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Estimating the Capacity of a Service Facility with Stochastic Demands

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Abstract- This study discusses how to estimate economically the service capacity that accommodates customers arriving at the service facility with stochastic demands. It is assumed that the relationship among cost factors and its demand probability distribution parameters during endurance life of a service facility are static. The cost to gain a service facility consists of fixed costs of any capacity and variable costs proportional to capacity. The optimal model to minimize the sum of overage cost and shortage cost of service capacity is constructed. The sensitivity analysis is performed to investigate the effects of a variety of parameters.
Keywords –Facility planning, Service facility, Capacity management, Single-period inventory

I. INTRODUCTION

Service facilities cannot retain surplus capacity in stock, so capacity is lost if not sold or utilized. Due to this perishability, capacity management, which adjusts the supply capacity of service facilities, becomes more important to respond to uncertain demand changes.If the capacity of a service facility does not meet the expected demand, there is a loss of opportunity due to oversupply or loss of service [1]-[3]. Therefore, in determining the capacity of service facilities, the estimation of economic capacity to cope with future demand is a very important decision problem.

The purpose of this study is to estimate the capacity of service facilities, which are of major interest in the design of service facilities, in terms of expected cost. This study proposes an estimation model that minimizes the sum of shortage and excess costs for service facilities in consideration of changes in demand and examines the effects of various related factors through sensitivity analysis.

In section Ⅱ, we propose an optimal model for capacity estimation that minimizes the total expected cost when a probability distribution for service demand is given. The comparison with the single-period inventory model and the results of the sensitivity analysis are summarized in section Ⅲ. Section Ⅳ summarizes the results of this study.

II. CAPACITY OF SERVICE FACILITY

Capacity estimation is animportant decision in service supply management. For example, in hospitals, hotels, restaurants, etc., the capacity to provide services is determined by the number of rooms, seats, and employees. However, because it is difficult to significantly change the capacity of a service facility, a service provider cannot provide services to some customers whose excess demand exceeds its supply capacity. One of the main tasks of service management is to manage service demands to meet their capacity.Optimal service capacity may increase service productivity.In general, service capacity means maximum capacity and is determined at the beginning of service.This study considers the following assumptions. First, it is difficult to adjust the capacity of a service facility whose capacity has been determined through the expansion, closure or lease of the facility according to the change in demand within the operation period. Second, the capacity of facilities not used within the unit period cannot be transferred to the next period, and the demand exceeding the capacity cannot be handled beyond the period by waiting or delay. Third, the service facility does not have a customer-specific policy and the minimum period of use is not fixed. Fourth, the demand for service facilities is independent of each other in time series and follows the same probability distribution.Capacity estimation problem has been applied to the solution of single-period inventory model. In the single-period inventory model, marginal analysis is applied to the costs incurred by the overcapacity of demand and the costs incurred by securing the capacity against the demand. This model uses equation (1) to find a capacity where the expected marginal gain and expected marginal loss are equal.In equation (1), c_e is the excess cost per unit of capacity, c_u is the minimum cost per unit of capacity, andS^* is the optimal capacity. $P(x\leq S^*)$ is the cumulative probability of demand for service facility use that does not exceed optimal capacity.

$$c_e P(x\leq s^*) = c_u (1 - P(x\leq s^*)) \quad (1)$$

Fitzsimmons and Fitzsimmons showed that a single-period inventory model could be applied to the strategy of canceling hotel room reservations [4]. An empirical study was conducted to estimate. Pfeifer and Smith et al. exemplified the possibility of analyzing overbooking with a single-term inventory model even for the issue of yield management of aircraft seats [5]-[6].

A single-period inventory model is limited in reflecting the uncertainty of demand and the various cost factors that need to be considered in capacity planning for service facilities. Therefore, this study intends to propose an optimal capacity estimation model that can be applied to various demand distributions while considering the cost factors in securing and operating capacity.

The cost for a service facility consists of the initial costs for securing the facility and the operating costs for the period of operation. Initial costs and operating costs consist of variable costs and fixed costs proportional to the size of service capacity. When the capacity of a service facility is S, the initial cost c_i can be expressed as $aS + b$, which consists of the variable cost a and the fixed cost b of the initial cost. The operating cost c_o per unit period is $cS + d$, which consists of the variable cost c of the operating cost and the fixed cost d of the operating cost. Therefore, if the service facility is operated for a unit period of n, the cost c_t of unit capacity per unit period is given by equation (2).

$$c_t = c_i / nS + c_o/S = (aS + b)/nS + (cS + d)/S = u_1 + u_2/S \quad (2)$$

where, $u_1 = (a + cn)/n$, $u_2 = (b + dn)/n$

The cost incurred when capacity exceeds demand is the disposal price minus the sum of the initial and operating costs. Assuming the disposal price of the unused storage facility is zero, the unit excess cost is given by equation (3).The unit cost incurred when the capacity of a service facility is insufficient is the selling price p of the unit capacity, minus the cost of the unit capacity, as shown in equation (4).

$$c_e = c_t - 0 = u_1 + u_2 / S \quad (3)$$

$$c_u = p - c_t = p - u_1 - u_2 / S \quad (4)$$

The cost per unit of capacity of a service facility can be expressed as the sum of the expected loss costs of excess capacity per unit period resulting from fluctuating demand and the expected loss costs of loss of service per unit period. Let x ($0 \le x \le \infty$) be the demand that occupies the capacity of a service facility per unit period. If the probability density function of demand is $f(x)$ and the mean μ follows a continuous distribution, the total expected cost per unit period can be expressed as shown in equation (5).

Looking at the random variables of demand, the first term in equation (5) is the monotonic reduction function, and the second term is the monotonically increasing function. The optimal capacity S^* that minimizes the total expected cost can be found using the differential method. Therefore, the optimal capacity can be obtained by differentiating TC with respect to S as shown in equation (6) and setting the derivative to 0.

$$TC = c_e \int_0^S (S - x) f(x) dx + c_u \int_S^\infty (x - S) f(x) dx$$
$$= \left(u_1 + \frac{u_2}{S}\right) \int_0^S (S - x) f(x) dx + \left(p - u_1 - \frac{u_2}{S}\right) \left\{\int_S^\infty x f(x) dx - S\left(1\right.\right. \quad (5)$$

$$\frac{dTC}{dS} = -\frac{u_2}{S^2}\int_0^S (S - x) f(x) dx + \left(u_1 + \frac{u_2}{S}\right)\int_0^S f(x) dx + \frac{u_2}{S^2}\int_S^\infty (x - S) f(x) dx$$
$$-\left(p - u_1 - \frac{u_2}{S}\right)\left\{1 - \int_0^S f(x) dx\right\}$$
$$= u_2 \int_0^\infty x f(X) dx - u_2 S \int_0^\infty f(x) dx + u_1 S^2 + u_2 S - p S^2 \left(1 - \int_0^S f(x) ds\right)$$
$$= u_2 \mu - u_2 S + u_2 S + S^2 \left\{u_1 - p\left(1 - \int_0^S f(x) dx\right)\right\}$$
$$= \quad (6)$$

Negative values in the square root in equation (7) result in losses in the operation of the service facility, which does not serve the purpose of supply management, which requires a profit. Therefore, the optimum capacity must satisfy equation (7) and can be obtained through the incremental-division search.

(7)

III. EXPERIMENT AND RESULT

Let's take a look at the process of estimating the optimal capacity of the method presented in this study with an example of determining capacity before building a service facility. Assume a fixed cost of \$ 250,000 in service facility construction costs, \$ 1,000 in variable cost per unit of storage space, \$ 10 fixed cost in service facility operating costs per day, and an additional \$ 1 per unit of service space. At this time, 100 units of service space per day are consumed in new service facilities, which is expected to follow $N(\mu, \sigma^2)$. The daily fee is \$ 20 per unit of service space and the operating day of the service facility is 300 days. The service facility is scheduled to operate for 10 years after the new construction.

Table 1 compares the capacity and total expected cost calculated by this study model and the marginal analysis method using equation (1) by changing the mean and standard deviation of demand. In both the study model and the marginal analysis method, increasing the mean and standard deviation of demand increases capacity and total expected cost. In all cases, however, the model found less capacity than total marginal cost compared to the marginal analysis method. However, as the standard deviation increased, the results of the two methods differed.Table 2 shows the capacity and total cost of this study model and the marginal analysis method for the different operating periods. In the case of shorter operating periods, the two methods are somewhat different, and the longer the operating period, the greater the optimal capacity and the lower the total expected cost.

In the case of applying the marginal analysis method used in the model presented in this study and the single-term inventory model, the estimated total cost and capacity were not significantly different. This may be due to the bowl effect of the gun cost function. However, the proposed model can effectively reflect the various cost factors and demand characteristics that need to be considered in capacity planning compared to single-term inventory models. Therefore, it has the advantage that it can be extended to model and applied to various situations.

Table 1 Capacity and total expected costs due to changes in average demand and standard deviation

Mean(μ)		80					90				
Standard deviation(σ)		5	10	15	20	25	5	10	15	20	25
Marginal analysis	Capacity	85.9	91.9	98.1	104.4	110.8	96	102.2	108.5	114.8	121.3
	Total cost	201.2	394	579.6	758.7	930.9	194.4	381.8	562.9	738.9	909.6
Proposed model	Capacity	86	92.2	98.8	105.6	112.8	96.1	102.5	109.1	115.9	122.9
	Total cost	201.2	393.8	578.9	757.5	928.8	194.4	381.7	562.6	737.9	908.1
Difference	Capacity	0.10%	0.30%	0.70%	1.10%	1.80%	0.10%	0.30%	0.50%	0.90%	1.30%
	Total cost	0.00%	0.10%	0.10%	0.20%	0.20%	0.00%	0.00%	0.10%	0.10%	0.20%
Mean(μ)		100					110				
Standard deviation(σ)		5	10	15	20	25	5	10	15	20	25
Marginal analysis	Capacity	106.1	112.3	118.8	125.2	131.9	116.2	122.6	129	135.6	142.1
	Total cost	188.8	371.7	549.1	722	890.4	184.1	363	537.4	707.5	874.3
Proposed model	Capacity	106.2	112.6	119.3	126.1	133.1	116.3	122.8	129.5	136.3	143.3
	Total cost	188.8	371.5	548.8	721.4	889.6	184.1	362.9	537.1	707.1	873.4
Difference	Capacity	0.10%	0.30%	0.40%	0.70%	0.90%	0.10%	0.20%	0.40%	0.50%	0.80%
	Total cost	0.00%	0.10%	0.10%	0.10%	0.10%	0.00%	0.00%	0.10%	0.10%	0.10%
Mean(μ)		120					130				
Standard deviation(σ)		5	10	15	20	25	5	10	15	20	25
Marginal analysis	Capacity	126.3	132.8	139.3	145.7	152.5	136.4	142.9	149.2	156.1	162.9
	Total cost	180.1	355.6	527.1	695.3	859.9	176.6	349.3	518.6	684.2	847.1
Proposed model	Capacity	126.4	132.9	139.6	146.5	153.5	136.4	143	149.3	156.6	163.6
	Total cost	180.1	355.6	527	694.8	859.3	176.6	349.2	518.5	684	846.8
Difference	Capacity	0.10%	0.10%	0.20%	0.50%	0.70%	0.00%	0.10%	0.10%	0.30%	0.40%
	Total cost	0.00%	0.00%	0.00%	0.10%	0.10%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 2 Capacity and Expected Expenses due to Changes in Operating Years

Operating year		1	5	10	15	20
Marginalanalysis	Capacity	96	109.7	112.3	113.4	114
	Total cost	736.1	494.2	371.7	324.8	300.1
Proposed model	Capacity	94.7	110.3	112.6	113.7	114.3
	Total cost	735.7	493.9	371.5	324.7	300.1

Difference	Capacity	1.40%	0.50%	0.30%	0.30%	0.30%
	Total cost	0.10%	0.10%	0.10%	0.00%	0.00%

IV. CONCLUSION

This study deals with the problem of estimating capacity of service facilities to minimize total cost. When the capacity of a service facility means the maximum available capacity and the probability distribution of demand is given, the capacity estimation problem is similar to the single-period inventory problem using marginal analysis.

However, a single-period inventory model is limited in reflecting the uncertainty of demand and the various cost factors that need to be considered in capacity planning for service facilities. Therefore, this study assumes that the initial investment cost of a service facility and the operating cost of a unit period for a given period of use consist of variable costs and fixed costs proportional to their capacity. A model that minimizes expected costs is presented. The proposed model was able to estimate the optimal capacity of storage facility through trial and error process in closed form.

Numerical experiments were conducted to compare the sensitivity analysis and the single-period inventory model to examine the effects of various factors on capacity. The greater the variance of demand and the shorter the period of use, the greater the difference in capacity obtained from this study model and marginal analysis. Therefore, it can be seen that the optimal model of this study can reflect the uncertainty of demand and the characteristics of operating period more effectively than the single period inventory model.

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