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Finite Element Analysis of Vibration in Rectangular Beam

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Abstract- Free and forced vibration analyses in simply supported and cantilever beams of rectangular cross section have been achieved numerically in this paper. Equations of vibration for free and forced analyses are solved numerically using Ansys Workbench. In forced vibration, the used external disturbance is vertical harmonic force of 700 N amplitude applied at beam mid-span in simply supported beam, and at free end in cantilever beam.

The obtained results from this analysis show that the maximum values of vibrational displacement, velocity, and acceleration for both simply supported and cantilever beams are occur at natural frequencies that produce vertical mode shape. While the natural frequencies that produce lateral and torsional vibration have no effect on the resulting vibration. Also, maximum generated stress under this harmonic load occurs when the forced frequency reach a value equal to the natural frequency of maximum mode shape in vertical direction. This stress must be avoided to prevent beam failure.

Keywords – Finite Element, Beam Vibration, Vibration analysis, Rectangular Beam

I. INTRODUCTION

Vibration can be defined as a system oscillation about some equilibrium position. Vibration is started when the system moved from its location of equilibrium by an external source such that the system attains energy and store it. Then the system will tend to return to its equilibrium position by action of a developed conservative force [1].

Beams vibration in mechanical components should be studied and examined carefully to avoid harmful vibration during operation. There are many researches deal with beam vibration since a longtime decades.

In 1976, Maurizi et al [2] studied a free vibration in horizontal beam constrained by a linear vertical spring at one end and hinged by a rotational spring at the other end. The fundamental frequencies in the beam were evaluated and listed for several values of ($K_L L^3/EI$) and ($K_r L/EI$), where K_L and K_r are linear and rotational stiffness constants of springs, respectively. L is beam length, EI represents flexural rigidity of the beam.

In 1985, Eisenberger et al [3] derived the matrices of consistent mass and exact stiffness in beams supported by elastic foundation in order to evaluate the natural frequencies and mode shapes in these beams with fully support or partial support (elastic foundation).

In 1990, Wu and Lin [4] introduced a combined (analytical and numerical) method to find natural frequencies and mode shapes of vibration in a uniform-fixed free beam having a number of concentrated masses. Equations of vibration derived analytically using the theorem of expansion, then natural frequencies and mode shapes computed using numerical method. The advantages of this method are cutting out the matrices of property for each element and replace them by overall property matrix, better accuracy, and more saving time for computer.

In 1997, Murphy [5] adopted a numerical solution for the equation of fundamental frequency in simply supported and over-hanged beams subjected to transverse vibration. he proved that numerical solutions converge to analytical results in case of simply supported beam, and converge to analytical results of over-hanged beam when the supports chosen at free vibration nodes.

In 2001, Liu and Wu [6] concentrated their study on free vibration of stepped Euler and multi-span beams, and beams having concentrated mass at end or intermediate of beam length. Conditions of compatibility and the related differential quadrature equations had been formulated in explicit scheme. The results showed that equal subdomain grids more accurate than unequal one.the results were obtained using special method called parameter expanding method.

In 2004, Kocaturk and Shimshek [7] used the beam theory of Bernoulli-Euler to study transverse vibration in simply supported beam of multiple supports, subjected to harmonic load at intermediate point. Dynamic response has been obtained using Lagrange equations to simplify the solution to a number of algebraic equations. These equations have been solved using Newmark's method of direct time integral.

In 2011, Sedighi et al [8] presented a new analytical method to study the dynamic behavior of a cantilever beam subjected to nonlinear boundary conditions with existence of preload spring. The results had been obtained using special method called parameter expanding method.

In 2013, Chopade and Barijibhi [9] focused their study on transverse vibration analysis in cantilever beam to evaluate the natural frequencies and mode shapes. The solution is obtained theoretically for the first five modes in the beam. The results compared with numerical solution of Ansys program for the first five modes to verify the validity of results.

In 2015, Akbash [10] studied static bending and free vibration in functionally graded beam mounting on elastic support using both Timoshenko and Bernoulli beam theories. The beam material has a uniform distribution of properties according to a special power low. Minimum potential energy and Navier methods had been used to derive the governing equations. The accuracy of results obtained from this analysis has been proved.

In 2015, Romaszko et al [11] studied the forced vibration in homogeneous fixed-free beam using vision method. Vibration of a number of defined points in the beam in longitudinal and transverse directions had been achieved theoretically and verified experimentally using a special vision system.

In 2016, Khan et al [12] presented a study of free vibration and static analysis in functionally graded beam using 1-D finite element. The model of the finite element is based on the theory of efficient zig-zag. The beam is considered as one element of two nodes. Each node has four degrees of freedom. For deflection, cubic hermit interpolation had been used. While for axial displacement, the linear interpolation had been used. Results of natural frequencies, mode shapes, shear stress, normal stress, and deflection were obtained and reported.

In 2019, Syed and Bishay [13] adopted a new design procedure based on finite element to straighten out the required attenuation frequency for structures of geometrically periodic beam. Both forward approach and reverse approach had been used in the analysis for start and end frequencies. The adopted method helps to specify the cell geometric parameters that give the required vibration results.

In 2020, Nguyen et al [14] adopted a method to detect the damage inside beams using free vibration analysis. They used natural frequency in free-free beam to monitor damage. 15 accelerometers were used to identify modal properties. A numerical solutions based on 1D, 2D, and 3D finite element were used to find the first five natural frequencies of bending, and the obtained results compared with experimental ones. The results verified that use of 2D and 3D finite element models give the same results with small error. While 1D models give very large error.

In 2021, Babaei and Arabghahestani [15] carried out a study on a transverse vibration in micro-beam subjected to simply supported boundary conditions using theory of modified couple stress such that the beam is modeled using theories of Timoshenko and Euler Bernoulli. Hamilton's approach was adopted to derive dynamic equations of vibration. The equations obtained by Timoshenko and Euler Bernoulli theory were solved using Galerkin's method. Also, Navier's method used for shear deformation effect. The obtained results showed how the slenderness ratio, angular, lateral, and axial velocity factors effects on the resulted fundamental frequency.

In 2021, Atiyah and Abdulsahib [16] studied the free vibration analysis in beams of functionally graded material with different boundary conditions. The eigenvalues equations are derived for all used boundary conditions. The effects of changing beam height, length, density, and Young's modulus on the resulting natural frequencies are studied and listed.

II. MATERIALS AND METHODS

A. *Free and forced vibration –*

For uniform beam, the general equation of motion (in free and forced vibration cases) can be written as [17]:

$$EI\frac{\partial^4 w}{\partial x^4}(x,t) + \rho A\frac{\partial^2 w}{\partial t^2}(x,t) = f(x,t) \qquad (1)$$

Where E is the elasticity modulus, I is area moment of inertia for beam cross section, ρ and A are beam density and cross section area respectively, x represents distance from one beam ends, t is the time, w represents beam deflection, and f(x,t) is the external disturbance force. f(x,t)=0 for free vibration, in this case the solution of Eq. (1) gives eigenvalues and eigenvectors of the beam.

In forced vibration case, Eq. (1) can be solved by assuming:

$$w(x,t) = \sum_{m-1}^{\infty} w_m(x)\, q_m(t) \qquad (2)$$

Where w_m (x) represents the mth normal mode and q_m (t) is the m^{th} mode generalized coordinate.

Eq. (1) can be solved using FEM to obtain vibration response (displacement, velocity, and acceleration of vibration) after applying the boundary conditions. For simply supported beam, the boundary conditions at (x=0 and x=L) are as the following:

$$w = 0 \quad \text{and} \quad Bending\ moment\ (M) = EI\frac{\partial^2 w}{\partial x^2} = 0 \tag{3}$$

While for cantilever beam, the boundary conditions are:

$$\text{Deflection } (w) = 0 \quad \text{and} \quad \text{slope} = \partial w/\partial x = 0 \quad \text{at } x = 0 \tag{4}$$

$$Bending\ moment = EI\frac{\partial^2 w}{\partial x^2} = 0 \quad \text{and} \quad shear\ force = EI\frac{\partial^3 w}{\partial x^3} = 0 \quad \text{at } x = L \tag{5}$$

In this manner, vibration response in the beam can be obtained.

Modal analysis system in Ansys Workbench is used to evaluate eigenvalues and eigenvectors in the beam (free vibration analysis). Achieving of this analysis is very important in order to know the natural frequencies in the beam for any number of modes to avoid these frequencies in forced vibration (avoid resonant cases). Since the beam is a rigid body, there are infinity number of vibration modes. But only six modes will be taken in the present work.

For forced vibration analysis, harmonic response method in Ansys Workbench will be used. The values of forced frequency can be chosen as required. The disturbance force is applied in vertical direction at beam end for cantilever beam, and at the beam mid-span for simply supported beam with 700 N amplitude.

B. Finite Element Analysis –

One of the most effective and useful method for vibration investigation in various mechanical components is the programmed finite element analysis (FEA) which can simulate the free and forced vibration easily and gives the vibration results and characteristics quickly. It has an ability to solve complex and nonlinear problems efficiently using a wide number of software programs in computer. In this paper, the analysis will be achieved using Ansys Workbench.

FEM can be applied to 1D, 2D, and 3D problems. For 2D and 3D problems, the domain is divided into a number of small areas or volumes, these called finite elements [18].

Main steps of finite element solution procedure can be listed as follows:

1. Discretize the solution region into a suitable number of finite elements. This step is performed in a preprocessor program. The mesh description includes many arrays, main of which are elements connectivity and nodal coordinates.
2. Choice the interpolation functions which are used to satisfy the variables of field through the element. Usually, the interpolation functions chosen as polynomials. The degree of polynomial is depend on the node number related to the element.
3. Evaluate the properties of element. The finite element matrix equation must be evaluated which relates the unknown function in nodal values to all parameters. To achieve this function, there are many approaches such as Galerkin method and variational approach.
4. Find the global finite element matrix for whole system by make an assembly to the elements equations using elements connectivity. Knowing that the boundary conditions must be applied before the solution since these conditions don't considered in elements equations.

III. Studied Cases and Results

A. Studied Cases –

There are two studied cases in this paper. The first case is simply supported rectangular beam subjected to harmonic vertical force at beam mid-span. The second case is fixed –free (cantilever) rectangular beam subjected to harmonic vertical force at free end. The cross section of the used beam specimens are 15 mm height, 30 mm width and 250 mm long.

Figure (1) shows the beam specimen meshed in Ansys workbench with 3mm for element size. The total resulted elements number is 4200 and 20929 nodes. The used material is structural steel of 7850 Kg density, 250 Mpa yield strength, 460 Mpa tensile ultimate tensile strength, 0.3 poison's ratio, and 200 000 Mpa elasticity modulus.

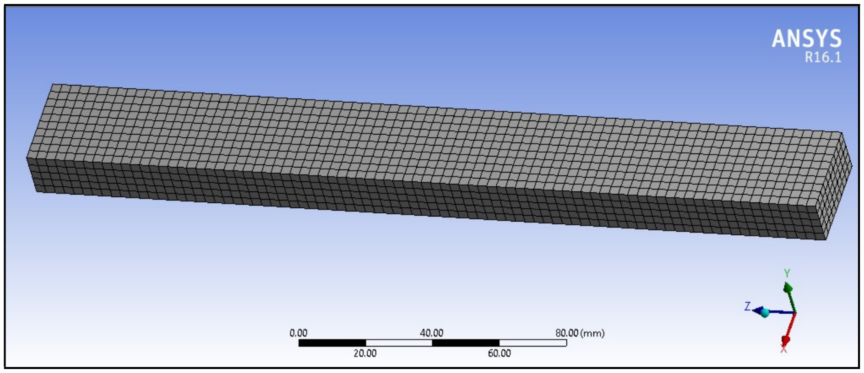
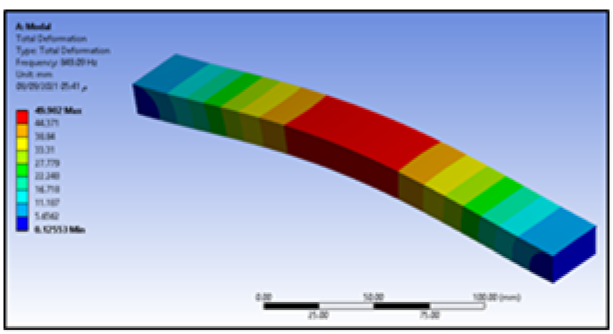


Figure 1. Rectangular beam meshed in Ansys workbench.

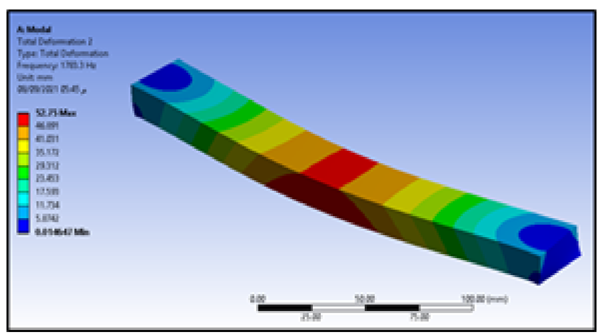
B. Results of Free Vibration –

Results of free vibration (modal analysis) in simply supported rectangular beam are shown in figure (2) for the first six modes. Figure (2a) shows the first mode with maximum mode shape at beam mid-span. This mode has one wave only in vertical direction. The second mode has one wave with maximum mode shape at beam mid-span but the wave is in lateral (horizontal) direction as shown in figure (2b). The third mode has two vibration waves in vertical direction with one vibration node at mid-span as shown in figure (2c). Fourth mode represents the torsion effect with one wave only (see figure (2d)). Fifth mode (figure (2e)) represents two waves mode shape in horizontal direction with a vibration node at beam mid-span. Finally, sixth mode appear as three vibration waves in vertical direction with two vibrational nodes at (L/3) and (2L/3) as shown in figure (2f). Table (1) shows the natural frequencies for the first sixth modes in simply supported beam.

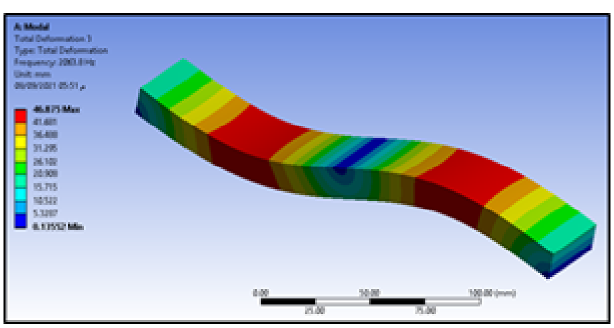
It is obvious from previous figures that first, third and sixth mode shapes and natural frequencies are related to vibration in vertical direction. While the fourth mode is related to torsional vibration, and the remaining modes (second and fifth modes) are related to lateral vibration.



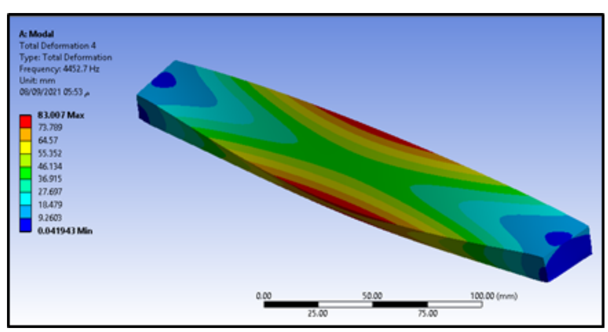
(a) First mode



(b) Second mode



(c) Third mode



(d) Fourth mode

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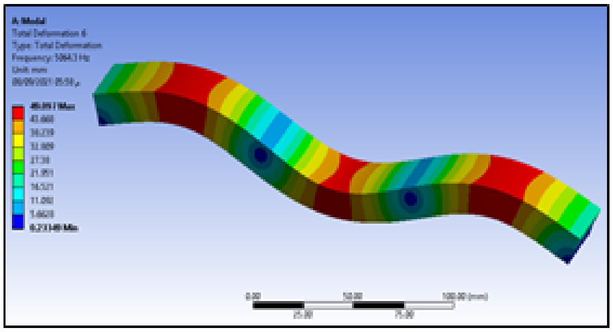
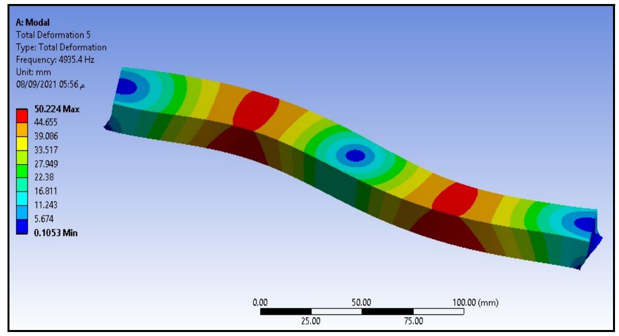


(e) Fifth mode



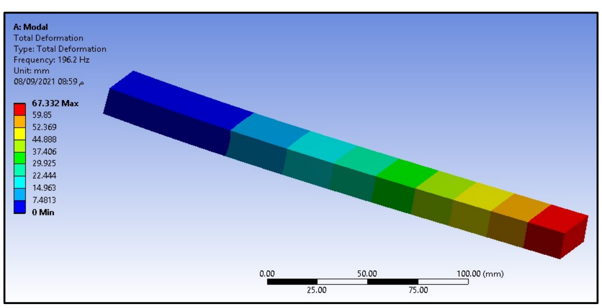
(f) Sixth mode

Figure 2. Modal analysis of simply supported rectangular beam (first sixth modes).

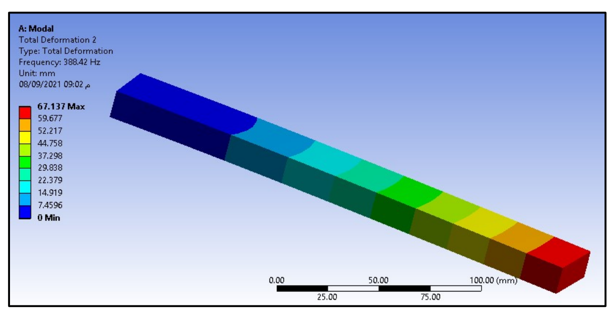
Table -1 Natural frequencies in simply supported beam (First six modes).

Mode	Natural Frequency, Hz
1	849.09
2	1783.3
3	2063.8
4	4452.7
5	4935.4
6	5064.3

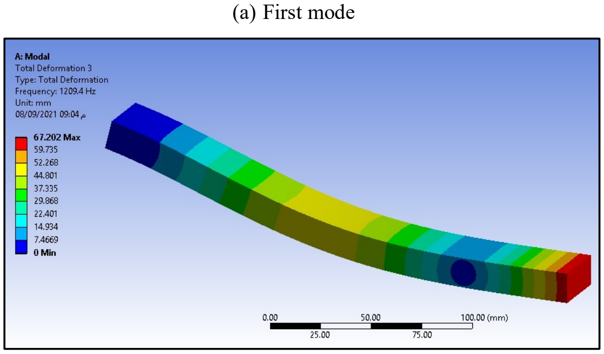
Figure (3) shows the results for the first six modes of free vibration in the same beam when the supports are fixed-free instead of simply supported. The resulting modes are as follows: the 1st mode represents one wave vibration in vertical direction, 2nd mode is one wave lateral vibration, 3rd mode represents two waves vertical vibration with vibration node locates near the free end, 4th mode is two waves lateral vibration with one vibration node near the free end, 5th mode is for torsional vibration, and finally the 6th mode is vertical vibration of three waves with two vibration nodes. Table (2) shows the natural frequencies for the first sixth modes in cantilever beam.



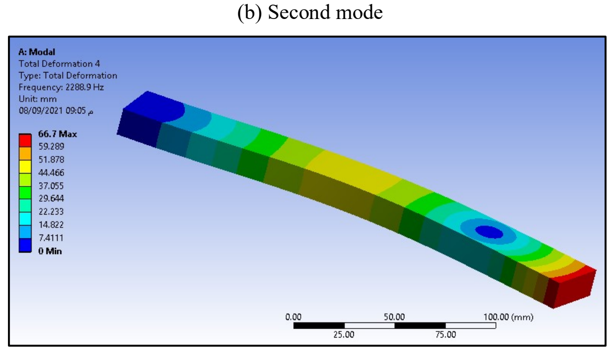
(a) First mode



(b) Second mode



(c) Third mode



(d) Fourth mode

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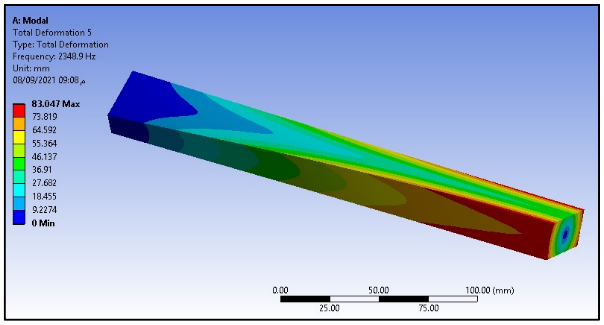
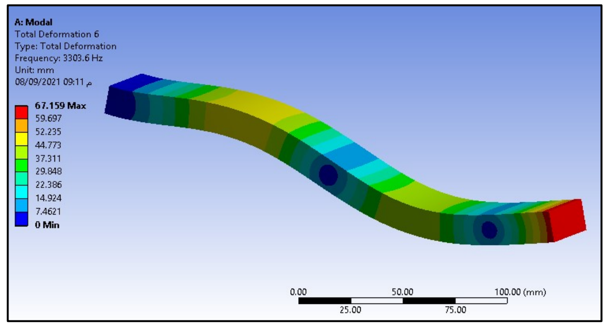


(e) Fifth mode



(f) Sixth mode

Figure 3. Modal analysis of cantilever beam (first sixth modes).

Table -2 Natural frequencies in cantilever beam (First six modes).

Mode	Natural Frequency, Hz
1	**196.2**
2	**388.42**
3	**1209.4**
4	**2288.9**
5	**2348.9**
6	**3303.6**

C. Results of forced vibration –

Figure (4) shows the results of forced vibration in simply supported rectangular beam when the external excitation is vertical harmonic force of 700 sin(ωt) Newton applied at beam mid-span such that the forced frequency ω is changed up to 6000 Hz. From this figure, it is obvious that the curve of vertical displacement amplitudes (y_max) reaches the maximum value at first natural frequency (849.09 Hz) as a result of resonance phenomena in beam with the vertical harmonic load, while at the sixth natural frequency (5064.3 Hz) the vertical displacement amplitude doesn't reach maximum value because of existence two vibrational nodes at (L/3) and (2L/3). It is noted that the resonance doesn't occurs in the third natural frequency (2063.8) although it is related to vertical mode because the external harmonic force is applied exactly at the same location of vibration node (which represents the location of zero vibration amplitude). The remaining natural frequencies don't affect the vibration amplitude because they are related to horizontal and torsional vibration. Results of vibration velocity ($\dot{y}_{max}$) and acceleration ($\ddot{y}_{max}$) in vertical direction are shown in Figures (5) and (6). The results of vibration velocity is similar to displacement results for the same reasons mentioned above, but vibration acceleration has a maximum value at sixth natural frequency.

Changings of vibration displacement, velocity, and acceleration in rectangular cantilever beam subjected to vertical harmonic force at free end are shown in Figures (7), (8) and (9), respectively. The amplitude of the applied force is 700N. The obtained results show that the peak values of vibration displacement, velocity, and acceleration are occur at frequencies of 196.2 Hz for displacement and velocity, and 3303.6 Hz for acceleration because these frequencies represent the resonant frequencies in vertical direction (direction of applied forced vibration).

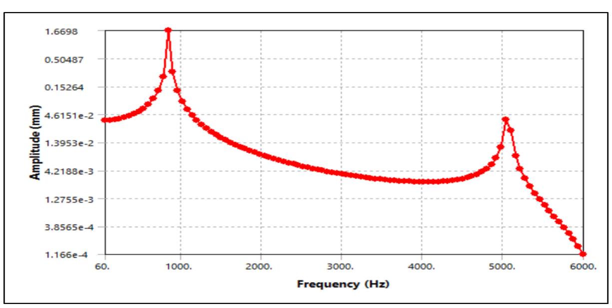


Figure 4. Vertical displacement of vibration versus forced frequency in simply supported rectangular beam.

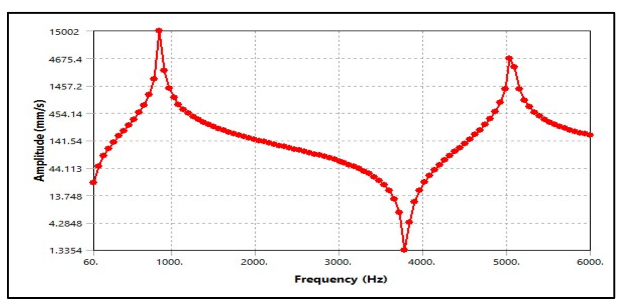


Figure 5. Vertical vibration velocity versus forced frequency in simply supported rectangular beam.

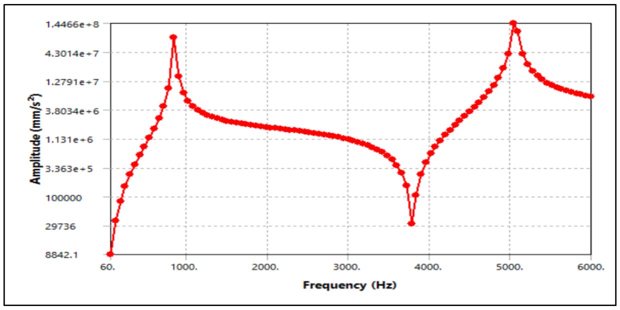


Figure 6. Vertical vibration acceleration versus forced frequency in simply supported rectangular beam.

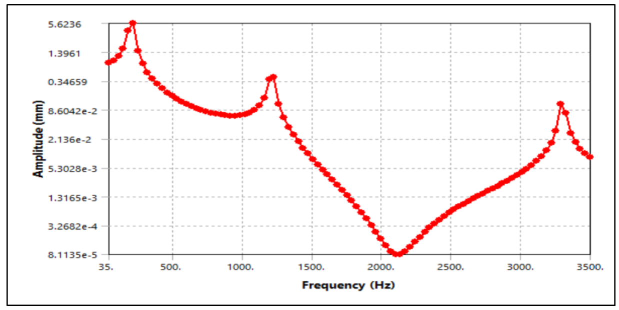


Figure 7. Vertical displacement of vibration versus forced frequency in cantilever rectangular beam.

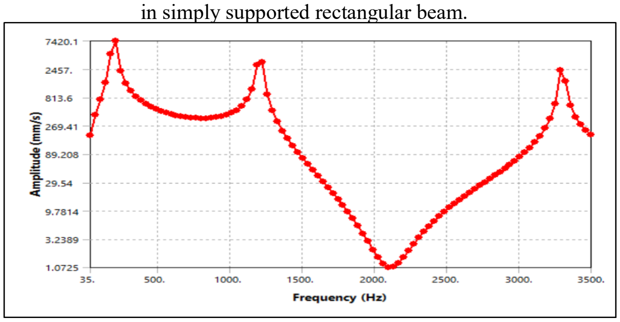


Figure 8. Vertical vibration velocity versus forced frequency in cantilever rectangular beam.

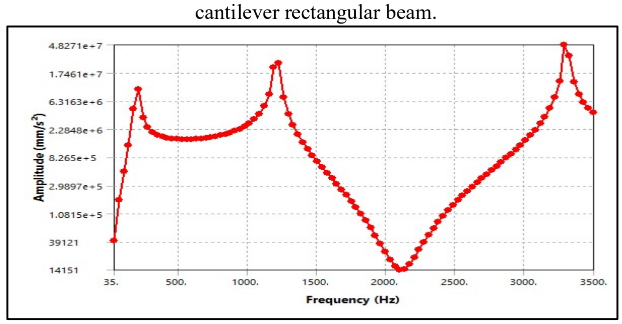


Figure 9. Vertical vibration acceleration versus forced frequency in cantilever rectangular beam.

Finally, figures (10) and (11) shows the equivalent von-Mises stresses in simply supported beam and cantilever beam respectively, at their natural frequencies of maximum displacement (849.09 Hz for simply supported beam and 196.2 Hz for cantilever beam) ,which give maximum stress results, under the same harmonic load. These stresses are very important because they are must not to be exceed the permissible stresses in the beam to avoid failure.

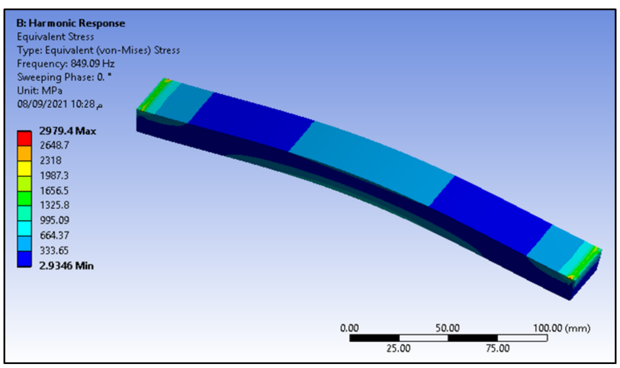


Figure 10. Von-Mises stress results in simply supported rectangular beam under forced frequency of 849.09 Hz.

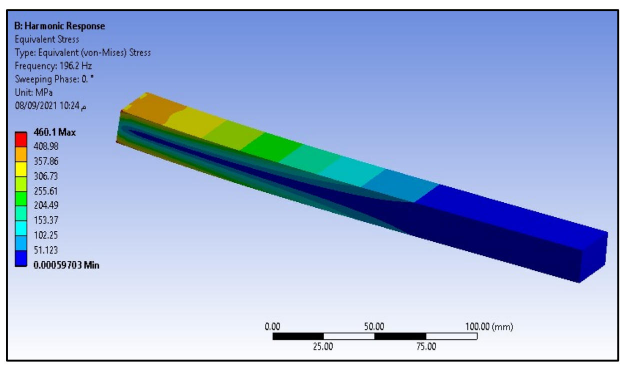


Figure 11. Von-Mises stress results in cantilever beam under forced frequency of 196.2 Hz.

IV.CONCLUSION

From the present work, the following conclusions can be summarized:

1. It is necessary to study free vibration (modal analysis) in any case study includes forced vibration to know the natural frequencies of the system.
2. The frequencies that must be avoided in forced vibration are the natural frequencies that cause a vibration parallel to vibration of external disturbance. For example, if the external harmonic force is vertical then the natural frequencies cause a vertical displacement must be avoided. Other frequencies are not affect.
3. To eliminate the vibration effect of any external disturbance (in forced vibration) at any natural frequency, the location of applied disturbance force must be chosen at the vibration node (zero vibration location).
4. Under forced harmonic vibration in vertical direction, both simply supported and cantilever beams have a maximum vertical displacement, velocity, and acceleration of vibration at natural frequencies that produce vertical mode shape.

5. Natural frequency of maximum vibration displacement gives maximum generated stress in the beam under harmonic force. This stress is very important because it must not to be exceed the permissible stress in the beam to avoid failure.

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