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Elementary Trigonometric equations and their solutions with complex numbers

Ram Swaroop Meghwal
Department of Physics
Government College, Kota, Rajasthan, India *

Surender Bharadwaj
Department of Mathematics
Government College, Kota, Rajasthan, India

Abstract- Solutions of sine, cosine , tangent and their squares were presented using complex numbers. The method is developed in such a way that there is no ambiguity in the answers and unique solution is obtained.

Keywords –Elementary TE, solutions of TE, solutions of squared TE

I. INTRODUCTION

Many problems in mathematics and physics involve geometry. A good teacher as a good scholar always puts why and makes the students to resolve it immediately. Keeping this thing in mind it is experienced that when teaching complex numbers or complex variable, we ask the students to ask the students to get familiar with Elementary Trigonometric equations and their solutions with complex numbers.
Complex variable plays major part in evaluating difficult integrals. Similarly complex numbers provide straight and lucid way of solving TE.

The rest of the paper is organized as follows. Proposed embedding and method involved is elucidated in section II. Concluding remarks are given in section III. References are given in section IV.

II. METHOD OF COMPLEX NUMBER TO SOLVE TRIGONOMETRIC EQUATIONS

A. We should assimilate the following geometrical diagram of geometry of trigonometric functions for requirement of the work ahead.

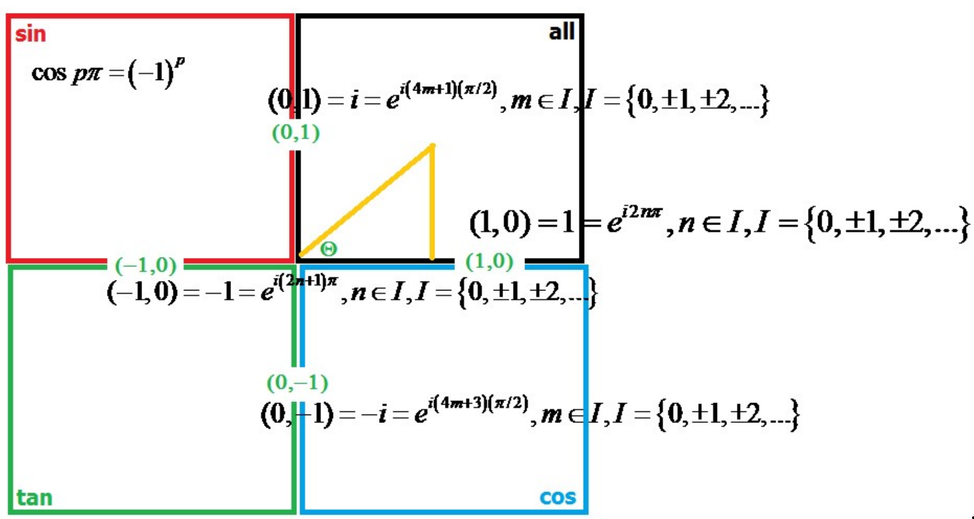

Figure 1. Geometry of Trigonometric Functions

On the basis of such considerations, we should draw the following inferences.

1. Any point on the positive real axis is well described by

$$(1,0)=1=e^{i(4m+0)(\pi/2)}=e^{i2n\pi},m,n\in I,I=\{0,\pm1,\pm2,...\} \quad ...(1)$$

2. Any point on the negative real axis is well described by

$$(-1,0)=-1=e^{i(4m+2)(\pi/2)}=e^{i(2n+1)\pi},m,n\in I,I=\{0,\pm1,\pm2,...\} \quad ...(2)$$

3. Any point on the positive imaginary axis is well described by

$$(0,1)=i=e^{i(4m+1)(\pi/2)},m\in I,I=\{0,\pm1,\pm2,...\} \quad ...(3)$$

4. Any point on the negative imaginary axis is well described by

$$(0,-1)=-i=e^{i(4m+3)(\pi/2)},m\in I,I=\{0,\pm1,\pm2,...\}$$

$$-i=e^{i(4m+3)(\pi/2)}=e^{i(4m+4-1)(\pi/2)}=e^{i(4m-1)(\pi/2)}\cdot e^{i2\pi}=e^{i(4m-1)(\pi/2)}$$

$$(0,-1)=-i=e^{i(4m-1)(\pi/2)},m\in I_0,I_0=\{\pm1,\pm2,...\} \quad ...(4)$$

5. Now we introduce two parameters ‘η’ and ‘λ’, to deal symmetry and signature of the trigonometric ratios in the following manner

$$e^{i\pi p}=\cos p\pi+i\sin p\pi\Leftrightarrow\left(e^{i\pi}\right)^{p}=\cos p\pi+i0\Rightarrow(-1)^{p}=\cos p\pi$$

$$\cos n\pi=(-1)^{n}=\eta=\begin{cases}1, & n-even\\ -1, & n-odd\end{cases}$$

$$\lambda=\pm1,\qquad \sin\lambda\theta=\lambda\sin\theta,\qquad \cos\lambda\theta=\cos\theta \quad ...(5)$$

B. Solutions of equations with vanishing value

1. We should call Euler ‘s formula for the purpose.

$$e^{i\theta}=(x,y)=(\cos\theta,\sin\theta)$$

$$\boxed{\sin\theta=0}\Rightarrow(\cos\theta,\sin\theta)=real\ axis=(\pm1,0)$$

$$=e^{i2n\pi}\ or\ e^{i(2n+1)\pi}=e^{in\pi}\Rightarrow\boxed{\theta=n\pi}\Leftrightarrow\boxed{\tan\theta=0} \quad ...(6)$$

The situation covers the entire real axis.

2. It is the case of cosine function the coverage goes to whole imaginary axis.

$$\boxed{\cos\theta=0}\Rightarrow(\cos\theta,\sin\theta)=imaginary\ axis=(0,\pm1)$$

$$=e^{i(4m+1)(\pi/2)}\ or\ e^{i(4m-1)(\pi/2)}\Rightarrow\boxed{\theta=(4m\pm1)(\pi/2)} \quad ...(7)$$

C. Solutions of equations with value equal to unity

$$\boxed{\sin\theta=1}\Rightarrow(\cos\theta,\sin\theta)=positive\,im.axis=(0,1)$$
$$=e^{i(4n+1)\pi/2}\Rightarrow\boxed{\theta=(4n+1)\pi/2}$$
$$\boxed{\cos\theta=1}\Rightarrow(\cos\theta,\sin\theta)=positive\,real\,axis=(1,0)$$
$$=e^{i(2n)\pi}\Rightarrow\boxed{\theta=2n\pi} \quad \ldots(8)$$

It is evident that negative parts of the axes are left untouched for positive values.

D. Solutions of equations with squared trigonometric functions

$$\boxed{\sin^2\theta=\sin^2\alpha}=\sin^2(\pm\alpha),\quad \boxed{\cos^2\theta=\cos^2\alpha},\quad \boxed{\tan^2\theta=\tan^2\alpha}$$
$$\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^2=\left(\frac{e^{i\alpha}-e^{-i\alpha}}{2i}\right)^2$$
$$\Rightarrow e^{i2\theta}+e^{-i2\theta}-2=e^{i2\alpha}+e^{-i2\alpha}-2\Rightarrow e^{i(2\theta)}=e^{i(2\alpha)}$$
$$\Rightarrow e^{i(2\theta)}=e^{i(2\alpha)}\cdot e^{i(2n\pi)}\Rightarrow\boxed{\theta=n\pi\pm\alpha} \quad \ldots(9)$$

It reveals that squared functions have the solutions at integral values of pi deviated at both sides by the principal value.

E. Solutions of equations with arbitrary values not related to axes

$$\boxed{\theta=n\pi+\lambda\alpha}\Rightarrow e^{i\theta}=e^{in\pi}\cdot e^{i\lambda\alpha}=(-1)^n(\cos\lambda\alpha,\sin\lambda\alpha)$$
$$=\eta(\cos\alpha,\lambda\sin\alpha)\Rightarrow\boxed{\cos\theta=\eta\cos\alpha},\ \boxed{\sin\theta=\lambda\eta\sin\alpha},\ \boxed{\tan\theta=\lambda\tan\alpha}$$
$$\lambda=1\Rightarrow\boxed{\tan\theta=\tan\alpha}\Rightarrow\boxed{\theta=n\pi+\alpha}$$
$$\eta=1\Rightarrow n=2m,\ \theta=2m\pi+\lambda\alpha\Rightarrow\boxed{\cos\theta=\cos\alpha}\Rightarrow\boxed{\theta=2m\pi\pm\alpha}$$
$$\lambda=\eta\Rightarrow\lambda\eta=1\Rightarrow\boxed{\sin\theta=\sin\alpha}\Rightarrow\boxed{\theta=n\pi+(-1)^n\alpha}$$

…(10)

Here the symmetry parameter appears in the solution in sine and signature in cosine.

III.CONCLUSION

In this method we see that trigonometric equations are very easily solved by the application of complex numbers with much devoting time to the underlying geometry of the problems. It is due to the fact that main object of geometry is to work with two quantities having no projections on each other. The similar thing goes with real and imaginary numbers as any of one has no projection on the other. Of course this the geometrical property of complex numbers. The separate- ability or non intermingle character is the main feature of coordinates. Due to it Murkowski space is one time described as real plane and other time as complex plane.

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* Here the refrences are just to match the results appreared in the paper , the entire methodology is newly developed by the corresponder.