A Study on Reliability Optimization of Selective Maintenance on Equipment

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Abstract - In the field of reliability optimization often comes the problem of selective maintenance. A system with series and parallel components often have situations where it takes small breaks or intervals and start functioning again. These intervals give the experts time to replace or repair deteriorating components of the system which is termed as selective maintenance. For such system, a decision making problem may be formulated as to optimize the reliability taking into consideration the time or cost spent on components. Further, there are cases where the reliability of individual components, cost and time involved are not crisp instead there is imprecision or variability of data. The problem of selective maintenance into a multi-objective nonlinear programming problem with fuzzy numbers and solve it using Fuzzy Programming Problem. Also, a numerical example is used to illustrate the applicability of the formulated problem for certain degree of membership at a particular cut.

Keywords: Fuzzy reliability, Fuzzy Weibull distribution.

I. INTRODUCTION

Reliability is the probability of an item to perform consistently a required function or mission without failure for a stated condition or interval of time. In a system, reliability of individual component in that system plays an important role for its proper functioning. There are many systems which accomplish a sequence of operation with finite breaks at regular intervals. These breaks give an opportunity to repair or replace deteriorated components in the system so as to improve the system reliability that eventually improve the functioning of the whole mission. Such kind of system go through “selective maintenance”.

II. LITERATURE SURVEY

The study of the selective maintenance in system was originally performed by Rice et al. (1998). They modeled a system of identical parallel series components and developed a decision making model to optimize. Cassady et al. (2001b) improved the selective maintenance by removing the structural restriction on subsystem and developing a general framework with binary state components. They further considered the components of the system to follow Weibull distribution with multiple maintenance actions such as minimal repair and corrective replacement in Cassady et al. (2001a). Lust et al. (2009) discarded the conventional enumeration method and developed three new methods viz. a construction heuristic, a heuristic based on the adaptation of Tabu search, and an exact method based on a branch and bound procedure for various system configurations. Several other authors who have studied numerous reliability optimization techniques and selective maintenance policies in reliability improvement are Tillman et al. (1980), Kuo et al. (1987), Chern (1992), Wang (2002), Iyoob et al. (2006), Nourelfath and Ait Kadi (2006), Rajagopalan and Cassady (2006), Nahas et al. (2008), Bartholomew-Biggs et al. (2009), Schneider et al. (2009), Liu and Huang (2010), Gupta et al. (2014) etc. Now, many systems involve uncertainties and imprecision in data where the estimation of precise values of probabilities is very difficult. In such scenario fuzzy reliability is of much help. Singer (1990), considered the fuzzy reliability of both serial and parallel systems using an approximation of a fuzzy binary operation with two L-R type fuzzy numbers.

Cheng and Mon (1993) used the a–cut of a triangular fuzzy number to get the interval and find the fuzzy reliability of the serial system and the parallel system. Chen (1994) likewise omitted the statistical approach when he used fuzzy numbers to find the fuzzy reliability of both the serial system and the parallel system. In Mon and Cheng (1994), the authors used the a–cut of fuzzy number to derive a non-linear program of the fuzzy system in both the serial and parallel cases. Other researchers who have worked in this field are Park (1987), Rao and Dhingra (1992), Cai et al. (1993), Ravi et al. (2000), Khasawneh et al. (2002), Verma et al. (2004), Mahapatra and Roy (2006), and Ali et al. (2012, 2011) has done research on different fuzzy approaches in system reliability.

Yao et al. (2008) in their paper discussed the problem of fuzziness in system reliability. They justified that reliability of a system may fluctuate at point estimate during a time interval. Statistical confidence interval may be used instead of the point estimate which was further transferred into a triangular fuzzy number. This paper deals with a problem of system reliability in selective maintenance model. Two types of components (repairable and replaceable) are
assumed to be involved. The system consists of three groups of subsystems viz. X, Y and Z and reliability of each subsystem is needed to be maximized. The system runs after a fixed interval of time. In this limited interval of time, maintenance is performed keeping in view the fixed budget for repair and replacement of components. The reliability of individual components, cost and time involved are considered to be fuzzy in nature. Moreover, the fuzziness is defined by triangular fuzzy numbers (TFNs). The selective maintenance problem is formulated as a multi-objective nonlinear programming problem with fuzzy cost, time and reliability. The compromise allocation of repairable and replaceable components is then obtained using fuzzy programming approach. The problem is also illustrated in a numerical example with assumed data.

III. FUZZY SELECTIVE MAINTENANCE MODEL

We consider a system which needs to perform a sequence of identical missions after every given (fixed) period. The system consists of several subsystems. These subsystems are divided into three groups viz. X, Y and Z.

Assumptions for the model of system are given below.

i) Two types of components are subject to the system
Functioning of one type of component (say type-I) is highly sensitive and hence deterioration of any such component leads to complete replacement.
Another type of component (say type-II) in the system are at low risk and after deterioration they can be repaired and placed back.

ii) Three groups X, Y and Z are connected in series
Group X has type-I components each of which are connected in parallel in subsystems with 1 to s number of subsystems connected in series.
Group Y has type-II components which are also connected in parallel in subsystems with m-s number of subsystems connected in series.
Group Z has type-I and type-II components connected in series with reliability same as in group X and group Y respectively. The series of duo are connected in parallel in subsystem. There are k-m number of subsystems connected in series.

iii) It is known that all components are subject to repair and replacement prior to the next run.

The model is shown below (Fig. 1) for better understanding.

Each component of the system has fuzzy reliability which is previously measured and given by triangular fuzzy number (see Yao et al. (2008)). The objective of tire decision maker is to maximize the reliability of each group X, Y and Z improving the reliability of overall system. Group X of the system is a series arrangement of s subsystems and its reliability can be defined as

\[
R_X = \prod_{i=1}^{s} 1 - (1 - \tilde{r}_i)^{n_i - a_i + d_i}
\]

, \( i = 1, \ldots, s. \) (1)

Group Y consists of m-s subsystems connected in series and the reliability can be defined as

\[
R_Y = \prod_{i=s+1}^{m} 1 - (1 - \tilde{r}_i)^{n_i - a_i + d_i}
\]

, \( i = s + 1, \ldots, m. \) (2)

Again group Z is a mixed model consists of k-m subsystems with both series and parallel connections. The reliability is given by

\[
R_Z = \prod_{i=m+1}^{k} 1 - (1 - \tilde{r}_i)^{2n_i - a_i + d_i + e_i}
\]

, \( i = m + 1, \ldots, k. \) (3)

where \( n_i - a_i + d_i \) = number of components left at the start of next run in X group,
n₁ - aᵢ + dᵢ = number of components left at the start of next run in Y group,
2nᵢ - aᵢ + dᵢ + dᵢ = number of components left at the start of next run in Z group,
ᵢ̃ = is the fuzzy reliability of the components that cannot be repaired,
ᵢ̃ᵣ = is die fuzzy reliability of the components that can be repaired,
nᵢ = number of components in ith subsystem,
aᵢ = number of failed components in ith subsystem,
dᵢ = number of components in ith subsystem that cannot be repaired and only replaced,
dᵢ = number of components in ith subsystem that can be replaced after repairing.

Now, while trying to maximize the reliability of the system the decision maker will also have to take care of the time limits and cost of repairable and replaceable components.

Let us consider before the next run, amount and time spent for replacing components are Tᵢ' and Cᵢ' respectively. And Tᵢ' and Cᵢ' for components that can be repaired.

Thus, the first two constraints are the time and cost constraint for components that can not be repaired and need replacement with a new one. The last two constraints are the time and cost constraint for components that can be repaired respectively.

\[
\sum_{i=1}^{s} t_i d_i + \sum_{i=m+1}^{k} t_i d_i \leq T_o,
\]
\[
\sum_{i=1}^{s} c_i d_i + \sum_{i=m+1}^{k} c_i d_i \leq C_o.
\]
\[
\sum_{i=m+1}^{k} t_i d_i' + \sum_{i=m+1}^{k} t_i d_i' \leq T_o',
\]
\[
\sum_{i=s+1}^{m} c_i d_i' + \sum_{i=m+1}^{k} c_i d_i' \leq C_o'.
\]

where \(t_i\) is the time needed to replace a component,
\(t_i'\) is the time needed to repair and replace a component,
\(c_i\) is the cost per component that can not be repaired and need immediate replacement with a new one,
\(c_i'\) is the cost per component that can be repaired.

Now, we find that the cost and time of replaceable items are known but the cost and time of repairing a component depends on the type of repairing needed because of several diverse situations like uncertain judgements, unpredictable conditions or human error, etc., due to which it is not always necessary that we get a precise data. Such type of imprecise data can be very well handled by fuzzy numbers, thus we may consider that the cost and time of repair are triangular fuzzy numbers, i.e., the component is subject to say three types of repairing.

Thus, the final multi-objective nonlinear programming problem (MONLPP) of the decision maker with fuzzy parameters will be as given below.

Maximize \(R_X = \prod_{i=1}^{s} [1 - (1 - \tilde{r}_i)^{n_i - a_i + d_i}]\)

Maximize \(R_Y = \prod_{i=s+1}^{m} [1 - (1 - \tilde{r}_i)^{n_i - a_i + d_i}]\)

Maximize \(R_Z = \prod_{i=m+1}^{k} [1 - (1 - \tilde{r}_i)^{2n_i - a_i + d_i + d_i}]\)

Subject to \(\sum_{i=1}^{s} t_i d_i + \sum_{i=m+1}^{k} t_i d_i \leq T_o,\)
\(\sum_{i=1}^{s} c_i d_i + \sum_{i=m+1}^{k} c_i d_i \leq C_o.\)
\[
\sum_{i=1}^{m} t_i d'_i + \sum_{i=m+1}^{k} t_i d'_i \leq T'_u,
\]
\[
\sum_{i=1}^{m} c_i d'_i + \sum_{i=m+1}^{k} c_i d'_i \leq C'_u,
\]
\[
2 \leq d_i, d'_i \leq a_i, i = 1,2,\ldots,k
\]
where \( \tilde{r}^i = \tilde{r}^i \) is considered for simplicity. The last two constraints are the time and cost constraint of components that are subject to three kind of repairing.

A. Converting Fuzzy Model into an Equivalent Crisp Model –

Let \( a(\tilde{R}) \) be the \( \alpha \)-cut of a fuzzy number \( \tilde{R} \) defined by

\[
a(\tilde{R}) = \{ r \in \text{supp}(\tilde{R}) \mid \mu_\tilde{R}(r) \geq \alpha, \alpha \in [0,1] \}
\]
where \( \text{supp}(\tilde{R}) \) is the support of \( \tilde{R} \). Let \( a(\tilde{R})^L \) and \( a(\tilde{R})^U \) be the lower bound and upper bound of the \( \alpha \)-cut of \( \tilde{R} \) respectively such that

\[
a(\tilde{R})^L \leq a(\tilde{R}) \leq a(\tilde{R})^U
\]

Then, for a prescribed value of \( \alpha \), objectives \( \tilde{R}_x \), \( \tilde{R}_y \) and \( \tilde{R}_z \) to be maximized can be replaced by the upper bound of their respective \( \alpha \)-cuts, that is

\[
a(\tilde{R}_x)^U = \prod_{i=1}^{n-k} \left(1 - \left(1 - a(\tilde{r}^i)^L\right)^{-a+1}d_i\right)^U
\]
\[
a(\tilde{R}_y)^U = \prod_{i=1}^{n-k} \left(1 - \left(1 - a(\tilde{r}^i)^L\right)^{-a+1}d_i\right)^U
\]
\[
a(\tilde{R}_x)^U = \prod_{i=m+1}^{k} \left(1 - \left(1 - a(\tilde{r}^i)^L\right)^{-a+1}d_i\right)^U
\]

For inequality constraints with TFNs
\[
\sum_{i=1}^{m} \tilde{t}_i d'_i + \sum_{i=m+1}^{k} \tilde{t}_i d'_i \leq T'_u
\]
\[
\sum_{i=1}^{m} \tilde{c}_i d'_i + \sum_{i=m+1}^{k} \tilde{c}_i d'_i \leq C'_u
\]
can be rewritten as follows
\[
\sum_{i=1}^{m} a(\tilde{t}^i)^L d'_i + \sum_{i=m+1}^{k} a(\tilde{t}^i)^L d'_i \leq T'_u
\]
\[
\sum_{i=1}^{m} a(\tilde{c}^i)^L d'_i + \sum_{i=m+1}^{k} a(\tilde{c}^i)^L d'_i \leq C'_u
\]

Therefore, the problem represented by (5) can be transformed into the following standard multi-objective nonlinear programming problem (MONLPP).

Maximize \( a(\tilde{R}_x)^U = \prod_{i=1}^{n-k} \left(1 - \left(1 - a(\tilde{r}^i)^L\right)^{-a+1}d_i\right)^U \),

Maximize \( a(\tilde{R}_y)^U = \prod_{i=1}^{n-k} \left(1 - \left(1 - a(\tilde{r}^i)^L\right)^{-a+1}d_i\right)^U \),

Maximize \( a(\tilde{R}_z)^U = \prod_{i=m+1}^{k} \left(1 - \left(1 - a(\tilde{r}^i)^L\right)^{-a+1}d_i\right)^U \),

Subjected to \( \sum_{i=1}^{m} t_i d_i + \sum_{i=m+1}^{k} t_i d_i \leq T'_u \),
\[
\sum_{i=1}^{k} a_i d_i + \sum_{i=1}^{k} c_i d_i \leq C_i,
\]
\[
\sum_{i=1}^{k} a_i (\tilde{r})_i^c d_i' + \sum_{i=1}^{k} a_i (\tilde{r})_i^c d_i' \leq T_i',
\]
\[
\sum_{i=1}^{k} a_i (\tilde{c})_i^c d_i' + \sum_{i=1}^{k} a_i (\tilde{c})_i^c d_i' \leq C_i',
\]
\[2 \leq d_i, d_i' \leq a_i, \quad i = 1, 2, \ldots, k. \]

Membership function for the objective functions of group X, Y and Z can be given by

\[
\mu_j^\alpha (R_j) = \frac{[\alpha (\tilde{R}_j)_1^U - \alpha (\tilde{R}_j)_1^N]}{[\alpha (\tilde{R}_j)_1^O - \alpha (\tilde{R}_j)_1^N]} \quad j = X, Y, Z
\]

where the aspired level \(\alpha(R_j)O\) and highest acceptable level \(\alpha(R_j)–\) are ideal and anti ideal solutions respectively, which can be obtained by solving each of the following problems independently.

\[
\min \prod_{i=1}^{k} 1 - \{1 - [r(1) + (r(2) - r(1))\alpha]\}_{n_i - d_i, d_i}, \quad \alpha(RX) = \frac{\xi}{\alpha(RX) - \alpha(RXO)}
\]

Subjected

\[
\sum_{i=1}^{k} t_i d_i + \sum_{i=1}^{k} t_i d_i' \leq T_i,
\]
\[
\sum_{i=1}^{k} c_i d_i + \sum_{i=1}^{k} c_i d_i' \leq C_i,
\]

Table 1: Numerical value of some parameters involved in selective maintenance model

<table>
<thead>
<tr>
<th>Subsystems</th>
<th>Group X</th>
<th>Group Y</th>
<th>Group Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3</td>
<td>4 5 6</td>
<td>7 8 9</td>
</tr>
<tr>
<td>ni</td>
<td>7 5 8</td>
<td>8 10 12</td>
<td>8 7 10</td>
</tr>
<tr>
<td>ai</td>
<td>3 2 4</td>
<td>3 4 5</td>
<td>12 8 11</td>
</tr>
</tbody>
</table>
IV. SOLUTION FOR FUZZY MODEL

The MONLPP formulated using (11) is

\[
\text{Max } R_1 = 1 - (1 - \bar{d}_1) \times 1 - (1 - (0.65) \times 1 - (1 - \bar{d}_3) \times 1 - (1 - \bar{d}_5) \times 1 - (1 - \bar{d}_7) \\
\text{Max } R_2 = 1 - (1 - \bar{d}_7) \times 1 - (1 - (0.55) \times 1 - (1 - \bar{d}_6) \times 1 - (1 - \bar{d}_4) \times 1 - (1 - \bar{d}_8) \\
\text{Max } R_3 = 1 - (1 - (0.525 \times 0.55) \times 1 - (1 - (0.360 \times 0.55)) \times 1 - (1 - (0.70 \times 0.60)) \times 1 - (1 - (0.525 \times 0.0675)) \times 1 - (1 - (0.85 \times 0.70)) \\
\]
The final compromise allocation of the repairable and replaceable components at a prescribed level of \( \alpha \), i.e., \( \alpha = 0.5 \) is obtained after solving the above FPP.

The compromise allocation obtained after solving the FPP are \( \bar{d}^c = (2,5,2,4,5,1) \) and \( \bar{d}'^c = (4,6,4,4,7,2) \) \( \rho = 6.38e-06 \), where \( \bar{d}^c \) is \((d_1,d_2,d_3,d_7,d_8,d_9)\) and \( \bar{d}'^c \) is \((d'_1,d'_2,d'_3,d'_7,d'_8,d'_9)\).

### A. Solution for Crisp Values

The MONLPP formulated using (11) is

\[
\begin{align*}
\text{Max } & \quad R_1 (1 - (0.65)^{d_1} + 1 - (0.55)^{d_2} + 1 - (0.70)^{d_3}) \\
\text{Max } & \quad R_2 (1 - (0.70)^{d'_1} + 1 - (0.55)^{d'_2} + 1 - (0.60)^{d'_3}) \\
\text{Max } & \quad R_3 (1 - (0.65)^{d_7} + 1 - (0.55)^{d_8} + 1 - (0.70)^{d_9}) \\
\text{s.t. } & \quad 6d_1 + 10d_2 + 7d_3 \leq 75 \\
& \quad 16d_1 + 12d_2 + 13d_3 \leq 150 \\
& \quad 3d'_1 + 4d'_2 + 3d'_3 \leq 40 \\
& \quad 8d'_7 + 7d'_8 + 8d'_9 \leq 115 \\
& \quad 6d_7 + 10d_8 + 7d_9 \leq 75 \\
& \quad 16d_7 + 15d_8 + 13d_9 \leq 150 \\
& \quad 3d'_7 + 4d'_8 + 3d'_9 \leq 40 \\
& \quad 8d'_7 + 7d'_8 + 8d'_9 \leq 115 \\
& \quad 1 \leq d_i, \quad d'_i \leq n_i
\end{align*}
\]

The ideal and anti-ideal solution for each individual objective function is given by

\[
\begin{align*}
\alpha_{(RX)O} = 0.9952518, \\
\alpha_{(RX)} = 0.9952518, \\
\alpha_{(RY)O} = 0.9996763, \\
\alpha_{(RY)} = 0.9996763.
\end{align*}
\]

Now, the final compromise allocation of the components with crisp values of time, cost and reliability is obtained after solving the following FPP.

\[
\begin{align*}
\text{max } & \quad \rho' \leq ((1 - (0.65)^{d_1} \times (1 - (0.55)^{d_2} \times (1 - (0.70)^{d_3})) - 0.995252

\text{s.t. } & \quad 0.043613
\\
\rho' \leq ((1 - (0.70)^{d'_1} \times (1 - (0.55)^{d'_2} \times (1 - (0.60)^{d'_3})) - 0.999676

\text{s.t. } & \quad 0.004792
\\
\rho' \leq ((1 - (0.65)^{d_7} \times (1 - (0.55)^{d_8} \times (1 - (0.70)^{d_9})) - 0.994769

\text{s.t. } & \quad 0.077824
\\
\text{s.t. } & \quad 6d_1 + 10d_2 + 7d_3 \leq 75 \\
& \quad 16d_1 + 12d_2 + 13d_3 \leq 150 \\
& \quad 3d'_1 + 4d'_2 + 3d'_3 \leq 40 \\
& \quad 8d'_7 + 7d'_8 + 8d'_9 \leq 115 \\
& \quad 6d_7 + 10d_8 + 7d_9 \leq 75 \\
& \quad 16d_7 + 15d_8 + 13d_9 \leq 150 \\
& \quad 3d'_7 + 4d'_8 + 3d'_9 \leq 40 \\
& \quad 8d'_7 + 7d'_8 + 8d'_9 \leq 115 \\
& \quad 1 \leq d_i, \quad d'_i \leq n_i
\end{align*}
\]

The compromise allocation obtained after solving the FPP are \( \bar{d}^c = (3,5,1,6,3,1) \) and \( \bar{d}'^c = (3,5,3,1,7,3) \) \( \rho = 2.70e-07 \), where \( \bar{d}^c \) is \((d_1,d_2,d_3,d_7,d_8,d_9)\) and \( \bar{d}'^c \) is \((d'_1,d'_2,d'_3,d'_7,d'_8,d'_9)\).

<table>
<thead>
<tr>
<th>Reliability for Fuzzy model</th>
<th>Group X</th>
<th>Group Y</th>
<th>Group Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9957569</td>
<td>0.9998694</td>
<td>0.9999698</td>
</tr>
<tr>
<td>Reliability for Crisp model</td>
<td>0.9952518</td>
<td>0.9996763</td>
<td>0.9947687</td>
</tr>
</tbody>
</table>
V. CONCLUSION

The cost and time required for the two types of components taken into consideration are also fuzzy in nature. Moreover, the decision maker has a hypothetical model with multiple objectives of maximizing reliability of different groups of subsystems. The objective of the decision maker is to simultaneously maximize the reliability of all the subsystems and find the optimal number of components required to be replaced or repaired after deterioration in the next run. Since the individual optimal solution of each group of subsystem might not happen to be optimal for another group, an optimal compromise allocation of repairable and replaceable components in the system is obtained. The problem is formulated as a multi-objective nonlinear programming problem with TFNs and solved using fuzzy programming problem at a prescribed level of cut.

VI. REFERENCES