,}6D - 79{,}8R - 103{,}1E - 34{,}4DE$ (mm)

VII. OPTIMUM CURVE OF STRAND STRESS DEPENDIND ON THE LENGTH

challenge is to propose a piecewise-linear curve, approximating to the one developed by the theoretical model and optimal against the one proposed by the calculation codes. We use the following points with this ordinates (See table 7) to approximate to the curve produced by the theoretical model.

	A1	A2	A3	A4	A5	A6
y	0	$\frac{\sigma_{pm0}}{4}$	$\frac{\sigma_{pm0}}{2}$	$\frac{3\sigma_{pm0}}{4}$	σ_{LT}	σ_{pm0}

Tableau7- optimum curve ordinates

The abscissa of point A1 is 1 cm. For points Ai $i \geq 2$, they belong to the straight line generated by the two points of the theoretical curve witch ordinates are near Ai ordinate.

By applying this principle to the 16 digital tests, we get points Ai for each curve. By means of Minitab software, we get the following impacts table:

Tableau8- Effets des différents points de la courbe optimale

factors	A1		A2		A3		A4		A5		A6	
Constant	0	0	23	348	34	695	49,1	1054	78,13	1316	150	1390
D	0	0	6,2	-1,9	6,3	-3,73	6,74	5,63	10,87	-10,5	0	-7,5
R	0	0	-2,5	-0,2	-3	-0,37	-3,4	-11,8	-7,25	0,358	0	-0,7
E	0	0	-9,3	4,66	-8	9,31	-7,2	2,74	-9,38	11,83	0	18,6
T	0	0	4,2	26,3	3,2	52,6	5,54	67,7	4,625	98,65	0	105
DE	0	0	-3,2	2,03	-3	4,06	-1,2	-5,14	-3,13	2,303	0	8,12

For each beam, the determination of the standardized values for D, R, E and T allows determining points Ai of the optimum curve by multiplying them by their respective impacts.

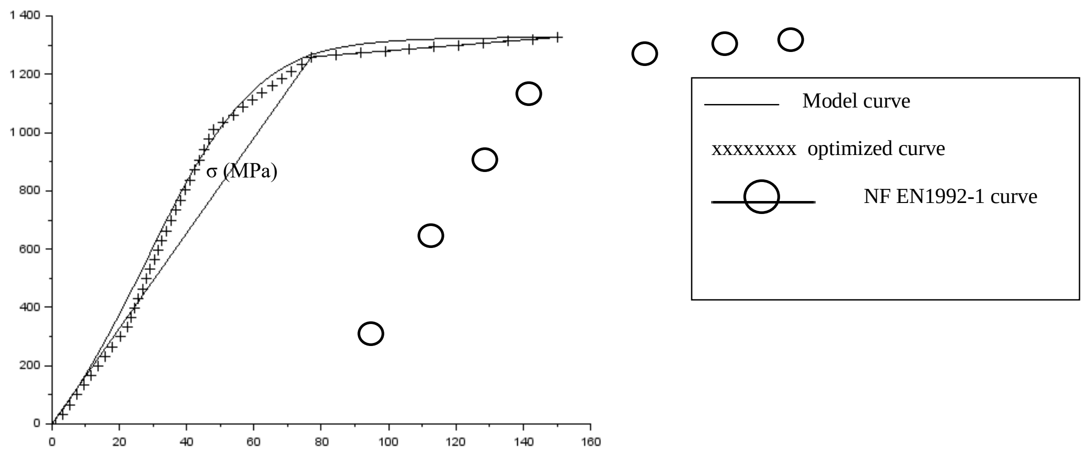

Figure 6-The optimum curve of beam M12NC3

VIII. CONCLUSION

To find a simple and robust model that can help predict the transfer length and the distribution of stresses, is not just a security matter but also an economic issue. To have a simple calculation formula for the transfer length thanks to a theory of experimental designs is an asset. By using Excel software, Engineers can determine an optimum evolution of the axial stress of the strand, depending on the length. These elements are helping tools for decision making by Engineers and Industrialists.

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