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Thermal Stresses of a Hollow Cylinder with Internal Heat Source: Direct Problem

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Abstract- The aim of this paper is to study thermal stresses of a circular cylinder, in which boundary conditions are of radiation type. We apply integral transform techniques and obtained the solution of the problem. Numerical calculations are carried out for a particular case and results are depicted graphically
Keywords – Thermoelastic response, Circular cylinder, integral transform, thermal stress

I. INTRODUCTION

Deshmukh et al. [1] have discussed quasi – static thermal stresses in a thick circular plate. Khobragade et al. [2] have studied transient thermoelastic problem for a circular solid cylinder with radiation. Nowacki [3] has discussed the state of stress in a thick circular plate due to temperature field. Roy Choudhary [6] have studied quasi – static thermal deflection of a thin clamped circular plate due to ramp type heating of a concentric circular region of the upper face. Wankhede [7] has discussed the quasi – static thermal stresses in a circular plate. Pakade et al. [8] have discussed transient thermoelastic problem of semi-Infinite circular beam with internal heat sources. Lamba et al [9] have studied integral transform methods for inverse problem of heat conduction with known boundary of a thin rectangular object and its stresses.

This paper is concerned with the transient thermoelastic problem of a hollow cylinder in which sources are generated according to the linear function of temperature, occupying the space $a \le r \le b$, $-h \le z \le h$ with radiation type boundary conditions.

II. STATEMENT OF THE PROBLEM-I

Consider a hollow cylinder of length 2h in which sources are generated according to linear function of temperature. The material is isotropic, homogeneous and all properties are assumed to be constant. The equation for heat conduction is [4]:

$$k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\chi(r,z,t)=\frac{\partial T}{\partial t} \tag{2.1}$$

where k is the thermal diffusivity of the material of the cylinder (which is assumed to be constant). Subject to the initial and boundary condition

$$M_t(T,1,0,0)=0 \tag{2.2}$$

$$M_r(T,1,k_1,a)=0 \text{ for all } -h \le z \le h,\ t>0 \tag{2.3}$$

$$M_r(T,1,k_2,b)=0 \text{ for all } -h \le z \le h,\ t>0 \tag{2.4}$$

$$M_z(T,1,k_3,h)=f(r,t) \tag{2.5}$$

$$M_z(T,1,k_4,-h)=g(r,t) \text{, for all } a \le r \le b,\ t>0 \tag{2.6}$$

The most general expression for these conditions can be given by

$$M_v(f,\bar{k},\bar{\bar{k}},\$)=(\bar{k}f+\bar{\bar{k}}\hat{f})_{v=\$}$$

where the prime $(\wedge)$ denotes differentiation with respect to $v:\delta(r-r_0)$ are the Dirac Delta functions having $a \le r_0 \le b$; $u(t)\ \delta(r-r_0)$ is the additional sectional heat available on its surface at $z=h$ and $\bar{k}$, $\bar{\bar{k}}$ are radiation constants on the upper and lower surface of cylinder respectively.

The radiation and axial displacement U and W satisfy the uncoupled thermoelastic equation as [3] are

$$\nabla^2 U - \frac{U}{r^2} + (1-2v)^{-1}\frac{\partial e}{\partial r} = 2\left(\frac{1+v}{1-2v}\right)\frac{\partial T}{\partial r} \tag{2.7}$$

$$\nabla^2 W + (1+2v)^{-1}\frac{\partial e}{\partial z} = 2\left(\frac{1+v}{1-v}\right)\alpha_t\frac{\partial T}{\partial z} \tag{2.8}$$

where

$e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilatation,

$$U = \frac{\partial \phi}{\partial r} \tag{2.9}$$

$$W = \frac{\partial \phi}{\partial z} \tag{2.10}$$

The thermoelastic displacement function $\phi(r,z,t)$ as [3] is governed by the Poisson's equation

$$\nabla^2\phi = \left(\frac{1+v}{1-v}\right)\alpha_t T \tag{2.11}$$

with $\phi = 0$ at $r = a$ and $r = b$. (2.12)

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$,

v and α_t are poisons ratio and the linear coefficient of thermal expansion of the material of the cylinder respectively.

The stress functions are given by

$$\tau_{rz}(a,z,t) = 0,\ \tau_{rz}(b,z,t) = 0,\ \tau_{rz}(r,0,t) = 0 \tag{2.13}$$

$$\sigma_r(a,z,t) = p_1,\ \sigma_r(b,z,t) = -p_0\ \sigma_z(r,0,t) = 0 \tag{2.14}$$

where p_1 and p_0 are the surface pressure assumed to be uniform over the boundaries of the cylinder.

The stress functions are expressed in terms of displacement components by the relations [99]:

$$\sigma_r = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left(\frac{U}{r} + \frac{\partial W}{\partial z}\right) \tag{2.15}$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) \tag{2.16}$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left(\frac{\partial W}{\partial z} + \frac{\partial U}{\partial r}\right) \tag{2.17}$$

$$\tau_{rz} = G\left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right) \tag{2.18}$$

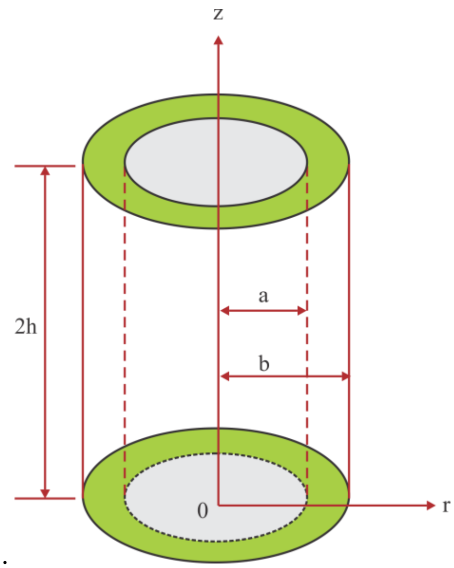


Figure shows the geometry of the problem

where $\lambda = \dfrac{2Gv}{1-2v}$ is the Lame's constant, G is the shear modulus and U and W are the displacement components.

Equations (2.1) to (2.18) constitute the mathematical formulation of the problem under consideration

III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Zgrablich transform [11] to the equations (2.5), (2.6) and (2.9) and using equation (2.7) and (2.8), one obtains

$$k\left[-\mu_n^2\overline{T}(n,z,t)+\frac{k\,\partial^2\overline{T}(n,z,t)}{\partial z^2}\right]+\overline{\chi}=\frac{\partial\overline{T}(n,z,t)}{\partial t} \tag{3.1}$$

$$M_t(\overline{T},1,0,0)=0 \tag{3.2}$$

$$M_z(\overline{T},1,k_3,h)=\overline{f}\,(n,t) \tag{3.3}$$

$$M_z(\overline{T},1,k_4,-h)=\overline{g}(n,t) \tag{3.4}$$

where $\overline{T}$ is the transformed function of T and n is the transformed parameter.

The eigen values μ_n are the positive roots of the characteristic equation

$$J_0(k_1,\mu a)\,Y_0(k_2,\mu b)-J_0(k_2,\mu b)\,Y_0(k_1,\mu a)=0$$

Further applying finite Marchi-Fasulo transform [2] to the equation (3.1) and using (3.3) and (3.4), one obtains

$$k\left[-(\mu_n^2+\xi_m^2)\,\overline{T}^*(n,m,t)+\left[\frac{P_m(h)\overline{f}}{k_3}-\frac{P_m(-h)\overline{g}}{k_4}\right]+\overline{\chi}^*\right]=\frac{d\overline{T}^*(n,m,t)}{dt} \tag{3.5}$$

$$M_t(\overline{T}^*,1,0,0)=0 \tag{3.6}$$

where $\overline{T}^*$ is the transformed function of $\overline{T}$ and m is the transformed parameter. The symbol (*) means a function in the transform domain and the nucleus is given by the orthogonal functions in the interval $-h\le z\le h$ as

$$P_m(z)=Q_m\cos(\xi_m z)-W_m\sin(\xi_m z)$$

In which

$$Q_m = \xi_m (k_3 + k_4)\cos(\xi_m h)$$
$$W_m = 2\cos(\xi_m h) + (k_3 - k_4)\,\xi_m \sin(\xi_m h)$$
$$\lambda_m = \int_{-h}^{h} p_m^2(z)\,dz = h[Q_m^2 + W_m^2] + \sin\frac{(2\xi_m h)}{2\xi_m}[Q_m{}^2 - W_m^2]$$

The eigen values ξ_m are the positive roots of the characteristic equation

$$[k_3 a\cos(ah) + \sin(ah)][\cos(ah) + k_4 a\sin(ah)]$$
$$= [k_4 a\cos(ah) - \sin(ah)][\cos(ah) - k_3 a\sin(ah)]$$

After performing some calculations on the equation (6.3.5), the reduction is made to linear first order differential equation as

$$\frac{d\bar{T}^*}{dt} + k(\mu_n^2 + \xi_m^2)\,\bar{T}^* = \left\{\left[\frac{P_m(h)\bar{f}}{k_3} - \frac{P_m(-h)\bar{g}}{k_4}\right] + \bar{\chi}^*\right\} \tag{3.6}$$

The transformed temperature solution is

$$\bar{T}^*(n,m,t) = \frac{\Omega(m,n)}{k(\xi_m^2 + \mu_n^2)}[1 - \exp\,(-k\,(\mu_n^2 + \xi_m^2)t] \tag{3.7}$$

where

$$\Omega(m,n) = \left\{\left[\frac{P_m(h)\bar{f}}{k_3} - \frac{P_m(-h)\bar{g}}{k_4}\right] + \bar{\chi}^*\right\} \tag{3.8}$$

Applying the inversion of transformation rules defined in equations (1.1. 24) and (1.1.30), the temperature solution is shown as follows:

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\,\mu_n)}{\lambda_m k(\xi_m^2 + \mu_n^2)} \times [1 - \exp\,(-k(\xi_m^2 + \mu_n^2)t)]P_m(z) \times S_0(k_1, k_2, \mu_n r) \tag{3.9}$$

The equation (6.3.10) represents the required temperature distribution at any instant and at all points of the hollow cylinder when there are conditions of radiation type.

IV. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting value of temperature distribution $T(r,z,t)$ from equation (3.10) in equation (2.15) one obtains the thermoelastic displacement function $\phi(r,z,t)$ as

$$\phi(r,z,t) = -\left(\frac{1+v}{1-v}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\,\mu_n)}{(\xi_m^2 + \mu_n^2)^2\lambda_m k} \times [1 - \exp(-k(\xi_m^2 + \mu_n^2)t)]\,P_m(z)] \times S_0(k_1, k_2, \mu_n r)\right] \tag{4.1}$$

Substituting the value of $\phi(r,z,t)$ from equation (4.1) in equations (2.13) and (2.14) one obtains

$$U = -\left(\frac{1+v}{1-v}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\,\mu_n)}{(\xi_m^2 + \mu_n^2)^2\lambda_m k} \times [1 - \exp(-k(\xi_m^2 + \mu_n^2)t)]\,P_m(z)\;\mu_n S_0'(k_1, k_2, \mu_n r) \tag{4.2}$$

$$W = -\left(\frac{1+v}{1-v}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\,\mu_n)}{(\xi_m^2 + \mu_n^2)^2\lambda_m k} \times [1 - \exp(-k(\xi_m^2 + \mu_n^2)t)]$$

$$\times(-\xi_m)\,[Q_m \sin(\xi_m z)+W_m \cos(\xi_m z)]\ \ \times S_0(k_1, k_2, \mu_n r)\ ] \tag{4.3}$$

Making use of two displacement components, the volume dilatation is established as

$$e=-\left(\frac{1+\nu}{1-\nu}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\,\mu_n)}{(\xi_m^2+\mu_n^2)^2\,\lambda_m k}\times[1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)$$
$$\times\left[\mu_n^2 S_0''(k_1,k_2,\mu_n r)+\frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r}-\xi_m^2 S_0(k_1,k_2,\mu_n r)\right] \tag{4.4}$$

V. DETERMINATION OF STRESS FUNCTIONS

The stress components can be evaluated by substituting the values of thermoelastic displacement from equations (4.2) and (4.3) in equations (2.19) to (2.22), one obtains

$$\sigma_r=-\left(\frac{1+\nu}{1-\nu}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\,\lambda_m\,k}\left[1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)\right]$$
$$\times\left[(\lambda+2G)\,\mu_n S_0''(k_1,\,k_2,\,\mu_n r)+\lambda\left(\frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r}-\xi_m^2 S_0(k_1,k_2,\mu_n r)\right)\right] \tag{5.1}$$

$$\sigma_z=-\left(\frac{1+\nu}{1-\nu}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\,\lambda_m\,k}$$
$$[1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)$$
$$\times\left[(-\lambda+2G)\,\xi_m^2 S_0(k_1,\,k_2,\,\mu_n r)+\lambda\left(\mu_n{}^2 S_0''(k_1,k_2,\mu_n r)+\frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r}\right)\right] \tag{5.2}$$

$$\sigma_\theta=-\left(\frac{1+\nu}{1-\nu}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=1}^{\infty}\frac{\Omega(\xi_m,\mu_n)}{(\xi_m^2+\mu_n^2)^2\,\lambda_m\,k}\,[1-\exp(-k(\xi_m^2+\mu_n^2)t)]\,P_m(z)$$
$$\times\left[(\lambda+2G)\,\frac{\mu_n S_0'(k_1,k_2,\mu_n r)}{r}+\lambda(-\xi_m^2 S_0(k_1,\,k_2,\,\mu_n r)+\mu_n^2 S_0''(k_1,k_2,\mu_n r)\right] \tag{5.3}$$

$$\tau_{rz}=-2G\left(\frac{1+\nu}{1-\nu}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\frac{\Omega(\xi_m,\,\mu_n)}{(\xi_m^2+\mu_n^2)^2\,\lambda_m k}\,[1-\exp(-k(\xi_m^2+\mu_n^2)t]$$
$$\times[(-\xi_m)\,(Q_m\sin(\xi_m z)+W_m\cos(\xi_m r))\times\mu_n S_0'(k_1,\,k_2,\,\mu_n r)] \tag{5.4}$$

VI. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(r,t)=(r-a)(r-b)e^{h}(1-e^{-t})$, $g(r,t)=(r-a)(r-b)e^{-h}(1-e^{-t})$

$$\chi(r,z,t)=\delta(r-r_0)\delta(z-z_0)\delta(t-t_0) \tag{6.1}$$

Modulus of Elasticity, E (dynes/cm2)	6.9 × 1011
Shear modulus, G (dynes/cm2)	2.7 × 1011
Poisson ratio, υ	0.281
Thermal expansion coefficient, αt(cm/cm-0C)	25.5 × 10-6
Thermal diffusivity, κ (cm2/sec)	0.86
Thermal conductivity,λ (cal-cm/0C/sec/ cm2)	0.48
Inner radius, a (cm)	10
Outer radius, b (cm)	12
Height, h (cm)	30

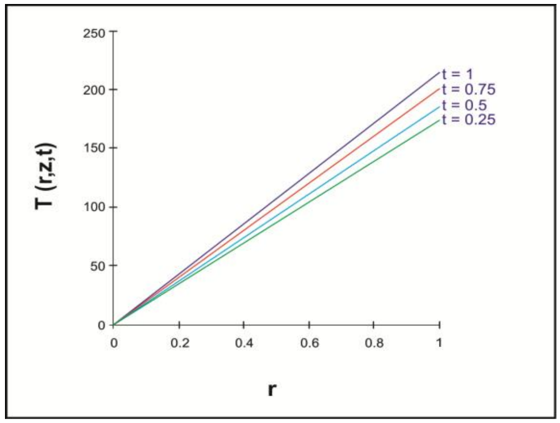
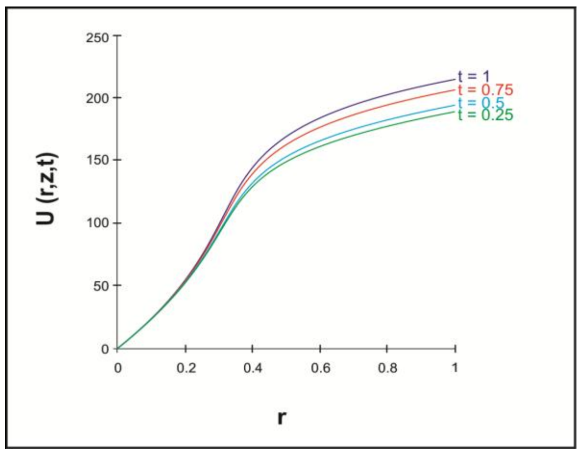
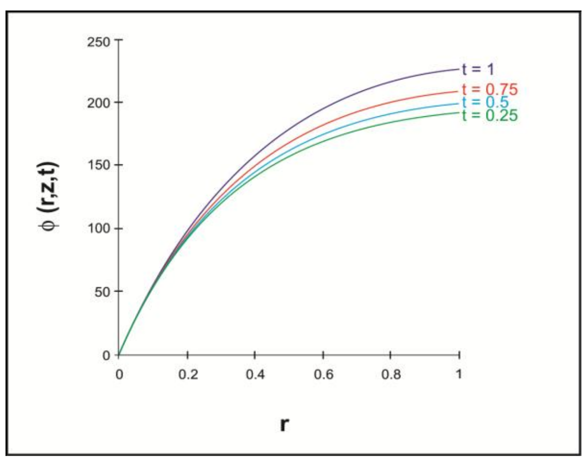


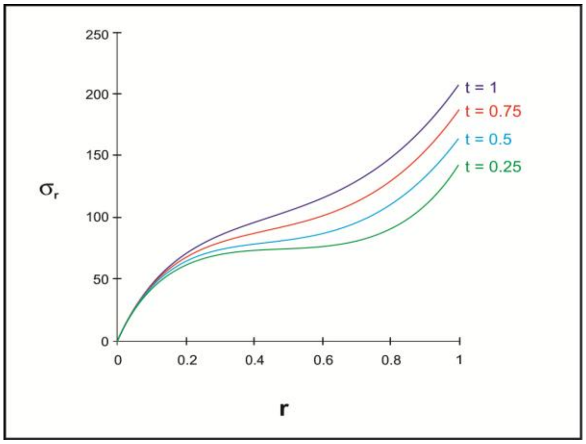
Figure (1). Graph of temperature distribution vs radius



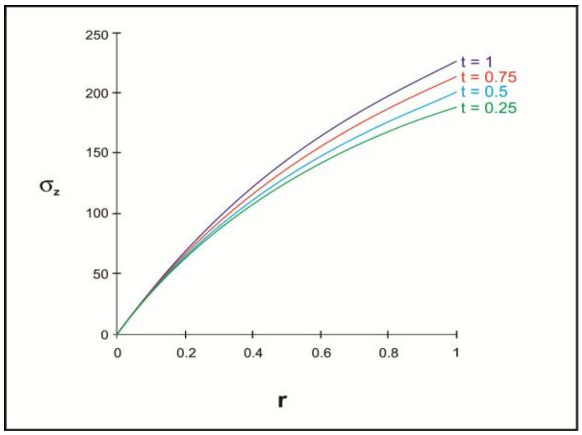
Figure(2). Graph of displacement component vs radius



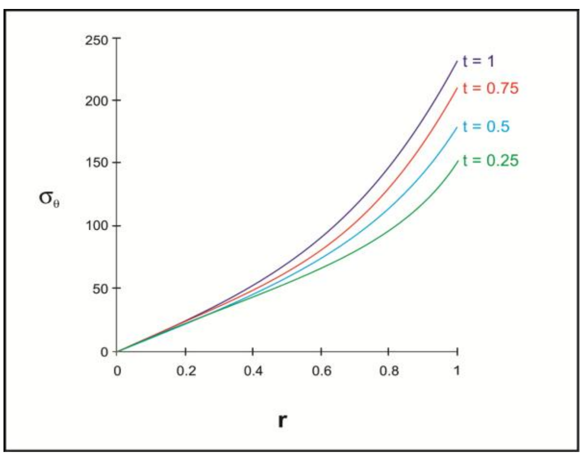
Figure(3). Graph of thermoelastic displacement function vs radius



Figure(4). Graph of radial stress vs radius



Figure(5). Graph of axial stress vs radius



Figure(6). Graph of tangential stress vs radius

VII. STATEMENT OF THE PROBLEM-II

Consider a hollow cylinder occupying the space $D = \{(x, y, z) \in R^3 : a \le (x^2 + y^2)^{1/2} \le b, -h \le z \le h\}$, where $r = (x^2 + y^2)^{1/2}$. The material of the hollow cylinder is isotropic, homogenous and all properties are assumed to be constant. Heat conduction with internal heat source and the prescribed boundary conditions of the radiation type, where the stresses are required to be determined. The equation for heat conduction in cylindrical coordinates [4] is:

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$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\chi(r,z,t)=\frac{\partial T}{\partial t} \qquad (7.1)$$

where κ is the thermal diffusivity of the material of the hollow cylinder (which is assumed to be constant), subject to the initial and boundary conditions

$$M_t(T,1,\ 0,0)=T_0 \text{, for all } a\le r\le b\ ,\ -h\le z\le h \qquad (7.2)$$

$$M_r(T,1,\ k_1,a)=F_1(z,t), \text{ for all } -h\le z\le h\ ,\ t>0 \qquad (7.3)$$

$$M_r(T,1,\ k_2,b)=F_2(z,t) \text{, for all } -h\le z\le h\ ,\ t>0 \qquad (7.4)$$

$$M_z(T,1,\ k_3,\quad h)=f(r,t) \qquad (7.5)$$

$$M_z(T,1,\ k_4,-h)=g(r,t) \text{, for all } a\le r\le b\ ,\quad t>0 \qquad (7.6)$$

The most general expression for these conditions can be given by

$$M_{\vartheta}(f,\bar{k},\bar{\bar{k}},\$)=(\bar{k}\ f+\bar{\bar{k}}\ \hat{f})_{\vartheta=\$}$$

where the prime (^) denotes differentiation with respect to ϑ ; T_0 is the reference temperature; $\bar{k}$ and $\bar{\bar{k}}$ are radiation coefficients respectively.

The Navier's equations without the body forces for axisymmetric two-dimensional thermoelastic problem can be expressed as [3]

$$\nabla^2 u_r-\frac{u_r}{r}+\frac{1}{1-2\upsilon}\frac{\partial e}{\partial r}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial T}{\partial r}=0 \qquad (7.7)$$

$$\nabla^2 u_z-\frac{1}{1-2\upsilon}\frac{\partial e}{\partial z}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial T}{\partial z}=0 \qquad (7.8)$$

where u_r and u_z are the displacement components in the radial and axial directions, respectively and the dilatation e as

$$e=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z} \qquad (7.9)$$

The displacement function in the cylindrical coordinate system are represented by the Goodier's thermoelastic displacement potential $\phi(r,z,t)$ and Love's function L as [4]

$$u_r=\frac{\partial\phi}{\partial r}-\frac{\partial^2 L}{\partial r\partial z}, \qquad (7.10)$$

$$u_z=\frac{\partial\phi}{\partial z}+2(1-\upsilon)\nabla^2 L-\frac{\partial^2 L}{\partial^2 z} \qquad (7.11)$$

in which Goodier's thermoelastic potential must satisfy the equation [4]

$$\nabla^2\phi=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t T \qquad (7.12)$$

and the Love's function L must satisfy the equation

$$\nabla^2(\nabla^2 L)=0 \qquad (7.13)$$

where

$$\nabla^2=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{\partial^2}{\partial z^2}$$

The component of the stresses are represented by the use of the potential ϕ and Love's function L as [4]

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2\phi}{\partial r^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left(\upsilon\nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{7.14}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial\phi}{\partial r} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left(\upsilon\nabla^2 L - \frac{1}{r}\frac{\partial L}{\partial r}\right)\right\} \tag{7.15}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2\phi}{\partial z^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left((2-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\} \tag{7.16}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left((1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\} \tag{7.17}$$

Where G and υ are the shear modulus and Poisson's ratio respectively. The boundary condition on the traction free surface stress functions are

$$\sigma_{zz}\big|_{z=\pm h} = \sigma_{rz}\big|_{z=\pm h} = 0 \tag{7.18}$$

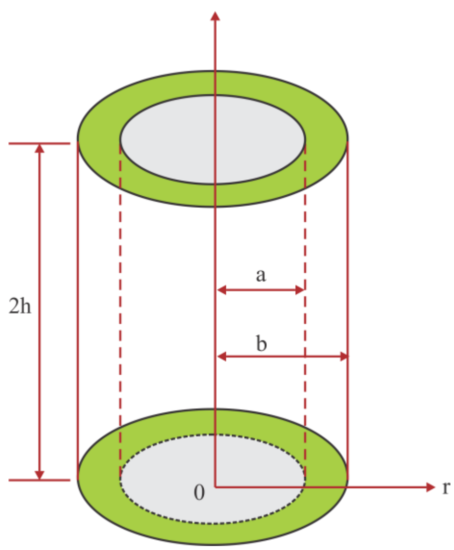

Figure Shows the geometry of the problem

The equations (7.1) to (7.18) constitute the mathematical formulation of the problem under consideration.

VIII. SOLUTION OF THE PROBLEM

Transient Heat Conduction Analysis:

Applying finite Marchi-Zgrablich transform [11] to the equations (7.3) to (7.5) and (7.7), and using equations (7.8) and (7.9), one obtains

$$\kappa\left[-\mu_n^2\overline{T}(n,z,t) + \frac{\partial^2\overline{T}(n,z,t)}{\partial z^2}\right] + \overline{\chi}(n,z,t) = \frac{\partial\overline{T}(n,z,t)}{\partial t} \tag{8.1}$$

$$M_t(\overline{T},1,\ 0,0) = \overline{T}_0 \tag{8.2}$$

$$M_z(\overline{T},1,\ k_3,\quad h) = \overline{f}(n,t), \tag{8.3}$$

$$M_z(\overline{T},1,\ k_4,-h) = \overline{g}(n,t) \tag{8.4}$$

where $\overline{T}$ is the transformed function of T and n is the transform parameter, and μ_n are the positive roots of the characteristic equation

$$J_0(k_1,\mu a)\, Y_0(k_2,\mu b) - J_0(k_2,\mu b)\, Y_0(k_1,\mu a) = 0$$

and F1, F2 are assumed to be zero.

Further applying finite Marchi-Fasulo transform [2] to the equations (8.1), (8.2)and using equations (8.3) and (8.4), one obtains

$$\frac{d\overline{T}^*}{dt} + \kappa(\Lambda_{n,m})\overline{T}^* = F(n,\, m_m) \tag{8.5}$$

where

$$\Lambda_{n,m} = \mu_n^2 + a_m^2$$

and

$$F(n,m) = \left\{ \frac{P_m(h)\overline{f}}{k_3} - \frac{P_m(-h)\overline{g}}{k_4} + \overline{\chi}^* \right\}$$

Where $\overline{T}^*$ denotes Marchi-Fasulo integral transform of $\overline{T}$ and m is the transform parameter.

The general solution of equation (8.5) is given by

$$\overline{T}^*(n,m,t) = \frac{F(n,m)}{\kappa(\Lambda_{n,m})} + \left[\overline{T}_0^* - \frac{F(n,m)}{\kappa(\Lambda_{n,m})} \right] \exp(-\kappa(\Lambda_{n,m})t) \tag{8.6}$$

Applying inversion theorems of transformation rules to the equation (8.6), one obtains

$$T(r,z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{1}{\lambda_m} [\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t)] \right\} \times P_m(z) S_0(k_1,k_2,\mu_n r) \tag{8.7}$$

where

$$\wp_{n,m} = \frac{F(n,m)}{\kappa(\Lambda_{n,m})}$$

Equation (8.7) represents the temperature at every instant and at all points of hollow cylinder when there are conditions of radiation type.

IX. THERMOELASTIC SOLUTION

Referring to the fundamental equation (7.1) and its solution (8.7) for the heat conduction problem, the solution for the displacement function are represented by the Goodier's thermoelastic displacement potential ϕ governed by equation (7.12) as

$$\phi(r,z,t) = \left(\frac{1+\upsilon}{1-\upsilon} \right) \alpha_t \times \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} [\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t)] P_m(z) \right\} \times S_0(k_1,k_2,\mu_n r) \tag{9.1}$$

Similarly, the solution for Love's function L are assumed so as to satisfy the governed condition of equation (7.18) as

$$L(r,z,t) = \left(\frac{1+\upsilon}{1-\upsilon} \right) \alpha_t \times \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} [\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t)] \right\} \times [B_{nm}\sinh(\mu_n z) + C_{nm} z \cosh(\mu_n z)] \times S_0(k_1,k_2,\mu_n r) \tag{9.2}$$

Using equations (9.1) and (9.2) in equations (7.14) and (7.15), one obtains

$$u_r = \left(\frac{1+\upsilon}{1-\upsilon} \right) \alpha_t \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{-1}{\lambda_m(\Lambda_{n,m})} [\wp_{n,m} + (\overline{T}_0^* - \wp_{n,m}) \exp(-\kappa(\Lambda_{n,m})t)] \right\}$$

$$\times\left[\{P_m(z)-[(B_{nm}\mu_n+C_{nm})\cosh(\mu_n z)+C_{nm}z\sinh(\mu_n z)\}\right]\times S_0'(k_1,k_2,\mu_n r) \tag{9.3}$$

$$u_z=\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{n,m})}[\wp_{n,m}+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$
$$\times\{[-a_m(Q_m\sin(a_m z)+W_m\cos(a_m z))-\mu_n^2(-1+2\upsilon)(B_{nm}\sinh(\mu_n z)+C_{nm}z\cosh(\mu_n z))]$$
$$-2(-1+2\upsilon)C_{nm}\sinh(\mu_n z)\mu_n]\,S_0(k_1,k_2,\mu_n r)$$
$$+\mu_n(2(1-\upsilon))[B_{nm}\sinh(\mu_n z)+C_{nm}z\cosh(\mu_n z)]\times[\mu_n S_0''(k_1,k_2,\mu_n r)$$
$$+\frac{S_0'(k_1,k_2,\mu_n r)}{r}\} \tag{9.4}$$

Thus, making use of the two displacement components, the dilatation can be obtained. Then, the stress components can be evaluated by substituting the values of thermoelastic displacement potential ϕ [4] from equation (9.1) and Love's function L from equation (9.2) in equations (7.14) to (7.17), one obtains

$$\sigma_{rr}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\sum_{n=1}^{\infty}\frac{1}{c_n}\times\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{n,m})}\left[\wp_{n,m}+(\overset{*}{T}_0-\wp_{n,m})\exp(-k(\Lambda_{n,m})t)\right]\right\}$$
$$\times\{-P_m(z)[\frac{S_0'(k_1,k_2,\mu_n r)}{r}-a_m^2\,S_0(k_1,k_2,\mu_n r)]$$
$$+\mu_n^2(\upsilon-1)S_0''(k_1,k_2,\mu_n r)[\mu_n\left(B_{nm}\cosh(\mu_n z)+C_{nm}(z\sinh(\mu_n z)+\cosh(\mu_n z)\right)$$
$$+\mu_n\upsilon[B_{nm}\cosh(\mu_n z)\mu_n+C_{nm}(z\sinh(\mu_n z)+\cosh(\mu_n z))]$$
$$\times[\frac{S_0'(k_1,k_2,\mu_n r)}{r}+\mu_n S_0(k_1,k_2,\mu_n r)]$$
$$+2\upsilon C_{nm}\mu_n^2\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r)]\} \tag{9.5}$$

$$\sigma_{\theta\theta}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\times\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=\ 1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{nm})}[\wp_{n,m}+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$
$$\times\{-P_m(z)[\mu_m^2 S_0''(k_1,k_2,\mu_n r)-a_m^2\,S_0(k_1,k_2,\mu_n r)]$$
$$+\frac{\mu_n(\upsilon-1)}{r}S_0^1(k_1,k_2,\mu_n r)\ \times[(\mu_n B_{nm}+C_{nm})\cosh(\mu_n z)+\mu_n C_{nm}z\sinh(\mu_n z)]$$
$$+\mu_n^2\upsilon[(\mu_n B_{nm}+C_{nm})\cosh(\mu_n z)+\mu_n C_{nm}z\sinh(\mu_n z)]$$
$$\times[S_0''(k_1,k_2,\mu_n r)+S_0(k_1,k_2,\mu_n r)]$$
$$+2\upsilon C_{nm}\mu_n^2\cosh(\mu_n z)S_0(k_1,k_2,\mu_n r)]\} \tag{9.6}$$

$$\sigma_{zz}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\times\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{nm})}[\wp_{n,m}+(\overline{T}_0^*-\wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$
$$\times\{-\mu_n P_m(z)[\mu_n S_0''(k_1,k_2,\mu_n r)+\frac{S_0(k_1,k_2,\mu_n r)}{r}]$$
$$+\mu_n^2[B_{nm}\cosh(\mu_n z)+C_{nm}z\sinh(\mu_n z)](2-\upsilon)$$
$$\times[\mu_n S_0''(k_1,k_2,\mu_n r)+r^{-1}S_0(k_1,k_2,\mu_n r)]+(1-\upsilon)\mu_n S_0(k_1,k_2,\mu_n r)$$

$$+\mu_n C_{nm}\cosh(\mu_n z)[(2-\upsilon)[\mu_n S_0''(k_1,k_2,\mu_n r)+\frac{S_0(k_1,k_2,\mu_n r)}{r}]$$
$$+(1-\upsilon)\mu_n^2 S_0(k_1,k_2,\mu_n r)] \tag{9.7}$$

$$\sigma_{rz}=2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t\times\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{-1}{\lambda_m(\Lambda_{nm})}[\wp_{n,m}+(\bar{T}_0^*-\wp_{n,m})\exp(-\kappa(\Lambda_{n,m})t)]\right\}$$
$$\times\{[-\mu_n a_m(Q_m\sin(a_m z)+W_m\cos(a_m z))]\times S_0'(k_1,k_2,\mu_n r)$$
$$+[(B_{nm}\sinh(\mu_n z)+C_{nm}z\cosh(\mu_n z))\mu_n^2[\upsilon\mu_n+\frac{(1-\upsilon)}{r}]$$
$$-2\upsilon\mu_n^2 C_{nm}\sinh(\mu_n z)]S_0'(k_1,k_2,\mu_n r)$$
$$+(1-\upsilon)[B_{nm}\sinh(\mu_n z)+C_{nm}z\cosh(\mu_n z)]\times[\mu_n^3 S_0'''(k_1,k_2,\mu_n r)-\frac{S_0(k_1,k_2,\mu_n r)}{r^2}]\} \tag{9.8}$$

X. DETERMINATION OF UNKNOWN ARBITRARY FUNCTION Bnm and Cnm

Applying boundary conditions (72.7)–(72.11) to the equations (9.1) and (9.2) one obtains

$$B_{nm}=\frac{P_m(h)\bar{X}[h\cosh(\mu_n h)\bar{Y}-\bar{Z}]-a_m\bar{f}\mu_n S_0'(k_1,k_2,\mu_n r)\bar{g}[(2-\upsilon)\bar{X}+(1-\upsilon)\mu_n S_0(k_1,k_2,\mu_n r)]}{[(2-\upsilon)\bar{X}+(1-\upsilon)\mu_n S_0(k_1,k_2,\mu_n r)]\{(h\mu_n)\cosh(\mu_n h)[h\cosh(\mu_n h)\bar{Y}-\bar{Z}]-\bar{g}\sinh(\mu_n h)\bar{Y}\}} \tag{10.1}$$

and

$$C_{nm}=\frac{P_m(h)\bar{X}[\sinh(\mu_n h)\bar{Y}]-a_m\bar{f}\mu_n S_0'(k_1,k_2,\mu_n r)\cosh(\mu_n h)[(2-\upsilon)\bar{X}+(1-\upsilon)\mu_n S_0(k_1,k_2,\mu_n r)]}{[(2-\upsilon)\bar{X}+(1-\upsilon)\mu_n S_0(k_1,k_2,\mu_n r)]\{(h\mu_n)\cosh(\mu_n h)[h\cosh(\mu_n h)\bar{Y}-\bar{z}]-\bar{g}\sinh(\mu_n h)\bar{Y}]\}} \tag{10.2}$$

where

$$\bar{X}=S_0''(k_1,k_2,\mu_n r)+\frac{S_0(k_1,k_2,\mu_n r)}{r}$$
$$\bar{Y}=\mu_n^2(\upsilon\mu_n+(1-\upsilon))\frac{S_0'(k_1,k_2,\mu_n r)}{r}+(1-\upsilon)\left(\mu_n^3 S_0'''(k_1,k_2,\mu_n r)-\frac{S_0(k_1,k_2,\mu_n r)}{r^2}\right)$$
$$\bar{f}=W_m\cos(a_m h)+Q_m\sin(a_m h)$$
$$\bar{g}=\cosh(\mu_n h)+(\mu_n h)\sinh(\mu_n h)$$
$$\bar{z}=2\upsilon\mu_n^2\sinh(\mu_n h)S_0'(k_1,k_2,\mu_n r)$$

XI. SPECIAL CASE

Set, Set $f(r,t)=(r-a)(r-b)e^h(1-e^{-t})$, $g(r,t)=(r-a)(r-b)e^{-h}(1-e^{-t})$

$$\chi(r,z,t)=\delta(r-r_0)\delta(z-z_0)\delta(t-t_0) \tag{11.1}$$

XII. NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computations, we consider material properties of Aluminum metal, which can be commonly used in both, wrought and cast forms. The low density of aluminum results in its extensive use in the aerospace industry, and in other transportation fields. Its resistance to corrosion leads to its use in food and chemical handling (cookware, pressure vessels, etc.) and to architectural uses.

Modulus of Elasticity, E (dynes/cm2)	6.9 × 1011
Shear modulus, G (dynes/cm2)	2.7 × 1011
Poisson ratio, υ	0.281
Thermal expansion coefficient, αt(cm/cm-0C)	25.5 × 10-6
Thermal diffusivity, κ (cm2/sec)	0.86

Thermal conductivity, λ (cal-cm/0C/sec/ cm2)	0.48
Inner radius, a (cm)	10
Outer radius, b (cm)	13
Thickness, h (cm)	30

Table 1: Material properties and parameters used in this study.

Property values are nominal.

The foregoing analysis are performed by setting the radiation coefficients constants, $k_i = 0.5(i = 1,2)$ and $k_i = 0.5(i = 3,4)$, so as to obtain considerable mathematical simplicities.
The derived numerical results from equation (8.71) to (9.8) has been illustrated graphically with internal heat source.

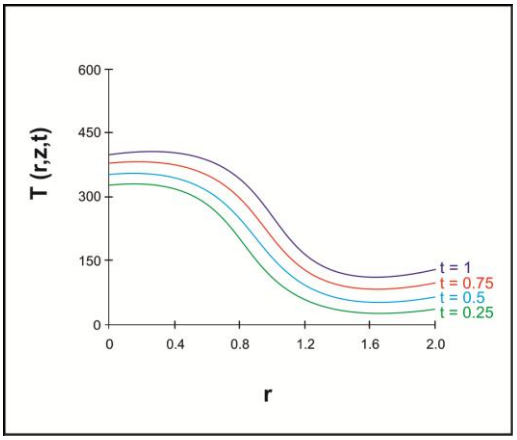

Figure (7). Temperature distribution with internal heat source

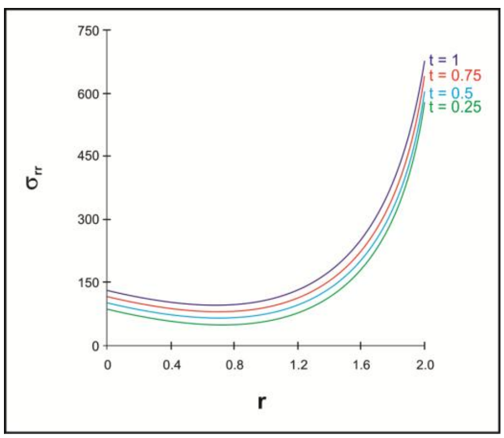

Figure (8). Radial stress distribution with internal heat source

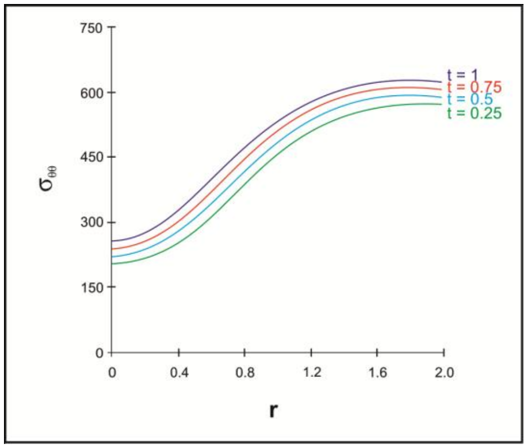

Figure (9). Tangential stress distribution with internal heat source

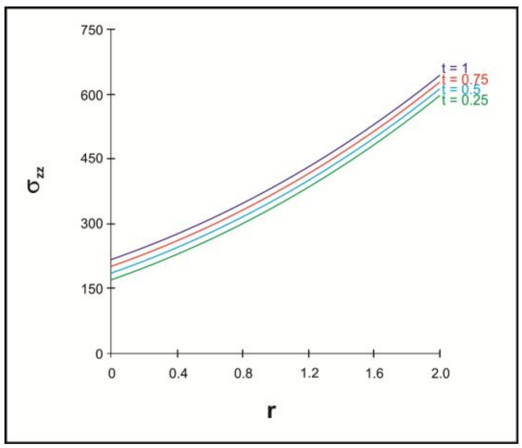

Figure (10). Axial stress distribution for varying along r-axis with internal heat source

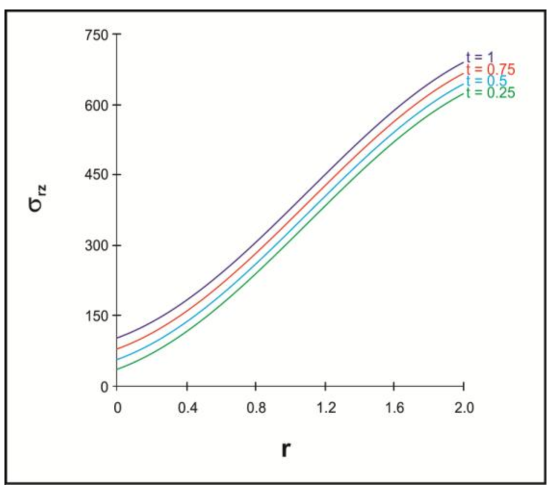

Figure (11). Shear stress distribution for varying along r-axis

XII. CONCLUSION

In this study, we treated the two-dimensional thermoelastic problem of a hollow cylinder in which boundary conditions are of radiation type. We successfully established and obtained the temperature distribution, displacements and stress functions of the hollow cylinder. Then, in order to examine the validity of two-dimensional thermoelastic boundary value problem, we analyze, as a particular case with mathematical model for

$\chi(r,z,t)=\delta(r-r_0)\delta(z-z_0)\delta(t-t_0)$ and numerical calculations are carried out. Moreover, assigning suitable values to the parameters and functions in the equations of temperature, displacements and stresses respectively, expressions of special interest can be derived for any particular case. We may conclude that the system of equations proposed in this study can be adapted to design of useful structures or machines in engineering applications in the determination of thermoelastic behaviour with radiation.

XIII. REFERENCES

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