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Optimal Inventory Model with Two Level Storage Under Ramp Type Demand

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Abstract- This paper presents an EOQ model for the two level of Inventory for deteriorating items with demand rate as a ramp type function of time and a two parameter Weibull distribution is used to represent the distribution of the time for deterioration both in OW (Own Warehouse) and RW (Rented Warehouse). As the holding cost is higher in RW than OW, 'k' units of RW is transferred in bulk from RW to OW at a time and the total amount is transferred in n shipments. A fixed cycle is considered and the shortages are partially backlogged. The objective of the paper is to find the optimal replenishment policyfor the problem.
Keywords: Two levels of storage, Ramp type demand, Bulk release

I. INTRODUCTION

In the study of inventory models, two important factors that need to be noted are demand and deterioration. Different demand patterns are used to reflect sales in the market. Most of the researchers considered either continuous increase or continuous decrease demand with time. But in practice this may not be true all the time. For example, in the case of newly launched products, fashionable garments, hardware devices, cosmetics, initially demand increases with time and becomes constant over a period of time. The Ramp type demand is suitable for such products where it increases up to a certain time and becomes constant thereafter.

Next important factor is deterioration. Most of the products like medicines, food items, chemicals, electronic components etc. deteriorate /damage/spoil over a period of time. Many mathematical models have been developed on the deteriorating items considering various functions of rate of deterioration like constant rate, exponentially decaying and so on.

Many researchers have studied about the Weibull deterioration with two or three parameters.Chakrabarty[16] mentioned in his introduction of the paper that the Weibull distribution for deterioration could be used explicitly for the products where the rate of deterioration increases with age, longer the items remained unused or the higher the rate at which they failed.

II. LITERATURE REVIEW

The inventory models for deteriorating items stored intwo warehouse have been discussed and explained by many authors and researchers in the past. KVS. Sarma[13] explained optimum release rule, A.K.Bhunia [2] discussed linear trend in demand and later included the shortages in [3] A. Goswamy [5] stock dependent rate, Hui-Ming Wee[6] partial backordering and Weibull deterioration under inflation, Ruxian Li[12] reviewed deteriorating inventory study, Deepa Khurana[4] considered time dependent demand under inflation, K.D Rathod [7] stock dependent demand and constant quantity release,

Many researchers started studying Ramp type demand and Weibull deterioration under different assumptions. Chakrabarty [16] discussed a single warehouse with trended demand and Weibull deteriorationSushil Kumar [15] considered Ramp Type demand underInflation, Li-Qunji [8] in his research, started with zero inventory level, that was taken forward by Ajay Singh Yadav [1] who formulated a model with Ramp Type Demand and Weibull Deterioration for two warehouse.

Neeraj Kumar [10] considered a multivariate demand i.e. a combination of linear time variable and on hand inventory, a two parameter Weibull deterioration is considered only in RW and constant rate is in OW, the objective of the study was to find that the quantity that can be stored in RW and the number of times the inventory should be transferred from RW to OW so that the net profit may be maximized.

Neeraj Kumar [11] investigated the effect of salvage value on an inventory problem of determining the optimal replenishment policy for deteriorating items with stock dependent demand and limited storage facility.

Snigdha Banerjee [14] a time dependent linear trend demand considered with three parameter Weibull distribution deterioration both in RW and OW and it was assumed that the goods in OW are consumed only when the inventory in RW is zero.Luis A San-Jose ramp type demand with nonlinear holding cost is considered for L_1 system i.e. for a single warehouse.

Although many researchers worked on Ramp type demand with Weibull distribution deterioration with two or three parameters some gaps have been observed. Neeraj Kumar [11] considered the demand that looks like a ramp type demand infact was the stock dependent demand and the effect of salvage value on inventory with constant deterioration was studied. But in this paper ramp type demand dependent on time is considered.

Even though a Weibull distribution deterioration for three parameters was discussed by Snigdha Banerjee [14]but it has not been worked on ramp type demand. Though the equations taken in this paper seems to be conceptually similar to Neeraj Kumar [10] but the calculations and the results are different. In fact,he considered demand as the combination of linear time variable and stock dependent Weibull distribution deterioration (only in RW) and bulk release and it has been noted that ramp type demand was not worked. So it is presumed that a two warehouse with ramp type demand,Weibull distribution deterioration with bulk release has not discussed in any of the above papers mentioned.

In this paper, with this presumption it is assumed that the Ramp Type demand with a two parameter Weibull deterioration, and the rate of deterioration is same in both RW and OW.This assumption is possible as the Weibull deterioration is applicable for the products for which the deterioration depends upon the time not on the facilities in the Warehouse. The stock in RW is transferred from RW to OW in bulk release. i.e. 'K' units are transferred from RW to OW at a time and the total amount in n shipments. A fixed cycle is considered and the shortages are partially backlogged. The objective is to find the optimum number of cycles to minimize the cost function.

III. ASSUMPTIONS

The mathematical model is developed based on the following assumptions:

- The demand rate is deterministic and Ramp type(fig1)
- The lead time is zero or negligible and replenishment is infinite
- There is no replacement or repair of deteriorating items during the period
- The Own House(OW) has fixed capacity of W units and Rented House(RW) has unlimited capacity
- The holding cost in RW is higher than that in OW
- The transfer of stock from RW to OW follows a K-release rule (fig2& fig3)
- Shortages are allowed and partially backlogged
- The rate of deterioration follows a two parameter Weibull distribution both in OW and RW with same shaping and scaling parameters. $\theta(t) = \alpha\beta t^{\beta-1}$

IV. NOTATIONS:

The following notations are used throughout the study

D(t): Demand rate and $D(t) = a + b\,[t-(t-u)\,H(t-u)]$ (fig.1)

- F, H: Holding cost in RW and OW resp. (F>H)
- I_R, I_o: Inventories in RW and OW resp.at any time't'
- C_2: Shortage cost per unit
- C_1: Deteriorated cost per unit
- T: The length of the cycle
- θ: The rate of deterioration
- α: Scaling parameter inboth OW and RW
- β: Shaping parameter in both OW and RW
- ρ: The fixed transport cost moving K units from RW to OW per one shipment

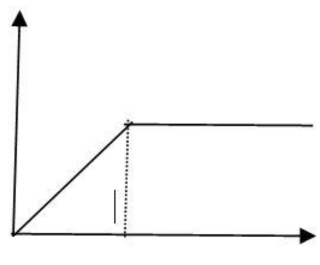

fig-1(Ramp type Demand)

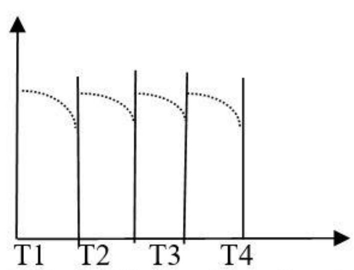

fig-2 (Graphical Representation of Inventory in OW)

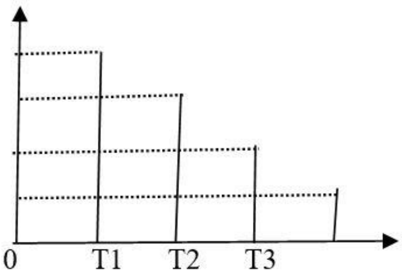

fig-3 (Graphical Representation of Inventory in OW)

V. FORMULATION AND SOLUTION OF THE MODEL

A company purchases S units, out of which W units are stored in OW and the remaining are stored in RW. Initially the demand is met from OW up to K units. After exhausting K units from OW, the same amount i.e. K units are transformed from RW to OW. As a result, the stock in OW becomes W units and in RW (S-W-K). After n steps all the units in RW will be exhausted. And from T_n to T the stock in OW will be used and there after shortages are allowed.

The inventory in OW decreases due to demand and deterioration. The demand increases initially up to t =u and thereafter the demand becomes constant. As the demand increases up to t=u and then becomes constant it depends upon the value of u hence different cases arises. Since u can be in any one of the periods $[T_i, T_{i+1}]$ for i = 0,1,2,3……n-1let itassumes that u is in$[T_j, T_{j+1}]$

The inventory in $[T_i, T_{i+1}]$ for i = 0, 1,2, …. j-1

$I_o^{'}(t) + \alpha\beta t^{\beta-1} I_o(t) = -(a+bt)$ with the boundary condition $I_o(T_{i+1}) = W$ (1)

The solution of the equation (1) is $I_o(t) = e^{-\alpha t^\beta}\left[\int_t^{T_{i+1}} (a+bx)e^{\alpha x^\beta}dx + W\right]$ (2)

The inventory in $[T_{j+1}]$ as u $\epsilon [T_j, T_{j+1}]$ we have two differential equations

$I_o^{'}(t) + \alpha\beta t^{\beta-1} I_o(t) = -(a+bt)$ for $T_j < t < u$ (3)

$I_o^{'}(t) + \alpha\beta t^{\beta-1} I_o(t) = -(a+bu)$ for $u < t < T_{j+1}$ (4)

With the boundary condition $I_o(T_{j+1}) = W$

The solution of equation (4) $I_o(t) = e^{-\alpha t^\beta}\left[\int_t^{T_{j+1}} (a+bu)e^{\alpha x^\beta}dx + W\right]$ (5)

And we know that $I_o(u) = e^{-\alpha u^\beta}\left[\int_u^{T_{j+1}} (a+bu)e^{-\alpha x^\beta}dx + W\right]$ (6)

The solution of equation (3) with the boundary condition equation (6) is

$I_o(t) = e^{-\alpha t^\beta}\left[\int_t^{u} (a+bx)e^{\alpha x^\beta}dx + \int_u^{T_{j+1}} (a+bu)e^{\alpha x^\beta}dx + W\right]$ (7)

The inventory in $[T_i, T_{i+1}]$ for i = j+1, j+2, …… n-1 is given by the differential equation

$I_o^{'}(t) + \alpha\beta t^{\beta-1} I_o(t) = -(a+bu)$ with the boundary condition $I_o(T_i) = W$ for all i = 1,2,3,…… n-1 (8)

The solution of equation (8) is $I_o(t) = e^{-\alpha t^\beta}\left[\int_t^{T_{i+1}} (a+bu)e^{\alpha x^\beta}dx + W\right]$ (9)

The inventory level at RW decreases only due to the deterioration. $I_R^{'}(t) = -\alpha\beta t^{\beta-1} I_R(t)$ (10)

With the boundary conditions $I_R(0)$ = S-W and $I_R(T_{i+1}) = I_R(T_i) - k$

The solution of equation (10) is $I_R(t) = C\, e^{-\alpha t^\beta}$ using the boundary conditions we get C = S-W .

So we get the solution as $I_R(t)$ = (S-W) $e^{-\alpha t^\beta}$ for $0 \le t \le T_1$

$I_R(t)$ = (S-W-K) $e^{\alpha\left(T_1^\beta - t^\beta\right)}$ for $T_1 \le t \le T_2$ continuing in the same way in i^{th} interval we get $I_R(t)$ = (S-W-iK) $e^{\alpha\left(T_i^\beta - t^\beta\right)}$ for $T_i \le t \le T_{i+1}$

The total inventory $I_o(t)$ at any time in OW is given by $\sum_{i=0}^{j} e^{-\alpha t^\beta}\left[\int_t^{T_{i+1}} (a+bx)e^{\alpha x^\beta}dx + W\right]$ +

$e^{-\alpha t^\beta}\left[\left(\int_t^{u} (a+bx)e^{\alpha x^\beta}dx + \int_u^{T_{j+1}} (a+bu)e^{\alpha x^\beta}dx\right) + W\right] + \sum_{i=j+1}^{n-1} e^{-\alpha t^\beta}\left[\int_t^{T_{i+1}} (a+bu)e^{\alpha x^\beta}dx + W\right]$ (11)

After expanding the infinite series e^x and neglecting the terms of $O(x^2)$ we get $e^x = 1+x$. Using this on simplifying the above equation we get

$$I_0(t) = \sum_{i=0}^{j-1} e^{-\alpha t^{\beta}}\left[aT_{i+1} - at + \frac{a\alpha}{\beta+1}\left(T_{i+1}^{\beta+1} - t^{\beta+1}\right) + \frac{b}{2}\left(T_{i+1}^{2} - t^{2}\right) + \frac{b\alpha}{\beta+2}\left(T_{i+1}^{\beta+2} - t^{\beta+2}\right) + W\right] +$$
$$e^{-\alpha t^{\beta}}\left[a(u-t) + \frac{a\alpha}{\beta+1}\left(u^{\beta+1} - t^{\beta+1}\right) + \frac{b}{2}\left(u^{2} - t^{2}\right) + \frac{b\alpha}{\beta+2}\left(u^{\beta+2} - t^{\beta+2}\right) + (a+bu)\left(T_{j+1} - u + \frac{\alpha}{\beta+1}\left(T_{j+1}^{\beta+1} - u^{\beta+1}\right)\right) + W\right]$$
$$+\sum_{i=j+1}^{n-1} e^{-\alpha t^{\beta}}\left[(a+bu)\left[T_{i+1} - t + \frac{\alpha}{\beta+1}\left(T_{i+1}^{\beta+1} - t^{\beta+1}\right)\right] + W\right] \quad (12)$$

The cost function consists of
Setup cost = A
Transportation cost moving from RW to OW is $n\rho$
Holding cost in OW = H $\int_0^{T_n} I_o(t)\,dt$
Holding cost in RW = F $\int_0^{T_n} I_R(t)\,dt$
The number of items deteriorated is S- $\int_0^{u}(a+bx)\,dx + \int_u^{T_n}(a+bu)\,dx = S + \frac{bu^2}{2} - aT_n - buT_n$ (13)
Shortages cost is $C_2 B_T = -C_2\int_{T_n}^{T}(a+bu)\,dx = C_2(a+bu)(T_n - T)$
The total cost per unit time is $\Psi = \frac{1}{T}$[A + $n\rho$+ H $\int_0^{T_n} I_o(t)\,dt$ + F $\int_0^{T_n} I_R(t)\,dt$ +C_1 $(S + \frac{bu^2}{2} - aT_n - buT_n)$+ $C_2(a+bu)(T_n - T)$]
The above cost function is a function of n, K. The aim is to find the optimal value of K for fixed value of n so that the cost function is minimized.

VI. NUMERICAL ILLUSTRATION

To illustrate the above theory, the following values for different parameters were taken.a=4, b=2, A=150, ρ=5, α=1, β=0.5, H=5, F= 8, S= 200, W=100, K=(S-W)/n,C_1=3, C_2 =5, T=3

Table-1 (The time values for different values of n)

n	T1	T2	T3	T4	T5	T6	T7	T8
2	0.75	1.5	2.25					
3	0.6	1.2	1.8	2.4				
4	0.5	1	1.5	2	2.5			
5	0.428	0.857	1.286	1.714	2.143	2.571		
6	0.375	0.75	1.125	1.5	1.875	2.25	2.625	
7	0.333	0.667	1	1.333	1.667	2	2.333	2.667

Table-2 (The cost function for different values of u and n and the optimal value is marked with * in each case.)

n		ψ		ψ		Ψ
2	u=0.25	473.1*	u=0.75	604.7	u=1.7	687.2
3		508.4		595.8*		639.9*
4		534.0		596.9		673.6
5		553.9		601.5		654.2
6		569.9		639.3		672.9
7		583.2		639.4		684.8

A graph is drawn with the parameter 'u' and the optimum cost per unit time and it is observed that as the value of u increases the cost function also increases.

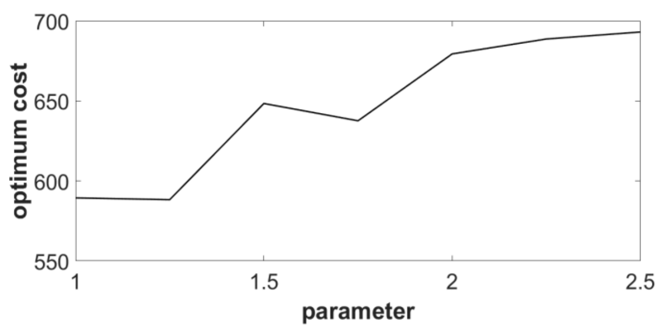

fig-4 (Parameter 'u' versus cost function)

VII. SENSITIVE ANALYSIS

Sensitive analysis is done for the parameters α, β,a,b when u=0.75 and the data in percentages were listed below in the Table -3 and able-4 respectively.

Table-3 (Sensitive Analysis for α and β)

%change in 'α'	%change in Cost(ψ)	%change in 'β'	%change in Cost(ψ)
10	-4.96	10	-0.4
20	-9.99	20	-1.02
30	-15.0	30	-1.7
-10	4.9	-10	0.3
-20	9.7	-20	0.5
-30	14	-30	0.6

Table-4 (Sensitive Analysis for a and b)

%change in 'a'	%change in Cost(ψ)	%change in 'b'	%change in Cost(ψ)
10	42.3	10	42.5
20	42.1	20	42.4
30	41.9	30	42.3
-10	42.8	-10	42.6
-20	43	-20	42.7
-30	43.2	-30	42.8

The above analysis (Table-3) concludes that the cost function is very sensitive when the scaling factor is changing. So it is very important to handle it whereas the shaping factor is less sensitive. Whereas the cost function is (Table-4) insensitive with respect to the parameters 'a' and 'b'

VIII. CONCLUSION

In this paper a two warehouse is studied with shortages. A ramp type demand is considered with a two parameter Weibull distribution deterioration both in OW and RW. Equal shaping and scaling factors are considered for convenience. Most of the researchers studied a similar type of demand and deterioration by ignoring the bulk release. This model is studied for different parameters up to seven cycles and the optimal solution is selected from the available solutions. The outcome has shown that the cost function will increase and stabilizes between 3 to 5 cycles but later again increases. It is also observed that as scaling parameter increases the cost function decreases and also that as shaping factor increases the cost function increases.

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