http://dx.doi.org/10.21172/ijiet.161.02

Integration Model for Probabilistic Project Duration Estimation

Seon-Gyoo Kim[1], Junyoung Kim[2]
[1]*Department of Architectural Engineering*
Kangwon National University, Chuncheon, Gangwondo, South Korea
[2]*Institute of Technology*
Mooyoung Construction Management Co. Ltd., Songpagu, Seoul, South Korea

Abstract- Project Evaluation and Review Technique (PERT) is a typical method in order to probabilistically establish a schedule for complex projects with uncertainty. However, it is impossible to evaluate each activity's relationship probabilistically when there are overlapping relationships because it is limited to Finish-to-Start (FS) relationship. In order to overcome this kind of limitation of PERT, Probabilistic Linkage Evaluation Technique (PLET) has been suggested to estimate construction schedule for various overlapping relationships for each activities. However, estimating construction time by PLET only considers uncertainty of relationship between activities and not activity time. It means that PLET is not the perfect schedule estimating method as well. The main objective of this study proposes an integrated model for estimating the project duration probabilistically by combining PERT that considers the uncertainty of the activity duration and PLET that considers the uncertainty of the relationship between the activities, and will verify through the sample network. This will help project managers or schedulers to establish more flexible project duration estimates in uncertain and complex construction environments

Keywords – Integration, Uncertainty, Overlapping Relationship, Probabilistic Estimation

I. INTRODUCTION

Construction projects are exposed to a number of risk events because of the long-term use of various resources in the external environment, and the process of wisely overcoming them is the construction process.(Smith 1999) These risk factors are derived and analyzed in various ways in the process of establishing a schedule of the construction project, but are not reflected in the fact that predicted risk factors are not guaranteed to occur, since there are many unexpected situations.(Edwards 1995) Especially for large complex projects or mega projects, there are numerous unpredictable and unexpected issues due to the complexity of the internal and external environment of the project. In high-risk project, the scheduling is the process of choosing the best road map while probabilistically assuming various scenarios considering various risk factors and uncertainty.(Bluce et al. 2005)

The risk factors to be considered when preparing a project schedule in a construction project are very diverse and extensive. Most of them exist in activity duration and the relationships between activities. Among them, probabilistic scheduling methods considering the uncertainty of activity duration have been proposed for a long time, and new methods are continuously proposed. However, few methodologies have been proposed to account for uncertainties in the relationships between activities. Therefore, the existing methods that depend only on the uncertainty of the activity duration are not complete. In order to improve the reliability of project duration estimation and improve the probability of success, it is necessary to search for a new probabilistic project duration estimation method considering the uncertainty of activity relationship.

The Project Evaluation and Review Technique (PERT) is a representative method for probabilistic scheduling.(Malcolm et al. 1959) Since PERT focuses only on the uncertainty of the duration and the relationship between activities and limited to the Finish-to-Start (FS), it is impossible to use PERT if probabilistic estimation of relationships between activities is required. In order to overcome this limitation of PERT, the Probabilistic Linkage Evaluation Technique (PLET) was proposed as a method for estimating the total project duration in consideration of the uncertainty of various overlapping relationships. However, the project duration estimation by PLET takes into account only the uncertainty of the relationship between activities, and does not consider the uncertainty of the activity duration in PERT.

This study proposes an integrated model for estimating the project duration probabilistically by combining PERT that considers the uncertainty of the activity duration and PLET that considers the uncertainty of the relationship between the activities, and will verify through the sample network. This will help project managers or schedulers to perform more flexible and comprehensive project duration estimates in uncertain and complex construction environments.

II. RESEARCH SCOPE AND METHODS

This study was carried out in the following way and in order. First, we review the existing probabilistic project duration estimation methods such as PERT, GERT, Monte Carlo Simulation, Scenario technique, PLET, and analyse their limitations and restrictions. Second, we suggest the necessity of integrating PERT and PLET among the methods of probabilistic project duration estimation. Third, we present an integrated model of PERT and PLET techniques, and perform probabilistic verification of an integrated model. Fourth, we propose a schedule calculation method of an integrated model of PERT and PLET and verify it through a sample network. Fifth, we present the expected effects and future direction of the integration model of PERT and PLET.

III. REVIEW OF EXISTING PROBABILISTIC PROJECT DURATION EVALUATION METHODS

A. PERT

The Project Evaluation and Review Technique (PERT), introduced in the US Navy's Polaris missile system program in 1958, is designed to support planning in the absence of empirical cost and schedule information for the overall project schedule and cost estimates.(Callahan et al. 1992) The PERT has the similarity as the CPM in the concept of centrally locating and managing the main line, expressed in the form of ADM (Arrow Diagramming Method). The difference between the CPM and the PERT is that the CPM assumes schedule with very small dispersion, but the PERT defines schedule with a distribution with a relatively large dispersion. Therefore, the PERT analyses the network as a probability concept and is therefore suitable for use as a lack of experience or as a schedule management tool for new R&D projects (Harris 1978). Recently, as the uncertainty of the construction environment has gradually increased, it has been widely applied as a technique for analyzing and managing time-related uncertainties in construction risk management.

The PERT network's activity duration has a probability distribution and is estimated with the following three estimated working hours: most likely time (m), most optimistic time (a), and most pessimistic time (b). Based on this, the average time (t_e), standard deviation (σ) and variance (σ^2) of the activity are calculated as follows.

$$\text{Average Time : } t_e = \frac{a+4m+b}{6} \quad (1)$$

$$\text{Standard Deviation : } \sigma = \frac{b-a}{6} \quad (2)$$

$$\text{Variance : } \sigma^2 = \left(\frac{b-a}{6}\right)^2 \quad (3)$$

The most significant feature of the PERT is that it provides more information than the CPM, which estimates the schedule stochastically, by probabilistically evaluating the period to reach the main completion point in order to comply with the overall project schedule. However, the PERT is based on the ADM network format, so that the interconnection of relationship is limited to the Finish-to-Start (FS) logic. If the logic between activities is confined to the FS only, the reliability of the PERT network analysis will suffer because the logic between actual operations is not accurately reflected.

B. GERT

The Graphical Evaluation and Review Technique (GERT) was proposed in 1966 as a network analysis technique for stochastic processing of network logic and activity duration estimation. The GERT approach provides a solution to the PERT/CPM limitations and allows loop logic between activities. The GERT is similar to the PERT but allows deterministic and probabilistic branching unlike the PERT.(Pritsker 1966) The most fundamental flaw of the GERT is that the process for modelling the GERT system is very complex and rarely utilized in practice.

C. Monte Carlo Simulation

Monte Carlo Simulation is a process of generating data through a random number generator, which models the problem involving uncertain variables with known or assumed probability distributions. This is a probabilistic analysis technique that is used to predict situations that cannot be realistically realized by repeatedly implementing real situations. This Monte Carlo simulation is widely used to estimate the probability distribution of project schedule (Hullet 2009) as well as the criticality of different activities and paths analysis (Forgionne 1986), estimation and allocation of project contingency time.(Barraza 2011)

D. Scenario Techniaue

The Scenario technique is a technique for modelling and analyzing a project stochastically in the presence of significant uncertainty in the overall project plan or some work environment. Uncertain network scenarios are represented by network scenarios with probability of occurrence where each network scenario is probabilistically branched and the network scenario with the largest expected value among the pruned network scenarios.(Bruce Pollack-Johnson et al. 2005)

E. PLET

The Probabilistic Linkage Evaluation Technique (PLET) is a method for estimating the project duration by probabilistic evaluating the relationship between the activities containing the uncertainty based on the Beeline Diagramming Method (BDM) technique.(Kim 2014) Figure 1 shows the linkage representation of PLET.

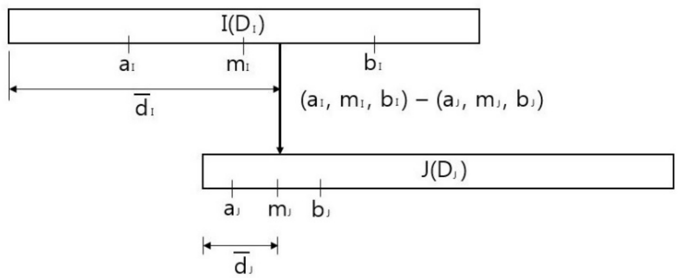

Figure 1. Linkage Representation of PLET

In Figure 1, the 'N-N' format of the BDM is changed to the form of a probabilistic linkage relationship '(a_I, m_I, b_I)-(a_J, m_J, b_J)' of the PLET. The front '(a_I, m_I, b_I)' indicates the uncertain elapsed days after the preceding activity 'I' is started. and the rear '(a_J, m_J, b_J)' indicates the elapsed days after the succeeding activity 'J' is started, where 'a' indicates the most optimistic days, 'm' indicates the most likely days, and 'b' indicates the most pessimistic days. In Figure 1, '$\bar{d}_I$' and '$\bar{d}_J$' represent the average elapsed days in the preceding activity 'I' and succeeding activity 'J' respectively. The formulas for calculating the average elapsed days ($\bar{d}$), standard deviation ($\bar{d}$) and variance ($\sigma_{\bar{d}}^2$) of activity 'I' and 'J' are shown in equations (4), (5), and (6) below.

$$\text{Average Elapsed Days : } \bar{d} = \frac{a+4m+b}{6} \quad (4)$$

$$\text{Standard Deviation : } \sigma_{\bar{d}} = \frac{b-a}{6} \quad (5)$$

$$\text{Variance : } \sigma_{\bar{d}}^2 = \left(\frac{b-a}{6}\right)^2 \quad (6)$$

F. Restrictions and Limitations of Existing Methods

Table 1 compares the existing probabilistic project duration estimation methods described above.

Table -1 Comparison of Probabilistic Project Duration Estimation Methods

No	Comparison Items	PERT	GERT	Monte Carlo Simulation	Scenario	PLET
1	Probabilistic Time Estimation	○	○	○	○	×
2	Overlapping Relationship	×	×	×	×	○
3	Probabilistic Linkage Estimation	×	×	×	×	○

First, whether or not the activity duration can be probabilistically estimated. The PERT, GERT, Monte Carlo Simulation, and Scenario methods allow probabilistic estimation of the activity duration, but the PLET does not allow it. Second, whether or not the overlapping relationship between activities can be expressed. The PERT, GERT, Monte Carlo Simulation, and Scenario methods cannot express the overlapping relationship between activities because they express the connection between activities only in Finish-to-Start (FS) logic. However, the PLET based on the BDM can express it freely. Third, whether or not the relationship between activities can be evaluated probabilistically. The PERT, GERT, Monte Carlo Simulation, and Scenario methods have a Finish-to-Start (FS) logic between activities, so probabilistic evaluation of the relationship is in principle impossible. On the other hand, the PLET can perform probabilistic estimation of the relationship expressed by the BDM.

As a result of comparison of the existing project duration estimation methods, the PERT, GERT, Monte Carlo Simulation, and Scenario techniques can estimate the activity duration probabilistically, but it is impossible to express the overlapping relationship and it is impossible to estimate probabilistically the relationships between activities. On the other hand, the PLET based on the BDM can freely express the overlapping relationships between activities, and it is also possible to perform probabilistically the relationships between activities.

G. Necessity of Integration of Probabilistic Project Duration Estimation Method

The risk factors to consider when establishing a schedule in a high-risk construction project are very diverse and extensive. However, these risk factors can also exist in the relationship between activities.

As described in a previous section, GERT, Monte Carlo Simulation, and Scenario techniques, including the PERT, can estimate the activity duration probabilistically. However, since the relationship between activities is estimated only by Finish-to-Start logic, it is impossible to estimate the uncertainty of the relationship between activities. When constructing the CPM schedule of the construction project based on the work breakdown structure (WBS), it is inevitable to express various overlapping relationships among the activities.(Kim 2015) However, PLET takes into account only the uncertainty of the relationship between activities, and thus cannot be considered as a complete project duration estimation method. Therefore, considering the uncertainty of the activity duration through the PERT as well as the uncertainty of the relationship between activities through the PLET at the same time can greatly improve the reliability of the probabilistic duration estimation of the high–risk construction project. Accordingly, it is necessary to search for a new integrated project duration estimation model that combines the PERT that considers the uncertainty of the activity duration and the PLET that considers the uncertainty of the relationships between activities.

IV. INTEGRATION MODEL OF PERT AND PLET

A. Expression of Integration Model of PERT and PLET

Figure 2 shows a generalized integration model of PERT and PLET. First, the durations of activity 'I' and 'J' are estimated by applying the PERT to three-points by following variables: the most likely duration (m), the most optimistic duration (a), and the most pessimistic duration (b). Therefore, the three-point estimation of the preceding activity 'I' is expressed as '$I(a_I, m_I, b_I)$' and the three-point estimation of the succeeding activity 'J' is expressed as '$J(a_J, m_J, b_J)$'. Second, the relationship between activity 'I' and activity 'J' applies the PLET. In other word, it is expressed by '$(la_I, lm_I, lb_I)\%(la_J, lm_J, lb_J)$' applying the 'N%N' type among the linkage representation methods of the BDM. The front '(la_I, lm_I, lb_I)' is a three-point estimation of the elapsed ratio after the preceding activity 'I' is started, and the rear '(la_J, lm_J, lb_J)' is a three-point estimation of the elapsed ratio after the succeeding activity 'J' is started. Here, 'la' indicates the most optimistic progress ratio, 'lm' as the most likely progress ratio, and 'lb' as the most pessimistic progress ratio. '$\bar{d}_I$' represents the average elapsed days since the preceding activity 'I' is started, and '$\bar{d}_J$' represents the average elapsed days since the succeeding activity 'J' is started.

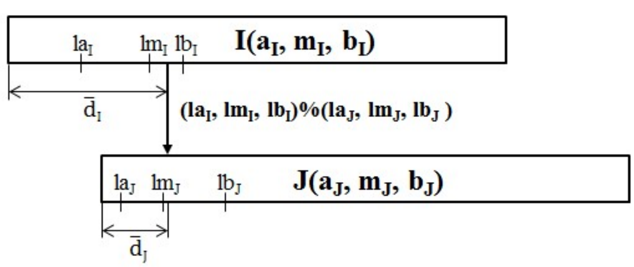

Figure 2. Integration Model of PERT and PLET

B. Application of 'N%N' Linkage Type of BDM

In the BDM, there are two ways of expressing the overlapping relationships; the 'N-N' type by the number of days elapsed days and the 'N%N' type by the elapsed ratio.(Kim 2010, 2018) In the PLET, the 'N-N' type among two kinds of the overlapping relationship is applied as the basic representation.(Kim 2014) Because it considers only the uncertainty of the relationship between the preceding and succeeding activities after assuming that the activity durations of the preceding and succeeding activities in the PLET does not change. That is, even if the relationship is expressed in the type of 'N-N', the average elapsed days ($\bar{d}_I$, $\bar{d}_J$) between the preceding and succeeding activities does not exceed the preceding activity duration D_I and the succeeding activity duration D_J. ($0 \leq \bar{d}_I \leq D_I$, $0 \leq \bar{d}_J \leq D_J$)

However, in the integration model of PERT and PLET, the average of activity durations continuously changes because they are estimated with three points provided by the PERT. Therefore, when the overlapping relationship between the preceding and succeeding activities is represented by the type of 'N-N', the average elapsed days may exceed the average activity durations.

However, in the integration model of PERT and PLET, the average of activity durations continuously changes because they are estimated with three points provided by the PERT. Therefore, when the overlapping relationship between the preceding and succeeding activities is represented by the type of 'N-N', the average elapsed days may exceed the average activity durations.

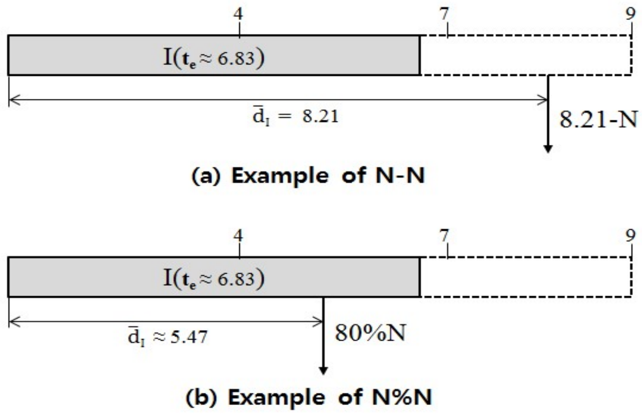

Figure 3. Difference of N-N and N%N

For example, as shown in Figure 3 (a), if the duration of the preceding activity 'I' is estimated to be three points (4, 7, 9) (dotted line), the average activity duration (t_e) is calculated to be about '6.83' (colour filling) by the equation (1). However, if the average elapsed days after the preceding activity 'I' is calculated as '8.21' and expressed as '8.21-N' in the type of 'N-N', there is a contradiction that the average elapsed days ($\bar{d}_I$) exceeds the average activity duration (t_e) as '($\bar{d}_I \approx 8.21$) > ($t_e \approx 6.83$)'.

Therefore, in the integration model of PERT and PLET, it is necessary to express the average activity duration in the form of 'N%N' by the elapsed ratio in order to avoid the contradiction because of the elapsed days of the overlapping relationships between the preceding and succeeding activities (0%≤N≤100%). Figure 3 (b) shows an example in which the average elapsed days until 80% of the average activity duration of the preceding activity 'I' estimated in Figure 3 (a) is expressed as '80%N'. In Figure 3 (b), the average elapsed days in the preceding activity 'I' is calculated to be approximately '5.47', which is 80% of the time elapsed since the activity was started, and is located within the average activity duration ($t_e \approx 6.83$).

In other words, since the average activity duration ($\bar{d}$) expressed in the form of 'N%N' cannot exceed the average activity duration (t_e), $0 \leq \bar{d} \leq t_e$, the overlapping relationship between the preceding and succeeding activities in the integration model of PERT and PLET must be expressed in 'N%N' format.

C. Calculation of Average Activity Duration and Elapsed Ratio at Integration Model of PERT and PLET

The method of calculating the average activity duration and the average elapsed days in the integration model of PERT and PLET is as follows. First, the average activity duration is calculated by applying equations (1), (2), and (3) to calculate the average activity time, standard deviation, and variance of the PERT. Second, the average elapsed days is calculated by applying equations (4), (5), and (6) to calculate the average elapsed days, the standard deviation, and variance of the PLET.

However, unlike Figure 1, which represents only the PLET, the relationship is changed to '(la_I, lm_I, lb_I)%(la_J, lm_J, lb_J)' in Fig 2. Since the average elapsed days ($\bar{d}$), the standard deviation ($\sigma_{\bar{d}}$), and variance ($\sigma_{\bar{d}}^2$) are calculated after multiplying the average activity duration (t_e) by the elapsed ratio (la, lm, lb) in the equations (4), (5), and (6), the equations (4), (5), and (6) should be changed to the equations (7), (8), and (9).

$$\text{Average Elapsed Days : } \bar{d} = \frac{t_e(la+4lm+lb)/100}{6} \qquad (7)$$

Standard Deviation : $\sigma_d = \frac{t_e(lb-la)/100}{6}$ (8)

Variance : $\sigma_d^2 = \left(\frac{t_e(lb-la)/100}{6}\right)^2$ (9)

D. Probability Distributions at Integration Model of PERT and PLET

In the PERT, the schedule computation of the network is simply computed by adding or subtracting the average activity duration of each work, calculating forward and backward, and calculating the variance of the entire network duration simply by adding the variance of each work. This is because each activity in the PERT network assumes that the probability distributions are completely independent. The PLET also applies the schedule computation method of the PERT because the probability distributions of the relationship are completely independent.

However, in the integration model of PERT and PLET, it is necessary to verify whether or not the average activity duration and the relationship are completely independent to each other. This is because activity duration is directly or indirectly influencing the relationship since the elapsed days of the preceding and succeeding activities are calculated as a ratio to the average activity duration of the preceding and succeeding activity. If the average activity duration and the relationship are joint probability distributions or conditional probability distributions, then the integration model of PERT and PLET will not be able to apply the existing schedule computation method of PERT or PLET as it is.

A case in which both events occur for any two cases A and B is called a 'joint event', the probability of occurrence of a joint event is called a 'joint probability' and is expressed as 'P(AB)'.(Cheong 2012) And, the probability that an event A occurs when an event B occurs is affected by the event B, which is called a 'conditional probability' and is expressed as 'P(A|B)'.(Walpole et al. 1985) If event A occurs but event B does not, the two events are independent and expressed as 'P(A|B)=P(A)'. Also, if both events occur simultaneously, they are multiplied by two probabilities. And if the probabilities of two events are independent, two probabilities are added.(Ahn 2014)

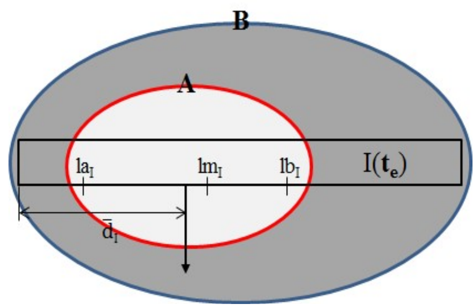

Figure 4. Probability Distribution of Activity Duration and Linkage

Figure 4 shows the probability distribution A of the average activity duration (t_e) of activity 'I' and the probability distribution B of the average elapsed days ($\bar{d}_I$) related to activity 'I' in the integration model of PERT and PLET. In Figure 4, after the probability distribution A of the average activity duration is determined, the probability distribution B of the average elapsed days is determined within the range A. Thus probability distribution A and B do not occur at the same time. Therefore, the probability distribution of the average activity duration and the probability distribution of the average elapsed days are not joint distributions. Also, the probability distribution A of the average elapsed days is conditional probability because it is influenced by the probability distribution B of the average activity duration. However, since probability distribution A occurs only within the range of probability distribution B, 'P(A|B)=P(A)' is established. Therefore, the probability distributions A and B are mutually independent.

From the above probabilistic distribution verification, since the probability distribution of the average activity duration and the probability distribution of the average elapsed days are conditional probabilities but mutually independent, the schedule computation methods of the PERT and PLET can be applied to the schedule computation method as the integrated model. That is, in the integration model of PERT and PLET, the schedule of the network can be calculated simply by adding or subtracting the average activity duration and the elapsed days with the forward and backward pass computations. And, the variance of the network can be calculated simply by adding the variances of the average activity durations and the average elapsed days, and then the standard deviation of the network is calculated as the square root of the variance of the network

V. Schedule Computation of Integration Model of PERT and PLET

A. Forwand Pass Computation

In the integration model of PERT and PLET, the forward pass computation calculates the early start date (ESD) and the early finish date (EFD), which are the early schedules of the succeeding activities, and the forward computation method of the PLET is applied.(Kim 2014)

First, the formula for calculating the ESD_J and EFD_J of the succeeding activity in a single versus single relationship where the average elapsed days '$\bar{d}_I$' after the start of the preceding activity 'I' and the average elapsed days '$\bar{d}_J$' after the start of the succeeding activity 'J' is the same as (10) and (11), where $(t_e)_J$ is the average activity duration of the succeeding activity 'J'.

$$ESD_J = ESD_I + \bar{d}_I - \bar{d}_J \quad (10)$$
$$EFD_J = ESD_J + (t_e)_J \quad (11)$$

Next, in the integration model of PERT and PLET, when multiple preceding activities merge into a single succeeding activity, a formula (12) that generalizes the forward pass calculation method of the PLET is applied, where $\forall I$ means all of activities 'I'.

$$ESD_J = \underset{\forall I}{Max}(ESD_I + \bar{d}_I - \bar{d}_J) \quad (12)$$

B. Backward Pass Computation

In the integration model of PERT and PLET, the backward pass computation of the PLET calculates the late start date (LSD) and the late finish date (LFD), which are late schedules of the preceding activities, and the backward computation method of the PLET is applied.(Kim 2014)

First, the formula for calculating the LSD_I and LFD_I of the preceding activity in a single versus single relationship where the average elapsed days '$\bar{d}_I$' after the start of the preceding activity 'I' and the average elapsed days '$\bar{d}_J$' after the start of the succeeding activity 'J' is the same as (13) and (14), where $(t_e)_I$ is the average activity duration of the preceding activity 'I'.

$$LSD_I = LSD_J + \bar{d}_J - \bar{d}_I \quad (13)$$
$$LFD_I = LSD_I + (t_e)_I \quad (14)$$

Next, in the integration model of PERT and PLET, when a single preceding activity bursts into multiple succeeding activities, a formula (15) that generalizes the backward calculation method of the PLET is applied, where $\forall J$ means all of activities 'J'.

$$LSD_I = \underset{\forall J}{Min}(LSD_J + \bar{d}_J - \bar{d}_I) \quad (15)$$

C. Computations of Free Float and Total Float

The free float (FF) of the integration model of PERT and PLET is the same as the free slack of the PERT and free float of the PLET that is defined as the margin time of the preceding activity without affecting the early start date of the succeeding activity. In the forward pass computation process of the network, there is a difference between the early start date (ESD) of the succeeding activity and the early finish date (EFD) of the preceding activity, which is called 'Link Lag'.(Harris 1978) Link Lag can be defined as the difference between the linking point time of the preceding and the succeeding activities, as shown in equation (16).

$$LAG_{IJ} = (ESD_J + \bar{d}_J) - (ESD_I + \bar{d}_I) \quad (16)$$

The free float (FF) is the minimum Link Lag value of the activity.(Harris 1978) If a preceding activity 'I' is associated with multiple succeeding activities J, then the free float FF_I of activity 'I' is the minimum LAG_{IJ} as shown in equation (17), where $\forall J$ means all of activities J.

$$FF_I = \underset{\forall J}{Min} LAG_{IJ} = \underset{\forall J}{Min}((ESD_J + \bar{d}_J) - (ESD_I + \bar{d}_I)) \quad (17)$$

In the integrated model of PERT and PLET, the total float (TF) is the same as the total slack of the PERT and the total float of the PLET, this is the amount of time that affects the early start date (ESD) of the succeeding activity but does not affect the completion time of the entire network.

http://dx.doi.org/10.21172/ijiet.161.02

The total float (TF) is calculated as the difference between the forward and backward pass computations, which is the maximum amount of time an activity can have. In other words, the TF_I of activity 'I' is the difference between LSD_I and ESD_I, or the difference between LFD_I and EFD_I as shown in equation (18).

$$TF_I = LSD_I - ESD_I = LFD_I - EFD_I \qquad (18)$$

D. Calculations of Average Duration, Variance, and Standard Deviation of Schedule Path

Figure 5 shows the schedule path H-I-J-K-L, which is the sequential activities in the integration model of PERT and PLET.

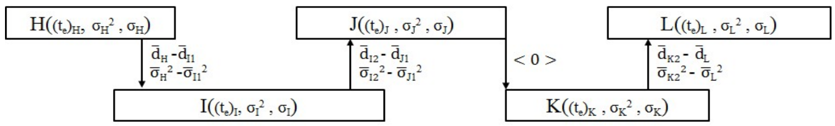

Figure 5. A Sample Path on Integration Model of PERT and PLET

In Figure 5, the activity durations of activity H, I, J, K, L was estimated to be three points by PERT, and the average activity duration, variance, and standard deviation of each is shown inside the activity bar in the form of 'activity description (average duration (t_e), variance (σ^2), standard deviation (σ))'. Also, the relationship between activities is estimated to be three points by the PLET, and each relationship is expressed in the form of 'average elapsed days ($\bar{d}_n$-$\bar{d}_m$), variance (σ_n^2-σ_m^2)' on the right side of the linkage.

The schedule computation of the integration model of PERT and PLET is similar to the calculation of the average duration (t_e), variance (σ^2), and standard deviation (σ) of the schedule path, which is the connection of activities in the PLET. However, the process of adding the average activity duration and variance calculated by the PERT to the average elapsed days and variance calculated by the PLET sequentially are added.

The process of calculating the average duration $(t_e)_{HL}$ of the schedule path H-I-J-K-L is as follows. First, it calculates the average activity duration (t_e) for all activities on the schedule path. Second, in the case of 'N%N' on the schedule path, ESD and EFD of each activity are calculated by applying the equations (10) and (11) continuously. Thirdly, if the linkage type '<0>' is included in the schedule path, it can be expressed as 'N-0' in 'N-N' format, so the average duration (t_e) of the preceding activity is added to the average duration of the schedule path. Fourth, add the last average activity duration of the schedule path. The above process can be expressed by equation (19).(Kim 2015)

$$(t_e)_{HL} = \sum \textit{Apply 'Eq.(10),(11)' (if 'N-N')} \\ + \sum \textit{Predecessor's Average Duration (if '<0>')} \\ + \sum \textit{Last Activity's Average Duration} \qquad (19)$$

Then, the variance (σ_{HL}^2) for the average duration in the schedule path H-I-J-K-L is calculated as the sum of the average activity duration's variance and the average elapsed day's variance for each activity connection line as shown in equation (20).

$$\sigma_{HL}^2 = \sigma_H^2 + \bar{\sigma}_H^2 + \sigma_I^2 + \bar{\sigma}_{I1}^2 + \bar{\sigma}_{I2}^2 + \sigma_J^2 + \bar{\sigma}_{J1}^2 + \sigma_K^2 \\ + \bar{\sigma}_{K2}^2 + \bar{\sigma}_L^2 + \sigma_L^2 \qquad (20)$$

The standard deviation (σ_{HL}) for the average duration of the schedule path H-I-J-K-L is equal to the square root of equation (20) as shown in equation (21).

$$\sigma_{HL} = \sqrt{\sigma_{HL}^2} \qquad (21)$$

VI. Verification of Integration Model of PERT and PLET

In order to verify the integration model of PERT and PLET proposed in this study, an integration model network of PERT and PLET is constructed as shown in Figure 6 for the finish activities of the apartment unit work.

The integration model network of PERT and PLET in Figure 6 consisted of a total of 10 activities with 11 linkages. In Figure 6, seven activities are estimated to be three points by PERT, and eight linkages are estimated to be three points by the PLET.

Table 2 shows the results of the schedule computation for the integration model network of PERT and PLET in Figure 6. In Table 2, the activity duration by the PERT is estimated by three points a, m, and b, and the average activity duration (t_e), variance (σ^2), standard deviation (σ) are calculated. And the probabilistic linkages for each activity are estimated by three points with the values of la, lm, and lb, and the average elapsed days ($\bar{d}$), variance (σ^2), standard deviation (σ) are calculated. Then, the results of calculating the early and late dates (ESD, EFD, LSD, LFD) and the float (FF, TF) of each activity are shown by applying equations (10) to (18).

As a result of schedule computations, the critical path (C.P.) where FF and TF are both '0' is 'Plastering → AL Door & Window → WD Door & Window → Papering → Flooring → Inspection', the average duration (t_e), variance (σ^2), and standard deviation (σ) of the critical path are calculated by applying the equations (19), (20), and (21) as follows.

$$t_e = 0 + 6.72 - 2.60 + 10.29 - 4.43 + 8.40 - 2.89 + 7.00$$
$$-2.20 + 14.17(< 0 > Linkage) + 9.00(Last\ Activity) \approx 46.48$$
$$\sigma^2 = 0.69 + 0.24 + 0.19 + 1.00 + 0.11 + 0.49 + 0.00 + 0.87 + 0.44$$
$$+1.00 + 0.05 + 0.06 + 0.83 + 0.00 + 0.00 + 0.00 \approx 5.97$$
$$\sigma = \sqrt{\sigma^2} = \sqrt{5.97^2} \approx 2.44$$

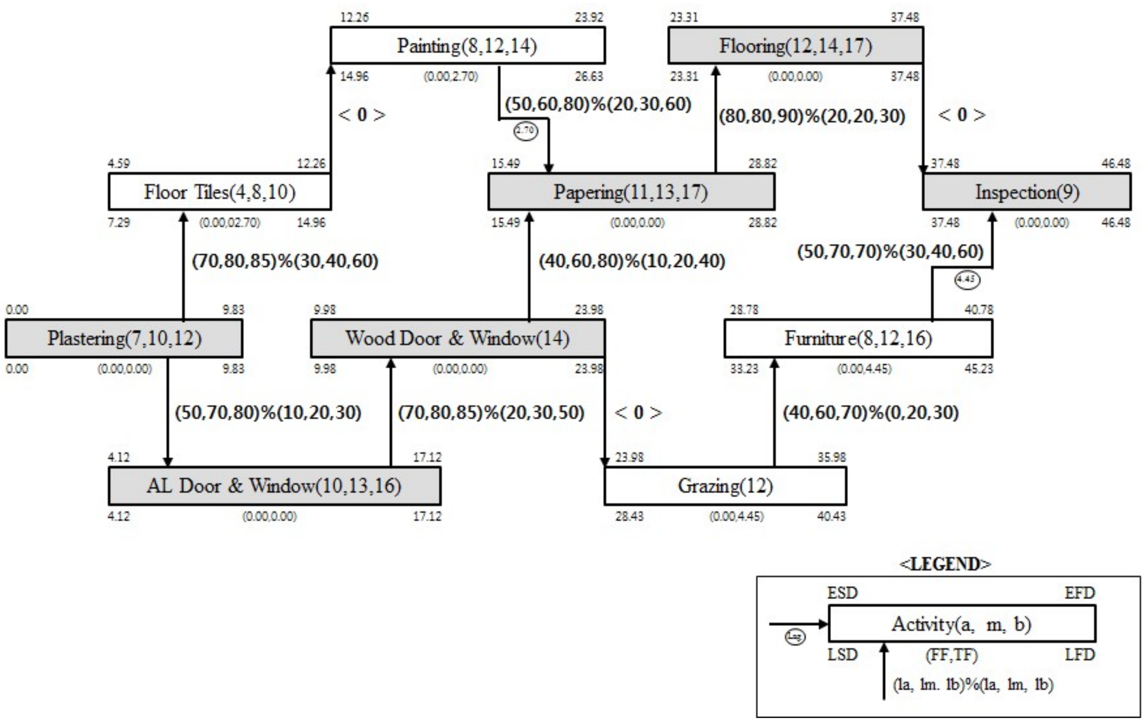

Figure 6. A Sample Network for Integration Model of PERT and PLET

Table -2 Schedule Computation Results of Sample Network

No	Activity Description └ Predecessors	Activity Duration						Linkages						Schedule Computation Dates and Floats						
		a	m	b	Avg	Var	S.D.	la	lm	lb	Avg	Var	S.D							
		days			t_e	σ^2	σ	%			$\bar{d}$	σ^2	σ	ESD	EFD	LSD	LFD	FF	TF	C.P.
1	Plastering	7	10	12	9.83	0.69	0.83	-	-	-	-	-	-	0.00	9.83	0.00	9.83	0.00	0.00	○
	└ None	-	-	-	-	-	-	-	-	-	-	-	-			2.70	12.54			
2	Floor Tiles	4	8	10	7.67	1.00	1.00	30	40	60	3.19	0.15	0.38	4.59	12.26	7.29	14.96	0.00	2.70	
	└ Plastering	7	10	12	9.83	0.69	0.83	70	80	85	7.78	0.06	0.25							
3	AL Door & Window	10	13	16	13.00	1.00	1.00	10	20	30	2.60	0.19	0.43	4.12	17.12	4.12	17.12	0.00	0.00	○
	└ Plastering	7	10	12	9.83	0.69	0.83	50	70	80	6.72	0.24	0.49							
4	Painting	8	12	14	11.67	1.00	1.00	0	0	0	0.00	0.00	0.00	12.26	23.92	14.96	26.63	2.70	2.70	
	└ Floor Tiles	4	8	10	7.67	1.00	1.00	0	0	0	0.00	0.00	0.00							
5	Wood Door & Window	14	14	14	14.00	0.00	0.00	20	30	50	4.43	0.49	0.70	9.98	23.98	9.98	23.98	0.00	0.00	○
	└ AL Door & Window	10	13	16	13.00	1.00	1.00	70	80	85	10.29	0.11	0.33			16.43	29.43			
6	Papering	11	13	17	13.33	1.00	1.00	10	20	40	2.89	0.44	0.67	15.49	28.82	15.49	28.82	0.00	0.00	○
	└ Wood Door & Window	14	14	14	14.00	0.00	0.00	40	60	80	8.40	0.87	0.93							
	Papering	11	13	17	13.33	1.00	1.00	40	50	60	6.67	0.20	0.44	12.78	26.12	15.49	28.82			
	└ Painting	8	12	14	11.67	1.00	1.00	50	60	80	7.19	0.34	0.58							
7	Grazing	12	12	12	12.00	0.00	0.00	0	0	0	0.00	0.00	0.00	23.98	35.98	28.43	40.43	0.00	4.45	
	└ Wood Door & Window	14	14	14	14.00	0.00	0.00	0	0	0	0.00	0.00	0.00							
8	Flooring	12	14	17	14.17	0.69	0.83	20	20	30	3.07	0.06	0.24	23.31	37.48	23.31	37.48	0.00	0.00	○
	└ Papering	11	13	17	13.33	1.00	1.00	80	80	90	10.89	0.05	0.22							
9	Furniture	8	12	16	12.00	1.78	1.33	0	20	30	2.20	0.36	0.60	28.78	40.78	33.23	45.23	4.45	4.45	
	└ Grazing	12	12	12	12.00	0.00	0.00	40	60	70	7.00	0.36	0.60							
10	Inspection	9	9	9	9.00	0.00	0.00	0	0	0	0.00	0.00	0.00	37.48	46.48	37.48	46.48	0.00	0.00	○
	└ Flooring	12	14	17	14.17	0.69	0.83	0	0	0	0.00	0.00	0.00							
	Inspection	9	9	9	9.00	0.00	0.00	30	40	60	3.75	0.20	0.45	33.03	42.03	-	-			
	└ Furniture	8	12	16	12.00	1.78	1.33	50	70	70	8.00	0.16	0.40							

Note: Avg.=Average; Var.=Variance; S.D.=Standard Deviation; C.P.=Critical Path

VII. CONCLUSION

It is impossible to accurately predict all the variables or risk factors that will arise in the future in the construction project. And also, it is almost impossible to fully and completely predict the external factors that are inherent in the project or affect the project, especially in high-risk construction projects with very high uncertainties. Nonetheless, reflecting the most predictable variables and risk factors when preparing and establishing a project schedule is a natural responsibility for project managers.

There are various ways to establish a schedule in view of the uncertainty of project. Among them, the PERT has been widely applied for a long time, and there are many techniques derived based on the PERT. Of these, integration of the PERT and the Monte Carlo simulation technique is the most popular probabilistic duration estimation model. However, since the PERT does not consider the redundancy or uncertainty of the inter-connection, only the uncertainty of the environment is considered by limiting the relationship to the Finish-to-Start(FS) logic, it is impossible to establish a schedule considering all the uncertainties of the project with the PERT alone. In order to overcome these limitations of the PERT, the PLET has been proposed. However, the PLET does not consider the uncertainty of the schedule considering the redundancy of the interworking and the uncertainty of the relationship.

This study proposes an integration model of PERT and PLET, which combines the estimation method of uncertainty of activity duration uncertainty of PERT and the uncertainty of relationship of PLET. The integration model of PERT and PLET can take into account not only the uncertainty of the activity duration but also the uncertainty of the relationship between activities, so that a probabilistic project duration estimation is possible considering all the variables and risk factors that can be predicted in the project.

Schedule management is a logical predictor of the progress of a project, but the future is always uncertain, so attempting to integrate schedule management with the concept of risk management, which manages uncertainty, are a natural phenomenon. The PERT, the Monte Carlo simulation, various other techniques, the PLET, and the integration model of PERT and PLET proposed in this study are all part of efforts to integrate schedule and risk management. However, due to the inherent nature of schedule management and risk management, integration is still incomplete. Therefore, further and dedicated research and effort will be needed to ensure the real integration of schedule and risk management.

REFERENCES

[1] Barraza, G. A., “Probabilistic Estimation and Allocation of Project Time Contingency” Journal of Construction Engineering and Management, ASCE, 259-265, 2011
[2] Bruce Pollack-Johnson, Matthew J. Liberatore, “Project Planning Under Uncertainty Using Scenario Analysis” Project Management Journal, PMI, 15-26., 2005
[3] Callahan, M. T., Quackenbush, D. G., Rowings, J. E., “Construction Project Scheduling”, McGRAW-HILL, USA, 1992
[4] Cheong, H. Y., “Probability, Random Variable and Random Process”, Hanbit Media Inc., Korea, 2012
[5] Edwards, Lesile, “Practical Risk Management in the Construction Industry”, Thomas Telford, England, 1995
[6] Forgionne, Guisseppi A., “Quantitative Decision Making”, Wadsworth Inc., USA, 1985
[7] Harris, R. B., “Precedence and Arrow Networking Techniques for Construction”, John Wiley & Sons, USA, 1978
[8] Hullet, David T, “Practical Schedule Risk Analysis”, Gower Publishing ltd., England, 2009
[9] Kim, S. G., “Advanced Networking Technique”, Kimoon-Dang, Korea, 2010
[10] Kim, S. G., “Estimation of Project Duration by Probabilistic Linkage Evaluation Technique (PLET)”, Korean Journal of Construction Engineering & Management Vol. 14 No. 5, Korea, 2014
[11] Kim, S. G., “BDM Scheduling”, Munwoon-Dang, Korea, 2015
[12] Kim, S. G., “A New Networking Technique-The Beeline Diagramming Method”, International Conference on Mechanical, Electronic, and Information Technology (ICMEIT 2018), Shanghai, China, 511-523, 2018.
[13] Kim, S. G., “Advanced Scheduling Theory”, Goomibook, Korea, 2018
[14] Leach, L., “Schedule and cost buffer sizing; How to account for the bias between project performance and your model”, Project Management Journal, USA, 2003
[15] Peebles, Peyton Z., “Probability, Random Variables and Random Signal Principles”, McGraw-Hill, USA, 2001
[16] Plotnick, F. L., “Relational Diagramming Method”, A Thesis of Ph.D, Drexel University, USA, 2008
[17] Pritsker, A. A. B., “GERT: Graphical Evaluation and Review Technique”, National Aeronautics and Space Administration, USA., 1966
[18] Smith, N.J., “Managing Risk in Construction Project”, Blackwell Science, USA, 1999