Comparison and Analysis of Solution of Inverse Fractional heat diffusion equation solution with the Conformable fractional derivative

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Abstract- In this paper a comparative analysis has been done among Conformable fractional derivative and Caputo derivative for obtaining its inverse problem. The Laplace decomposition method is employed for obtaining the solution for both derivatives. We assembled that the temperature distribution in the case of Caputo's derivative increases as the value of α decreases from 1 to 0. Adomian decomposition method has been used for this research as it is the most efficient method. Several illustrative examples are given to demonstrate the effectiveness of the present method.
Keywords – Caputo Derivative, Conformable Derivative, Heat diffusion equations, Decomposition method, Laplace Transform, Inverse Problem.

I. INTRODUCTION

In the past few years, fractional calculus and their derivatives met across ample attainment in several fields of research viz. science and technology [2-4]. Fractional derivatives are quite elastic in elaborating viscoelastic nature.[70] Therefore, in the past ten years the analysis of fractional diffusion equations has fascinated extreme devotion.[66,67] Consider that if the preliminary concentration distribution and boundary conditions are known, a whole retrieval of the unidentified solution can be obtained by solving a well-posed forward problem.[68,69] Though, in few practical implementation problems, the boundary data can only be estimated on a share of the boundary or few facts in the result province.[64,65] This may give rise to an ill-posed problem of the fractional heat diffusion equation . In the paper, we analyze an inverse heat conduction problem (IHCP) given in the form of fractional derivative [62-63]. This type of ill-posed problem is crucial in many fields of engineering sciences [6-8]. This is known as the ill-posed backward determination problem that is in behavior unstable due to the unidentified solution. The inverse problems related to fractional differential equations are quite difficult. The individuality of an inverse problem for one-dimensional fractional diffusion equation was given in [9]. Zheng et al. and Wei et al. [10] have given the regularization method for Cauchy problem of the time fractional advection-dispersion equation in a space-unbounded region. Numerical results by utilizing different methodologies were being proposed in [11, 12]. Tuan et. al [13, 14] have analyzed the inverse problems related to spectral for the fractional diffusion equation.

Though, with the recent advancement in the area of fractional differential equations [3-6]; there can be a lot of diffusion problems which can be studied that can feature fractional order derivatives in aspect of time or space variables or both. Therefore, a lot of methods have been realized over a time to find the solution of many models which includes both the analytical solution and the numerical solution was given by Yan et al. [7], an estimated decomposition technique for non-integer diffusion-wave model [8] and the most famous and efficient method that is Adomian decomposition technique [9-13]. There are several other techniques which is given by many renowned authors that is the symmetry technique for the solution of nonlinear heat equation by Ahmad et al. [14], the Aboodh decomposition technique for nonlinear and time –fractional diffusion equations by Nurudeen et al. [15-16], the q-homotopy analysis technique [17], the double Laplace transform method for fractional heat equation by Anwar et al. [18] and finally the Wiener-Hopf method [19-21] some heat problems as compare to different methods.

Further, in the present research work, the time-fractional diffusion models in one and two dimensions will be studied and investigated. There are two possibilities which we have considered here is defining the time-fractional diffusion models in Caputo's derivative [4] sense and the new evolved derivative that is Conformable fractional derivative sense [6]. Here we employ the famous decomposition method of Laplace integral transform that is Adomian [10] as the technique to analyze, see [23-32, 34], also see [33] for the advancement of the conformable fractional derivative which is applicable for its alteration by [6] in this research work. [Base Paper References] So the present research has been motivated by [38] and the results of the direct problem and the inverse problem have been validated in case of both derivatives that is Caputo derivative, Conformable fractional derivative.

The paper is organized as follows: Section 2 presents the basic definitions for the two fractional derivatives. In section 3, we summarize the introduction of inverse problems for fractional heat diffusion equation. Various decomposition methods are studied in Section 4 Section 5 deals with the numerical results. Section 6 elaborates the attained results whereas In Section 7 the conclusion has been given.

II. BASIC DEFINITION OF THE TWO-FRACTIONAL DERIVATIVES

The following part includes the two most crucial definitions of fractional derivative that is Caputo fractional derivative and conformable fractional derivative that will be discussed in the following section and considered for the research.

Caputo's Derivative

The fractional derivative given by Caputo for a function v (t) [3-4] as follows:

$$D_t^{\alpha} v(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^{\infty} (t-s)^{-\alpha} v'(s) ds, (0 < \alpha \leq 1), \quad (2.1)$$

with $\Gamma(.)$ the gamma function which has the integral illustration as follows:

$$(x-1)! = \Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt.$$

The following properties are satisfied by Caputo fractional derivative:

$D_t^{\alpha} t^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)} t^{r-\alpha},$

$D_t^{\alpha}\left(cv(t)\right) = cD_t^{\alpha} v(t), where\ c\ is\ the\ constant,$

$D_t^{\alpha} c = 0,$

$D_t^{\alpha}\left(cv(t) + k\,x(t)\right) = cD_t^{\alpha}\left(v(t)\right) + kD_t^{\alpha}\left(x(t)\right),$

$D_t^{\alpha}\left(v(t)x(t)\right) = x(t)D_t^{\alpha}\left(v(t)\right) + v(t)D_t^{\alpha}\left(x(t)\right).$

To see more properties of Caputo Derivative see ref. [1-2].

Laplace Transform for Caputo's Derivative

The Laplace integral Transform for Caputo's basic definition for fractional order derivative as given in equation (2.1) which can be transformed as follows:

$$\mathcal{L}\{v^{\alpha}(t)\} = s^{\alpha}\mathcal{L}\{u(t)\} - \sum_{k=0}^{n-1} s^{\alpha-k-1} u^k(0), \alpha \in (n-1, n). \quad (2.2)$$

Mittag-Leffler Function

The Mittag-Leffler function [4] for fractional derivative is defined as follows:

$$E_{\alpha}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + 1)}. \quad (2.3)$$

Conformable Derivative

The new definition of the fractional derivative that is conformable derivative was given by Jawad et al. [6] states the two definitions that is left and right definitions of the fractional conformable derivatives which can be stated as follows:

$$T_{\alpha}(f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon} \quad (2.4)$$

$for\ all\ t > 0, \alpha\epsilon\ .If\ f\ is\ \alpha - differentiable\ in\ some\ (0, \alpha), a > 0, \log_{t\to 0^+} f^{(\alpha)}(t)\ exists,$

then define,

$$f^{(\alpha)}(0) = \lim_{t\to 0^+} f^{(\alpha)}(t).$$

In some cases $write\ f^{(\alpha)}(t) for\ T_{\alpha}(f)$ to specify the conformable fractional derivatives of order α. additionally, if the conformable fractional derivative of f of order α exists, then simply we can transform the definition and call f is α-differentiable.

We should remark that $T_\alpha(t^p) = pt^{p-\alpha}$. Furthermore, it can be defined from this definition that conformable fractional derivative coincides with R-L derivative for a constant multiple and gives the following definition:

$$aD_t^\alpha\big(v(t)\big) = (t-a)^{1-\alpha}v'(t), \tag{2.5}$$

and

$$bD_t^\alpha\big(v(t)\big) = (b-t)^{1-\alpha}v'(t), \tag{2.6}$$

where v is a differentiable function. It should be noted that the definitions (2.5)-(2.6) are local derivatives, see reference [5-6] for more details and [33] for the original research work. [39]

Some Useful Properties of Conformable Derivative

Assume T_α be the operator which is known as the fractional derivative of order α. For α=1, T1 satisfies the following properties:

$T_1(af+bg) = aT_1(g) + bT_1(f), for\ all\ a, b\ \in \mathbb{R}\ and\ f, g\ in\ the\ domain\ of\ T_1.$

$T_1(t^p) = pt^{p-1}\ for\ all\ p \in \mathbb{R}.$

$T_1(fg) = fT_1(g) + gT_1(f).$

$T_1\frac{f}{g} = \frac{gT_1(f) - fT_1(g)}{g^2}.$

$T_1(\lambda) = 0, for\ all\ constant\ functions\ f(t) = \lambda.$

To see more properties of Caputo Derivative see ref. [1-2].

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$$\mathcal{L}\{v^\alpha(t)\} = s^\alpha \mathcal{L}\{u(t)\} - \sum_{k=0}^{n-1} s^{\alpha-k-1}u^k(0), \alpha \in (n-1, n). \tag{2.2}$$

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Assume T_α be the operator which is known as the fractional derivative of order α. For α=1, T1 satisfies the following properties:
$T_1(af+bg)=aT_1(g)+bT_1(f), for\ all\ a,b\ \in\mathbb{R}\ and\ f,g\ in\ the\ domain\ of\ T_1.$
$T_1(t^p)=pt^{p-1}\ for\ all\ p\in\mathbb{R}.$
$T_1(fg)=fT_1(g)+gT_1(f).$
$T_1\frac{f}{g}=\frac{gT_1(f)-fT_1(g)}{g^2}.$
$T_1(\lambda)=0, for\ all\ constant\ functions\ f(t)=\lambda.$

III. LAPLACE DECOMPOSITION METHODS

Let us consider the initial-value problem of nonhomogeneous time-fractional partial differential equation of the state is illustrated as:

$$v_t^\alpha(x,t) = L\big(v(x,t)\big)+N\big(v(x,t)\big)+f(x,t), \alpha\epsilon(n-1,n), \tag{3.1}$$

with

$$D_o^k v(x,0)= g_k(x),\ \ (k=0,1,3,\dots .n-1),$$

(3.2)

$D_0^n(x,0)=0,\ \ n=[\alpha],$

where v_t^α denotes for the derivative of v fractional order α; f(x,t) is the source term, and L and N are the linear and nonlinear fractional differential operators, respectively. Therefore, we assume conditions of α utilizing the above definitions as below:

When derivative is in Caputo's derivative sense

Let us consider that α to be illustrated in the Caputo's fractional derivative sense, the following theorems are given as:

Theorem 1. Consider the initial-value problem (1.10) with the case (1.11) declares the following solution in Caputo's derivative sense:

$$\begin{cases} v_0(x,t)=\sum_{k=0}^{n-1}\frac{t^k}{\Gamma(k+1)}g_k(x)+\mathcal{L}^{-1}\left(\frac{1}{s^\alpha}\Big(\mathcal{L}\big(f(x,t)\big)\Big)\right), m=0, \\ \qquad v_{m+1}(x,t)=\mathcal{L}^{-1}(\frac{1}{s^\alpha}\mathcal{L}\big(L(v_m(x,t)+A_m)\big), m\geq 0. \end{cases}$$

Proof.

Taking the Laplace Transform in t to both sides of (1.10) with situation (1.10) illustrates

$$\mathcal{L}\{v(x,t)\}=\sum_{k=0}^{n-1}s^{-k-1}\ g_k(x) + \frac{1}{s^\alpha}\mathcal{L}\left(L\left(v(x,t)N\big(v(x,t)\big)+\big(f(x,t)\big)\right)\right). \tag{3.3}$$

Applying the inversion formulation of Laplace on (1.12) gives

$$v(x,t)=\sum_{k=0}^{n-1}\frac{t^k}{\Gamma(k+1)}g_k(x)+\ \mathcal{L}^{-1}\left(\frac{1}{s^\alpha}\Big(\mathcal{L}\big(f(x,t)\big)\Big)\right) + L^{-1}\left(\frac{1}{s^\alpha}\mathcal{L}\Big(L\big(v(x,t)\big)+N\big(v(x,t)\big)\Big)\right) \tag{3.4}$$

Now the function v(x,t) and N(v(x,t)) by the solution of the series represented as follows:

$$v(x,t)=\sum_{m=0}^{\infty}v_m(x,t), N\big(v(x,t)\big)=\sum_{m=0}^{\infty}A_m \tag{3.5}$$

where Am's are the adomian polynomials see reference [10]; we obtain

$$\sum_{m=0}^{\infty} v_m(x,t) = \sum_{k=0}^{n-1} \frac{t^k}{\Gamma(k+1)} g_k(x) + \mathcal{L}^{-1}\left(\frac{1}{s^\alpha}\left(\mathcal{L}\left(f(x,t)\right)\right)\right) + \sum_{m=0}^{\infty} \mathcal{L}^{-1}\left(\frac{1}{s^\alpha}\mathcal{L}\left(L\left(v_m(x,t)\right) + A_m\right)\right). \quad (3.6)$$

Thus, corresponding v0(x,t) with the expressions from non-homogenous and initial condition, and the rest vm(x,t) follow serially as illustrated below:

$$\begin{cases} v_0(x,t) = \sum_{k=0}^{n-1} \frac{t^k}{\Gamma(k+1)} g_k(x) + \mathcal{L}^{-1}\left(\frac{1}{s^\alpha}\left(\mathcal{L}\left(f(x,t)\right)\right)\right), \\ v_{m+1}(x,t) = \mathcal{L}^{-1}(\frac{1}{s^\alpha}\mathcal{L}\left(L\left(v_m(x,t) + A_m\right)\right) \quad (3.7) \end{cases}$$

Therefore, the estimated logical solution of equations (1.12) and (1.13) is obtained by the series

$$v(x,t) = \lim_{M \to \infty} \sum_{m=0}^{M} v_m(x,t) \qquad (3.8).$$

When derivative is in Conformable derivative sense

Let us consider the condition of α to be illustrated in the conformable fractional derivative sense. Also, before illustrating the approach it should be noted that the application of the general conformable derivative definition in (1.5) for α ∈ (n1,n) , (a=0).

Theorem 2.

Let us consider the initial value problem (1.10 with the case (1.11) agrees the below solution in the conformable derivative sense:

$$\begin{cases} v_0(x,t) = \sum_{k=0}^{[\alpha]-1} \frac{t^k}{\Gamma(k+1)} g_k(x) + \mathcal{L}^{-1}\left(\frac{1}{s^{[\alpha]}}\left(\frac{1}{t^{[\alpha]-\alpha}}\mathcal{L}\left(f(x,t)\right)\right)\right). m = 0. \\ v_m(x,t) = \mathcal{L}^{-1}\left(\frac{1}{s^{[\alpha]}}\mathcal{L}\left(\frac{1}{t^{[\alpha]-\alpha}}\left(L\left(v_m(x,t) + A_m\right)\right)\right)\right) m \geq 0. \end{cases}$$

Proof.

Firstly, equation (4.1) utilizing general definition of conformable derivative that is equation (2.4) as

$$t^{[\alpha]-\alpha} v^{[\alpha]}(x,t) = L\left(v(x,t)\right) + N\left(u(x,t)\right) + f(x,t) \qquad (3.9)$$

or

$$v^{[\alpha]}(x,t) = \frac{1}{t^{[\alpha]-\alpha}}\left(L\left(v(x,t)\right)\right) + N\left(v(x,t)\right) + f(x,t) \qquad (3.10)$$

Here [α] is the least integer of α. As in above, the Laplace transform arise from (4.10) and the case (4.2) the below expression as:

$$\mathcal{L}\{v(x,t)\} = \sum_{k=0}^{[\alpha]-1} s^{-k-1} g_k(x) + \frac{1}{s^{[\alpha]}}\mathcal{L}\left(\frac{1}{t^{[\alpha]-\alpha}}\left(L\left(v(x,t)\right)\right) + N\left(v(x,t)\right) + f(x,t)\right) \qquad (3.11)$$

Then by taking the inverse Laplace transform of the above equation and further as in exceeding equation the regressive formulation illustrated as follows:

$$\begin{cases} v_0(x,t) = \sum_{k=0}^{[\alpha]-1} \frac{t^k}{\Gamma(k+1)} g_k(x) + \mathcal{L}^{-1}\left(\frac{1}{s^{[\alpha]}}\left(\frac{1}{t^{[\alpha]-\alpha}}\mathcal{L}\left(f(x,t)\right)\right)\right), m = 0. \\ v_{m+1}(x,t) = \mathcal{L}^{-1}\left(\frac{1}{s^{[\alpha]}}\mathcal{L}\left(\frac{1}{t^{[\alpha]-\alpha}}\left(L(v_m(x,t) + A_m)\right)\right)\right) \end{cases}$$

(3.12)

Here the estimated analytical solution of (4.1)-(4.2) is illustrated by the following series:

$$v(x,t) = \lim_{M\to\infty} \sum_{m=0}^{M} v_m(x,t). \quad (3.13)$$

IV. COMPARISON OF RESULTS

In this section our main goal is to make comparisons between Caputo's and fractional conformable derivatives solutions of direct and inverse problems presented in Examples 1-3. The temperature distributions in each case would be investigated by learning the outcome of changing α ranging from 0 to 1. The comparison tables are presented in Tables 1-3 with conforming graphical representations represented in Fig.7-9.

4.1 Example One

In example one, it was perceived from Fig. 1 that the temperature distribution in the case of Caputo's derivative increases as the value of α decreases from 1 to 0 ; while opposite trend is perceived in the case of fractional conformable derivative as shown in Fig. 2. However it should be noted that the two are the same at α=1 and also the temperature distribution is seen when α=0 in case of Caputo's fractional derivative while it diverges in conformable sense. For this see Table 1. and Fig. 7. The Table 1. illustrates that the difference between Direct problem and inverse problem in case of both derivative is observed and the absolute difference between both derivative is also depicted and validated by the graph of direct problem.

Table.1 Comparison of Caputo's and Fractional Conformable derivative solutions of example.1 at M=11, t=2, x=π/3.

Values of alpha	Direct Problem	Direct Problem	Inverse Problem	Inverse Problem	Inverse Problem	Relationship between Direct Problem and Inverse Problem	Relationship between Direct Problem and Inverse Problem
α	Caputo Derivative	Conformable Derivative	Caputo Derivative	Conformable Derivative	Absolute Difference	Absolute Difference (Caputo Derivative)	Absolute Difference (Conformable Derivative)
0.0	0.500000	-	0.500000	-	-	-	-
0.1	0.487485	-1.5625	0.36568	-1.6589	2.0247	0.121805	0.0964
0.3	0.348997	-0.306019	0.2500	-0.4	0.65	0.098997	0.1
0.5	0.2150	-0.038	0.2156	-0.028	0.2436	-0.0006	-0.01
0.7	0.1998	0.084	0.1956	0.078	0.1176	0.0042	0.006
0.9	0.183940	0.1573	0.1876	0.1452	0.0424	-0.00366	0.0121

Validation of Results

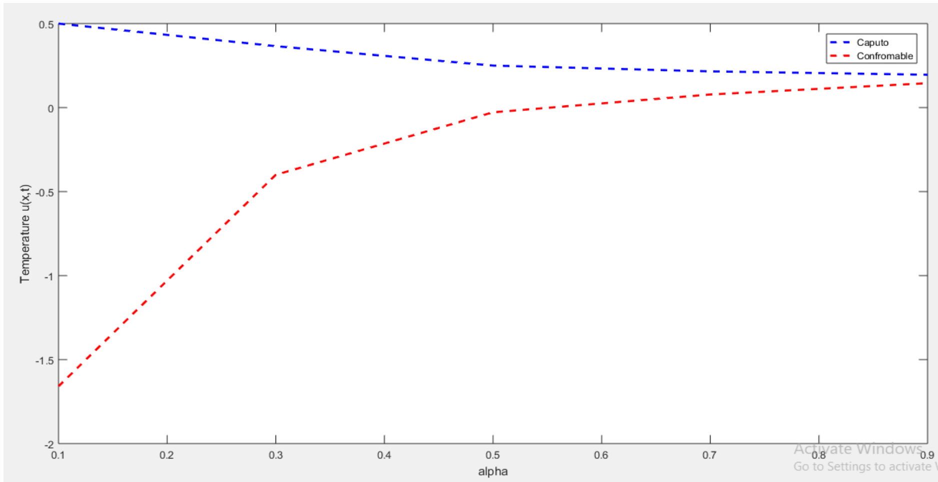

Fig. 7. : Temperature plots for comparison of solutions of Example One for Inverse Problem

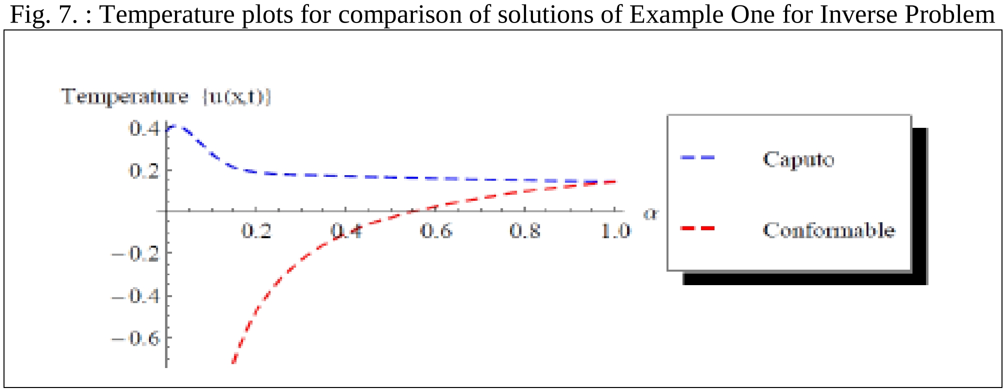

Fig. 8. : Temperature plots for comparison of solutions of Example One for Direct Problem of

Example One

In example two, it was perceived from both Fig. 3 and 4. that the temperature distribution in the case of Caputo's and fractional conformable derivative decreases as the value of α increases from 0 to 1. However it should be noted that the two are the same at α=1 and the temperature distribution is also seen when α=0 in case of Caputo's fractional derivative when it diverges in conformable sense. See Table 2. and Fig. 8. and Fig. 8.The Table 2. illustrates that the difference between Direct problem and inverse problem in case of both derivative is observed and the absolute difference between both derivative is also depicted and validated by the graph of direct problem.

Table.2 Comparison of Caputo's and Fractional Conformable derivative solutions of example.2 at M=21, t=0.6, x=0.7.

Values of alpha	Direct Problem	Direct Problem	Inverse Problem	Inverse Problem	Inverse Problem	Relationship between Direct Problem and Inverse Problem	Relationship between Direct Problem and Inverse Problem
α	Caputo Derivative	Conformable Derivative	Caputo Derivative	Conformable Derivative	Absolute Difference (Caputo and Conformable for Inverse Problem)	Absolute Difference (Caputo Derivative for Direct and Inverse Problem)	Absolute Difference (Conformable Derivative for Direct and Inverse Problem)
0.0	1.15866 X	-	1.2563X	-	-	-	-

	1018		1015				
0.1	1.43653 X 1017	1.129X 1019	2.567 X 108	1.35 X 1017	1.217X 1017	-1.13047 X109	-0.088
0.3	2.38508 X 1013	1.63626 X 1013	2.35 X 1013	1.4256 X 108	0.9244 X 1013	0.03508	0.21066
0.5	2.67164 X 1011	300.13	1.5 X 1018	350	1.54 X 1018	1.17164 X 10-7	-49.87
0.7	561.961	0.28394	512.693	0.3256	512.3674	49.5936	-0.04166
0.9	0.326371	0.313401	1.2563X 1015	-	-	-0.929929	0.313

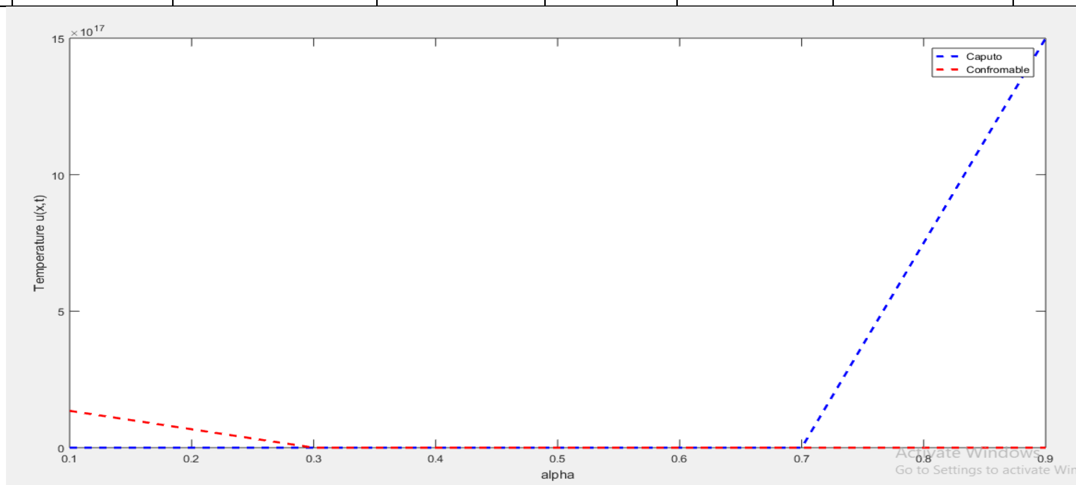

Fig. 9 Temperature plots for comparison of solutions of Example Two for Inverse Problem

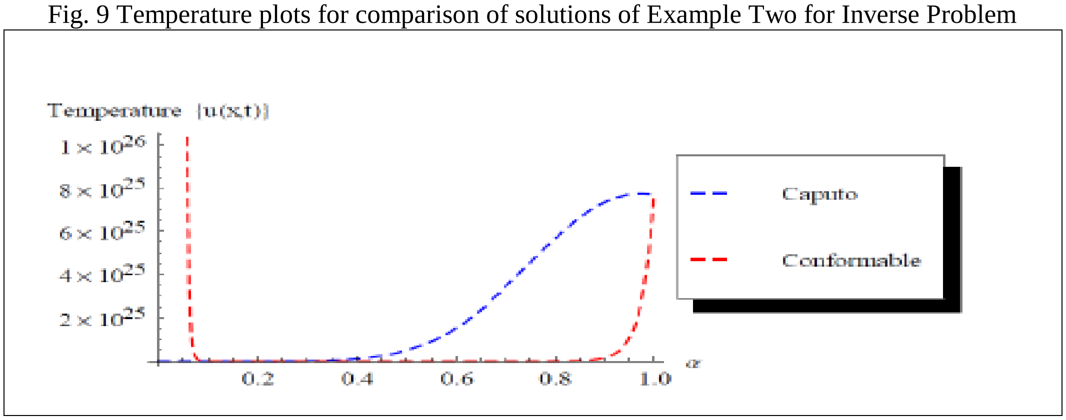

Fig. 10 Temperature plots for comparison of solutions of Example Two for Inverse Problem

Example Three
Example three has the same elucidation with example one. One can easily see that there is a very little difference and upon fixing y variable we reached at example one's solution. Therefore, see Table 3. and Fig.11 respectively.
Table 3. Comparison of Caputo's and Fractional Conformable derivative solutions of example.3 at M=21, t=0.6, $x=\frac{6\pi}{4}$, $y=\frac{5\pi}{4}$.

Values of alpha	Direct Problem	Direct Problem	Inverse Problem	Inverse Problem	Inverse Problem	Relationship between Direct Problem and Inverse Problem	Relationship between Direct Problem and Inverse Problem
α	Caputo Derivative	Conformable Derivative	Caputo Derivative	Conformable	Absolute Difference	Absolute Difference	Absolute Difference

				Derivative	(Caputo and Conformable for Inverse Problem)	(Caputo Derivative for Direct and Inverse Problem)	(Conformable Derivative for Direct and Inverse Problem)
0.0	0.241845	-	0.231763	-	-	-	-
0.1	0.175102	-1.61978	0.19	-1.65	1.84	-0.014898	0.04
0.3	0.110547	-0.391489	0.12	-0.3856	0.5056	-0.009453	-0.005889
0.5	0.103409	-0.126223	0.11	-0.12563	0.23563	-0.006591	-0.000593
0.7	0.0966441	-0.00544053	0.09256	-0.005485	0.098045	0.0040841	0.00004447
0.9	0.0909496	0.0642243	0.0905	0.0685698	-		

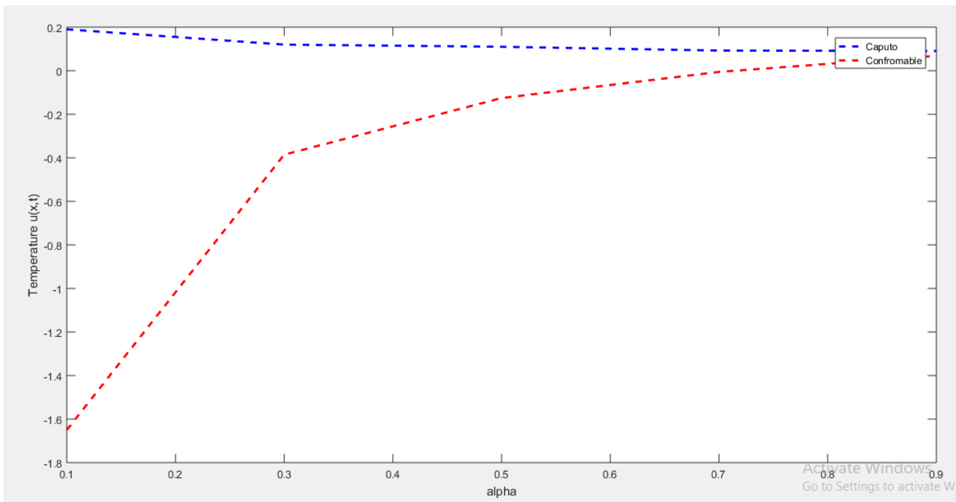

Fig. 11 Temperature plots for comparison of solutions of Example Three.

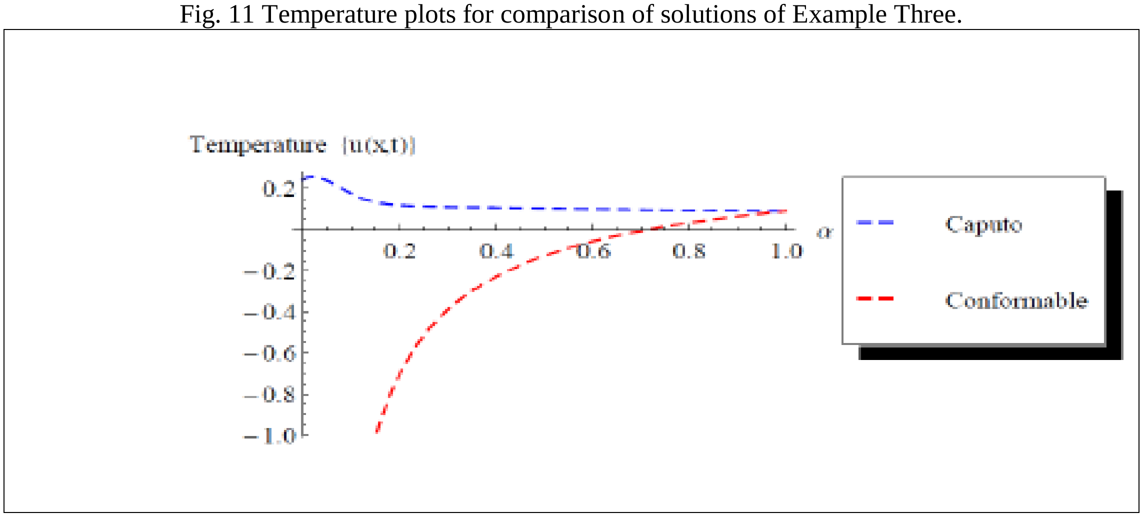

Fig. 12 Temperature plots for comparison of solutions of Example Three for Inverse Problem

V. CONCLUSION

In conclusion, the one and two-dimensional fractional heat diffusion models including fractional order derivative in time have been investigated. The fractional orders assumed here involve the Caputo's and the new fractional conformable derivatives. Here the Laplace integral transform method in union with the decomposition method by Adomian is employed as a tool for this research. We assembled that the temperature distribution in the case of Caputo's derivative increases as the value of α decreases from 1 to 0; while opposite trend is perceived in examples 1 and 3. We have considered here the inverse problem of the for conformable and Caputo's derivative. The opposite trend has been obtained in comparison with the direct problem and has been validated with the graph exist. Though it should be noted that the two are the same at α=1 and also the temperature distribution is realized when α=0 in case of Caputo's fractional derivative while it diverges in conformable sense in all the three examples. We observed that in case of Caputo fractional derivative the solution is positive and in case of conformable fractional derivative the solution is negative.

VI. REFERENCE

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