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Thermal Effect on infinite impermeable porous flat plate in liquid metal

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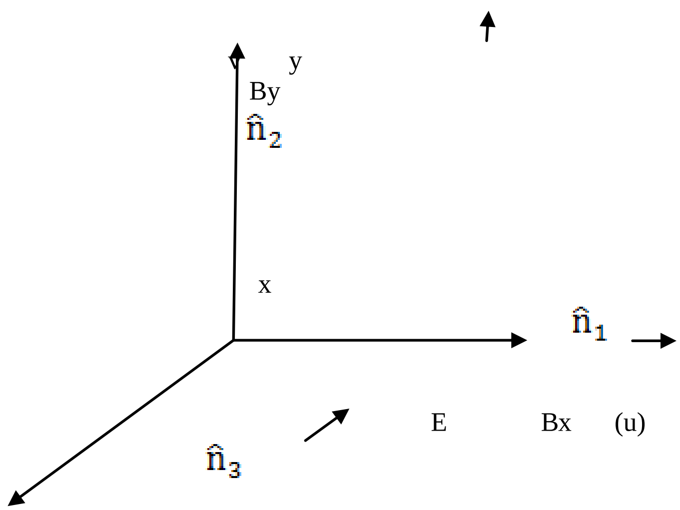
Abstract- The aim of the paper is to study the temperature distribution in liquid metals.(For this purpose we have to take liquid metal with low Prandtl number). In MHD flow past an infinite permeableflat plate at zero incidence under uniform transverse magnetic field with suction and injection. The equations of continuity momentum and energy are transformed into ordinary differential equation and solved numerically by the help of shooting method. Here, we have also used Ohm's law, which is the life line of conductivity. The temperature distribution and velocity are taken into account and presented through graphs. Prandtl number and Hartmann number have played an important role in numerical analysis. In particular flow of Bismuth lead (56.5% + 48.5%) for Pr = 0.0450 at 150oC, Pr = 0.0364 at 200oC has been studied.

Geometrical Interpretation



z Figure-1

I. INTRODUCTION

In brief, MHD is the study of interaction between the magnetic field and moving conducting fluid like liquid metals and Plasma. If a charged fluid moves in a magnetic field electric currents are induced in the fluid as a result of its motion. Thus induce current will in turn modify the field. At the same time the flow of charged fluid across the magnetic field will produce mechanical forces which will modify the motion of fluid. This is the way by which interaction take place between electromagnetic field and hydrodynamics motion.

In some aspects, Bismuth possesses the same propertiesas lead and is frequently associated with lead in its ore Bismuth is used as an impurity in refining the lead and vice-versa. The equilibrium diagram of the bismuth lead system has shown in figure (2) [1].

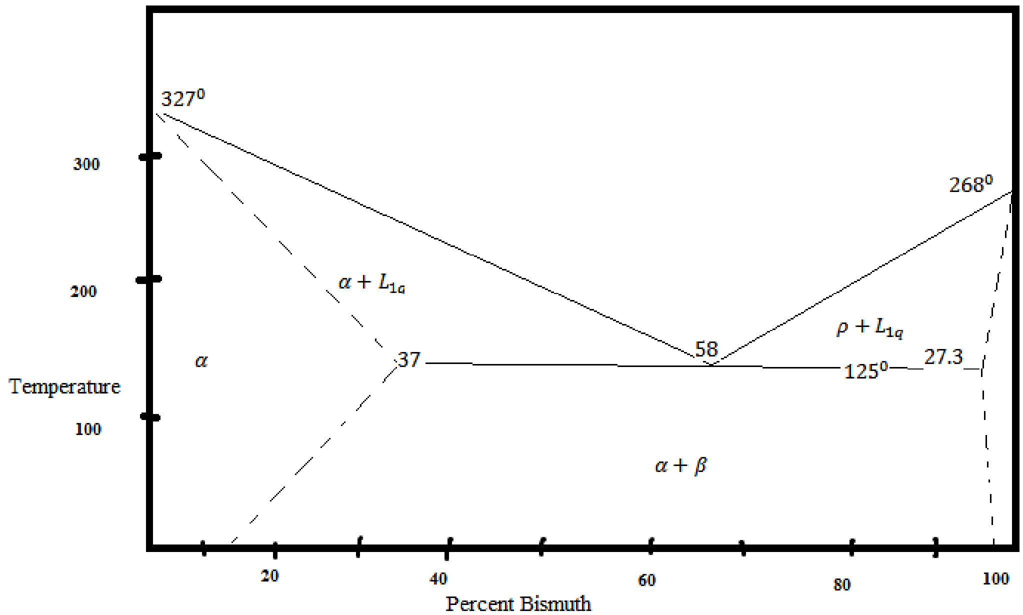


Figure 2- Equilibrium diagram of the system lead-bismuth

Generally, pure Bismuth (99.99 Percent) and pure lead (99.8 percent) are used in preparation of the alloys and numerous analyses have shown that the total number of impurities in the alloys was number of impurities in the alloys was usually less than 0.2 percent.

Palhauisen has investigated the thermal boundary layer on the basis of thermal energy integral equation [2]. The temperature field has also been studied for extreme values of Prandtl number [3]. Some scientists have employed suitable polynomial to obtain temperature distribution of the problem for free convection from a heated vertical plate [4-6]. These studies are based on non conducting fluid.

Cheng and Yen have discussed the case of an infinite flat insulated plate set impulsively into uniform motion in its own plane, in the presence of transverse magnetic field [7]. Soundalgekar and Jahagirdar studied the free convection effects of transversely applied magnetic field, on the flow of an electrically conducting viscous incompressible fluid, past an infinite vertical and uniformly accelerated plate [8]. Amakiri and Ogulu reported that, the increase in the viscous dissipative heat leads to increase in both the flow velocity and, temperature distribution [9]. The increase in the magnetic parameter is accompanied by an increase in the velocity but does notaffect the temperature distribution. The energy equation of two dimensional thermal boundary layer flows has been discussed.

II. ENERGY EQUATION OF TWO DIMENSIONAL THERMAL BOUNDARY LAYER FLOWS

For two dimensional flows, the velocity has only two components Le and v. The magnetic flied has also two components Hxand Hy. The electric current "J" and electric field $"\vec{E}"$ have only z-component.

For small electrical conductivity $"\sigma"$,

We have

$$H = H_0 + \sigma H_1 + \sigma^2 H_2 \tag{1a}$$

$$E = E_0 + \sigma E_1 + \sigma^2 E_2 \tag{1b}$$

In the boundary layer over a flat plate at y= 0, the x- component of velocity "Le" is much larger than y- component of velocity ϑ.

We have also taken as unit vectors in the directions of x, y, and z axis respectively.

So,

$$\vec{v} = u\vec{n_1} + v\vec{n_2}$$

(2a)

$$\vec{H_0} = Hx_0\vec{n_1} + Hy_0\vec{n_2}$$

(2b)

Than

$$\vec{V} \times \vec{H_0} = (uHy_0 - vBx_0)\vec{n_3}$$

(3a)

Similarly

$$\vec{V} \times \vec{H_1} = (uHy_0 - vBx_1)\vec{n_3}$$

(3b)

From Ohm's law,

$$\vec{J} = \sigma\left(\vec{E_0} + \mu_0\vec{V} \times \vec{H_0}\right) + \sigma^2\left(\vec{E_1} + \mu_0\vec{V} \times \vec{H_1}\right) + \cdots \ldots\ldots\ldots\ldots\ldots\ldots$$

(4)

By the geometry

$$\vec{J} = \{Ez_0 + \mu_0(uHy_0 - vHx_0)\} + \sigma^2\{E_{z1} + \mu_0(uHy_1 - vHx_1)\}$$

(5)

The circuit of electrical current must be closed. Then, we have boundary condition

$$hE = -RAJ_z$$

(6)

Where, h is distance between electrodes in Z- direction. R is resistance of the circuit and "A" is area.
Substituting the equation (1a), (1b) and (5) into (6) and equating the co-efficient of different powers
We have

$$E_{z0} = -\frac{\beta}{1+\beta}\{\mu_0(uHy_0 - VHx_0)\}$$

(7a)

$$E_{z1} = -\frac{\beta}{1+\beta}\{\mu_0(uHy_1 - VHx_1)\}$$

(7b)

Substituting the values of E_{z0} and E_{z1} from (7) to (5) we get

$$J_z = \frac{\beta}{1+\beta}\{\sigma\mu_0(uHy_0 - VHx_0) + \sigma^2\mu_0(uHy_1 - VHx_1) + \dots\dots\dots\}$$

(8)

$$Hx_0 = 0.$$

If the externally applied magnetic field is perpendicular to the wall,
So, x- component's Pondermotive force

$$E_{e_x} = -\frac{\beta}{1+\beta}(4\sigma\mu_0^2 Hy_0 + 2\sigma^2 u\mu_0^2 Hy_1 + \dots\dots\dots\dots)$$

And y- components of Pondermotive force

$$F_{e_y} = \frac{\beta}{1+\beta}(\sigma^2 uHy_0 Hy_1 + \dots\dots\dots\dots\dots)$$

σ

For small value of ,

$$F_{e_x} = -\sigma^2\mu_0^2\beta_0^2 u$$

(9)

H_0

If the externally applied magnetic field is denoted by , then

$$\left.\begin{aligned} F_{e_x} &= -\sigma^2\mu_0^2\beta_0^2 u \\ &\text{and} \\ F_{e_y} &= 0 \end{aligned}\right\}$$

(10)

So, the equation of motion becomes

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial p}{\partial x} + Fe_x + \mu_0\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

(11)

and

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\frac{\partial p}{\partial x}+Fe_x+\mu_0\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

(12)

Using boundary layer assumptions and simplifications, equation (11) becomes

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho}\mu_0^2H_0^2u$$

(13)

For the steady two dimensional hydromantic flows, the equation of continuity is,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

(14)

And boundary layer equations is,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho}\mu_0^2\beta_0^2u$$

(15)

Corresponding to equation (15), the energy equation is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{1}{\rho c_p}\frac{\partial p}{\partial x}+\frac{\sigma\mu_0^2H_0^2u^2}{\rho c_p}$$

(16)

$$\alpha$$

Where " " is constant.

The boundary conditions are $y=0, u=0, v=v_s, T=T_\infty, v_s>0$ for infection and $v_s<0$ for Suction

$$u=u(x),\ T=T_\infty,\ \frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}=0$$

; (17)

$$\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial y^2}=0$$

And also (18)

After applying equation (18), the equation (15) gives

$$0 = -\frac{U}{\rho c_p}\frac{\partial p}{\partial x} + \frac{\sigma \mu_0^2 H_0^2 u^2}{\rho c_p}$$

$$\frac{\partial p}{\partial x} \quad (19)$$

Substituting (19) into (16)
We have

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{\sigma}{\rho c_p}\mu_0^2 H_0^2 U_u + \frac{\sigma \mu_0^2 H_0^2 u^2}{\rho c_p} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (20)$$

For an infinite flat plate, physical variables would not depend on x, so, $\frac{\partial u}{\partial x} = 0.$
Putting this value in equation (14)
We get

$$\frac{\partial v}{\partial y} = 0 \quad (21)$$

Using the condition, at y= 0. It is observed that $v = v_s$ for every y. So, equation (20) becomes

$$V_s\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\sigma \mu_0^2 H_0^2 U_0^2}{\rho c_p}\left\{\frac{u}{U_0}\left(1 - \frac{u}{U_0}\right)\right\} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (22)$$

Schlichting's monopanometric family of velocity profile is given by

$$\frac{u}{U} = F_1(n) + kF_2(\eta)$$

$$\eta = \frac{y}{\partial(x, H_{M,}^2 \lambda)}$$

When

$$F_1(n) = 1 - e^{-n}$$

Where

$$F_2(n) = F_1 - \frac{\sin \pi n}{\rho}\ ;\ \ 0 \leq \eta \leq 3$$

$$F_2(n) = F_1 - 1\,; \quad \eta \leq 3$$

For MDH thermal velocity profile

$$\frac{u}{U} = 1 - e^{-\frac{h\eta}{2}}$$

(23)

Where,

$$h = -\overline{V_S} + \sqrt{\overline{V_S^2} + 4H_a^2}$$

$$\eta = \frac{y}{\delta t}$$

$$\overline{V_S} = \frac{\overline{V_S}\,\delta t}{v} \qquad \overline{V_S} > 0$$

; Injection

< 0

Suction

and

$$H_a = \sqrt{\frac{\sigma\mu_0^2 H_0^2 \delta t^2}{\rho}}$$

= Hartmann number

For temperature distribution

Using equation (23) in (22)

We get

$$V_\delta \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{\sigma}{\rho cp}\mu_0^2 H_0^2 U_0^2 \left\{\left(1 - e^{\frac{-h\eta}{2}}\right) e^{\frac{-h\eta}{2}}\right\} + \frac{\mu}{\rho cp} U_0^2 \frac{h^2}{4\delta t} e^{-h\eta}$$

On integration under boundary conditions

$$T(\eta) = \frac{\mu_0 U_0^2}{hk}\left(\frac{2H_a^2 e^{-h\eta}}{\frac{h}{2}+P_r\overline{v_s}} - \frac{H_a^2 + h^2/_4}{h+P_r\overline{v_s}} e^{-h\eta}\right) + T_\infty \tag{24}$$

Hence

$$T_\infty - T_\infty = \frac{\mu_0 U_0^2}{hk}\left(\frac{4H_a^2}{h+2P_r\overline{v_s}} - \frac{4H_a^2+h^2}{4(h+P_r\overline{v_s})}\right) \tag{25}$$

Therefore

$$T = \frac{\frac{4H_a^2}{h+2P_r\overline{v_s}} e^{\frac{-h\eta}{2}} - \frac{4H_a^2+h^2}{4(h+P_r\overline{v_s})} e^{\frac{-h\eta}{2}}}{\frac{4H_a^2}{h+P_r\overline{v_s}} - \frac{4R_m^2+h^2}{4(h+P_r\overline{v_s})}} \tag{26}$$

This gives the temperature distribution in MHD flow in porous infinite Permeable flat plate.

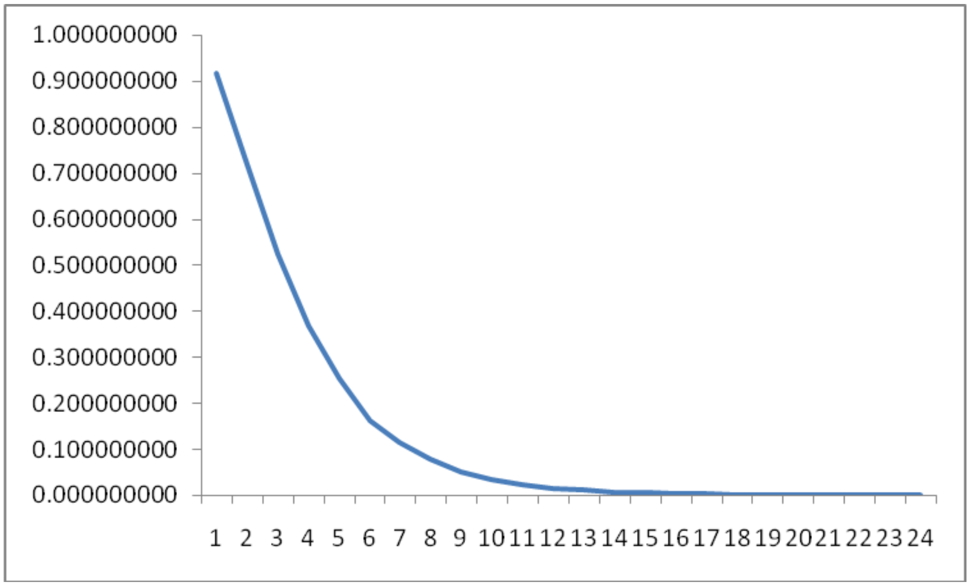
III. RESULTS AND DISCUSSION

In order to investigate the physical significance of the problem, the numerical values of Prandtl number, Hartmann number, Suction velocity and temperature have been computed for different values of various parameters. To obtain the steady state solutions, the computations have been carried out up to T = 0 to T = 1 and T = 0 to T = ∞. It is seen that, numerical values of η, v_s show little changes after $\eta = 5.$ we have also observed that, as the magnetic field increases, the thermal boundary layer thickness decreases. In the case of suction, asymptotic state is reached earlier whereas in the case of injection it reaches late. The boundary conditions are satisfied for all values of less than or $\overline{v_s}$ greater than zero with Rm is not equal to zero.

Table -1

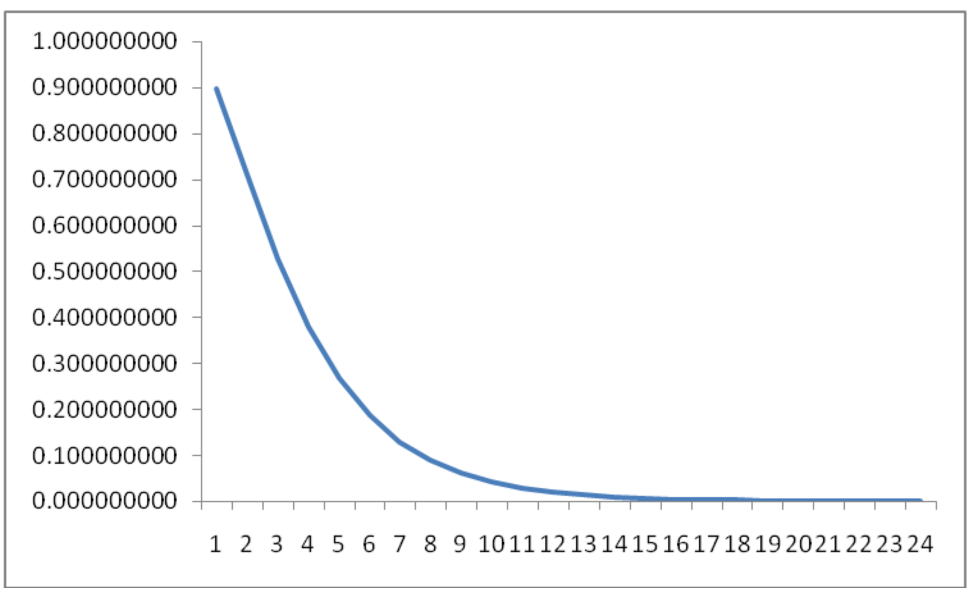
η	T		
	$\overline{v_s}$	$\overline{v_s}$	$\overline{v_s}$
0.0	0.000000000	0.000000000	0.000000000
0.5	0.869972918	0.859407822	0.852978244
1.0	0,650531887	0.650703396	0.653842034
1.5	0.454494248	0.463929507	0.475161477
2.0	0.306363463	0.216633382	0.335080641
2.5	0.202332959	0.144899673	0.232056822
3.0	0.132006901	0.144899673	0.158897840
3.5	0.085482955	0.096206042	0.108015638
4.0	0.550098659	0.063576170	0.730804930
4.5	0.035410470	0.041886305	0.492908160
5.0	0.022715299	0.027542210	0.033176992
5.5	0.014554413	0.018087239	0.022300478
6.0	0.009318498	0.011868208	0.014975990
6.5	0.005933448	0.007783279	0.010883150
7.0	0.003815062	0.005102536	0.006743000

7.5	0.002440219	0.003344327	0.004522464
8.0	0.001560638	0.002191623	0.003032621
8.5	0.000998024	0.001436082	0.002033331
9.0	0.000638201	0.000940946	0.001363209
9.5	0.000408094	0.000616498	0.000913888



η $\bar{v}_s$

Figure-3 (Graph is plotted Tagainst , for)



η $\overline{v_s}$

Figure-4 (Graph is plotted Tagainst , for)

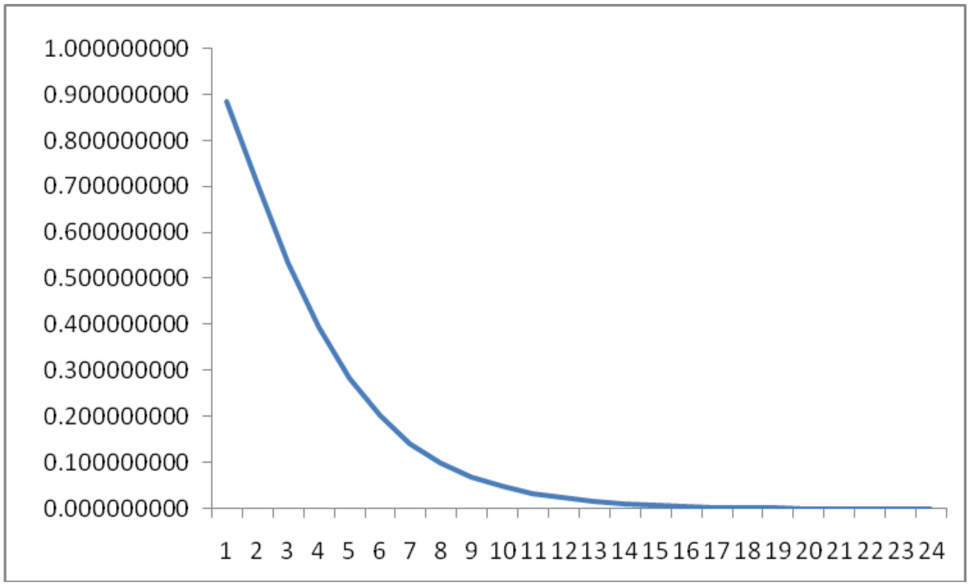


Figure-5 (Graph is plotted Tagainst η,for $\overline{v_s}$)

IV. CONCLUSION

It is therefore observed that, the Prandtl number serves as direct measure of the ratio of thickness of the two layers in forced flow. Here, the investigation for temperature distribution in liquid metal for which Prandtl number is very small.

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